# Detection and utility of quantum work and heat statistics

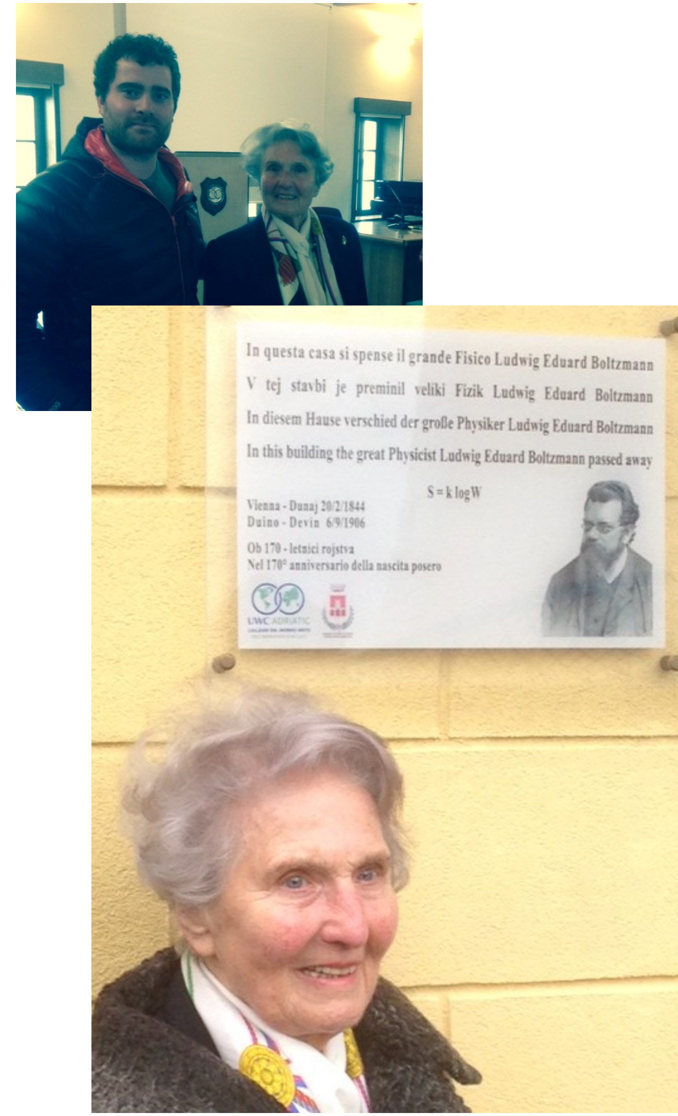
Grenoble, 30 September 2014

John Goold (ICTP, Trieste)

The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA International Atomic Energy Agency




Available energy is the main object at stake in the struggle for existence and the evolution of the world.

Ludwig Boltzmann

## Far from equilibrium statistical mechanics

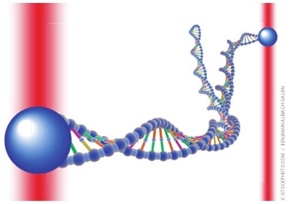
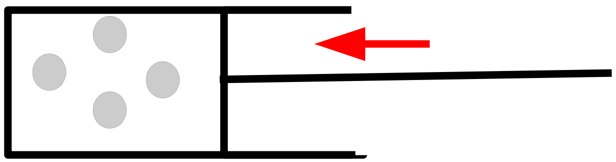
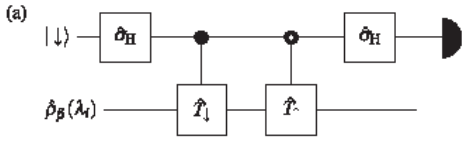
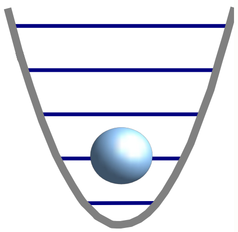


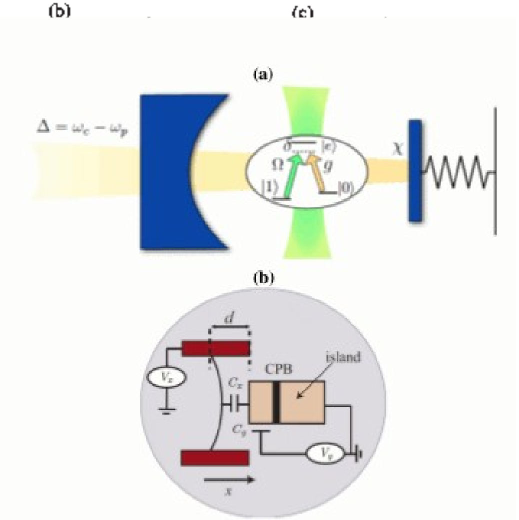
Figure 1 | DNA at a stretch. Measurements of the fluctuating extension of molecules as force is applied by optical tweezers can be used to reconstruct equilibrium free-energy profiles.



## Explorations in Quantum Systems





# Introduction

- Defining Work and Heat

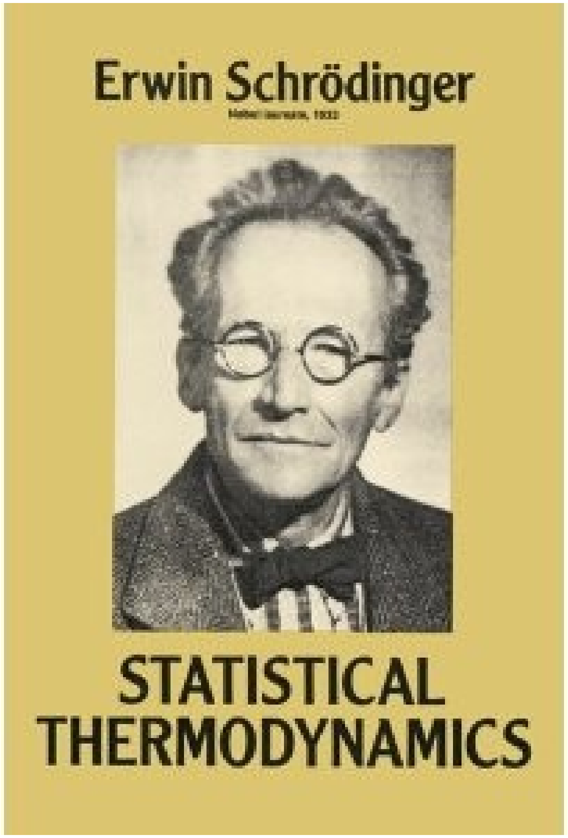
- Motivations to study the energy information conversion in quantum systems

- Distribution of dissipated heat in a generic protocol

- Experimental proposal to measure heat in a quantum system

- A curious bound on the heat

# Quantum Work and Heat – definitions under 'quasi-static' transformation




$$dU = dQ + dW$$

$$dU = \sum_n \underbrace{E_n dP_n}_{\text{heat}} + \sum_n \underbrace{P_n dE_n}_{\text{work}}$$

heat

work

**The quantum open system as a model of the heat engine**

*R Alicki 1979 J. Phys. A: Math. Gen. 12 L103*

$$U = \mathrm{tr}[\mathcal{H}\rho]$$

$$\frac{dU}{dt} = \mathrm{tr}\left[\mathcal{H}\frac{d\rho}{dt}\right] + \mathrm{tr}\left[\rho\frac{d\mathcal{H}}{dt}\right]$$

$$\Delta U = \int_{t_i}^{t_f} dt \left\{\mathrm{tr}\left[\mathcal{H}\frac{d\rho}{dt}\right] + \mathrm{tr}\left[\rho\frac{d\mathcal{H}}{dt}\right]\right\}$$

Heat

Work

# Closed processes

# or

# Unitary processes

# All work and no heat

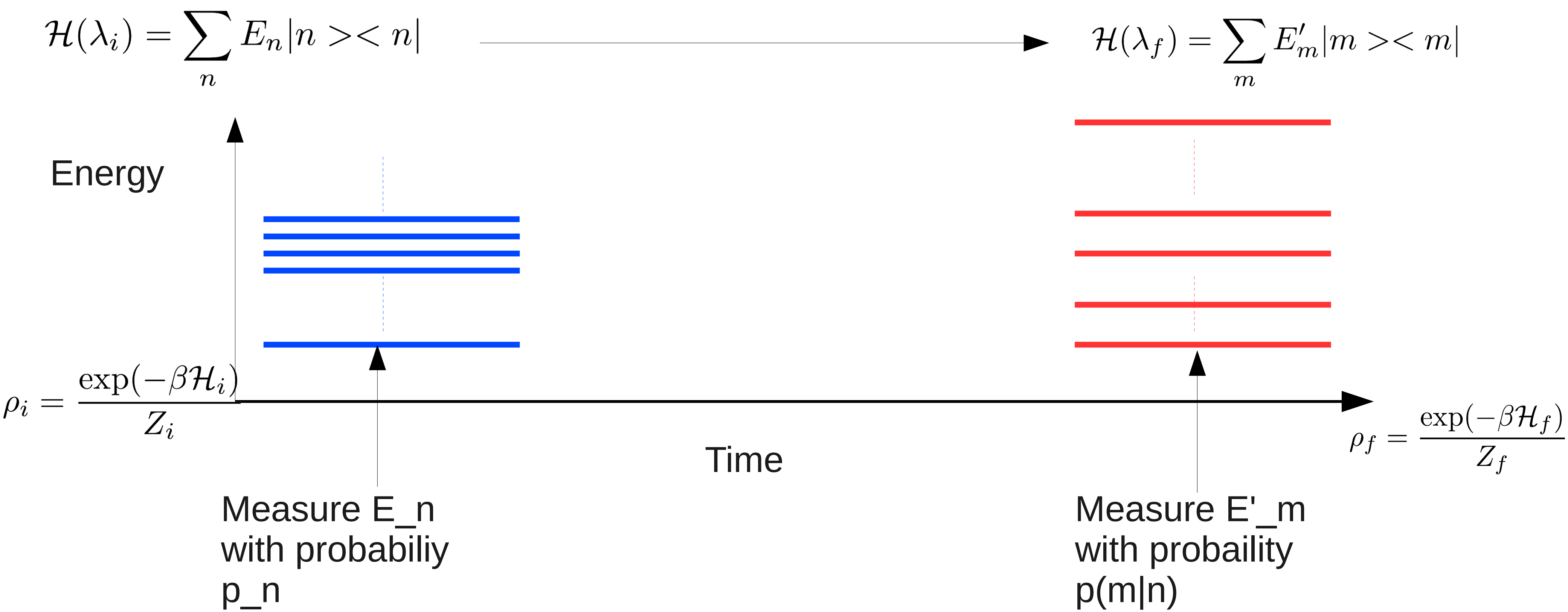
makes it a unitary process

$$\langle Q \rangle = \int_{t_i}^{t_f} \mathrm{tr}[\mathcal{H}[\rho, \mathcal{H}]] dt$$

$$\boxed{\Delta U = \langle W \rangle}$$

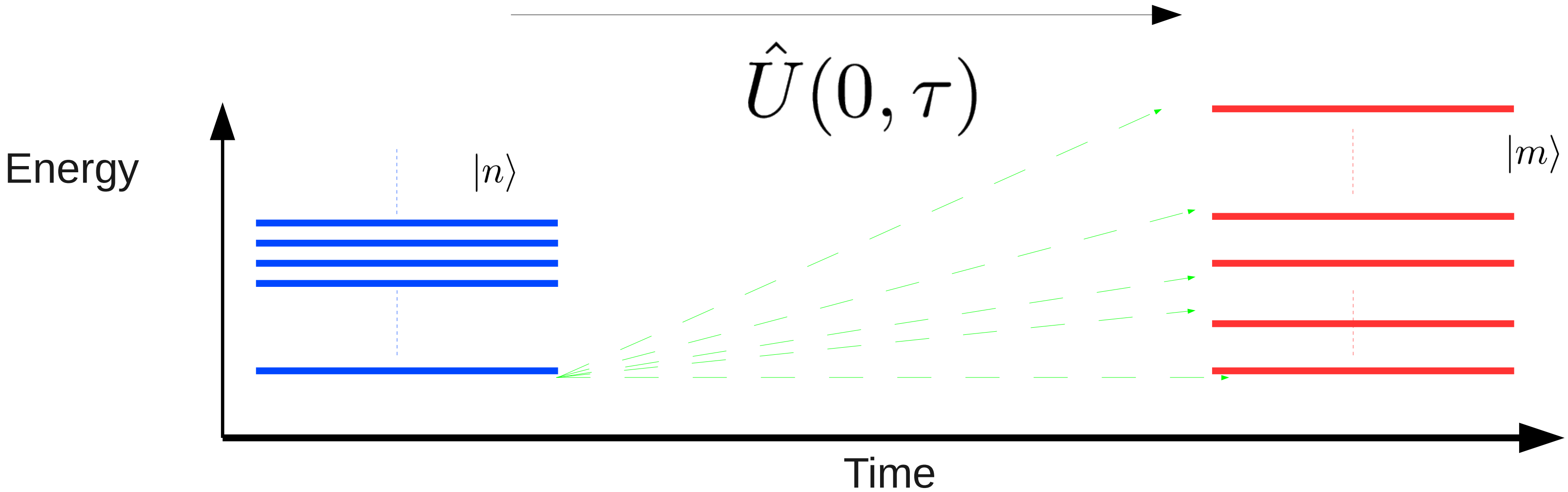
# Quantum Systems : thermodynamics in finite time

$$\hat{H}(\lambda(t)) \qquad t \in [0, \tau]$$



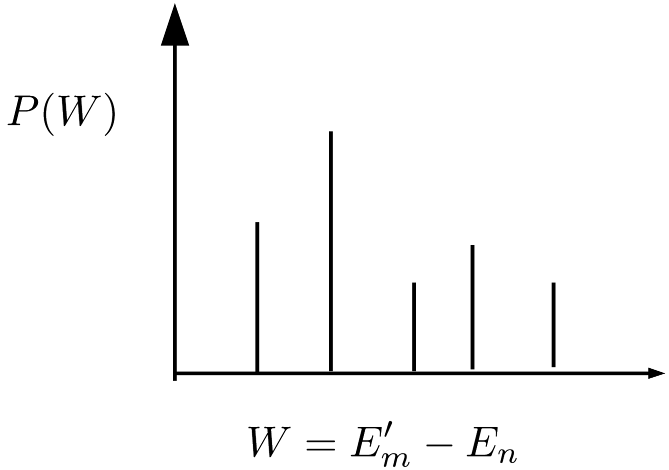

**1. Prepare →2. Measure →3. Evolve →4. Measure – 5. Build statistics**

# Work Distribution for closed quantum systems (Unitary)




Work Distribution :

$$P_F(W) := \sum_{n,m} p_n^0 \, p_{m|n}^\tau \delta\left[W - (E'_m - E_n)\right],$$




$$p_n^0 p_{m|n}^\tau = \frac{e^{-\beta E_n(\lambda_0)}}{Z} |\langle n|U(\tau,0)|m\rangle|^2$$

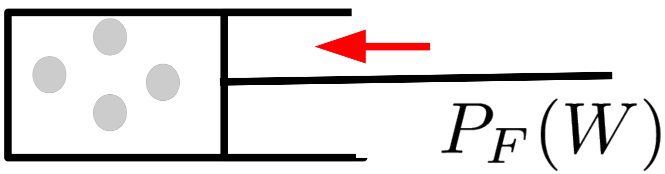
Heights of delta peaks

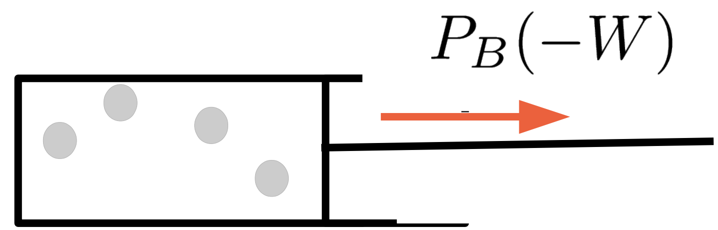
# Fluctuation Relations

Remarkable relations exist between the fluctuations in the work done in a non-equilibrium transformation and the equilibrium properties of the system.

$$\frac{P_F(W)}{P_B(-W)} = \exp(\beta(W - \Delta F))$$

Tasaki-Crooks Relation




$$\Delta F = -\beta^{-1} \ln \frac{\mathcal{Z}(t_f)}{\mathcal{Z}(t_i)}$$

$$\langle \exp(-\beta W) \rangle = \exp(-\beta \Delta F)$$

Jarzynski equality

Campisi et al, Rev. Mod. Phys. 83, 771 (2011)

M. Esposito et al, Rev. Mod. Phys. 81, 1665 (2009).

# Emergent Irreversibility

Irreversible entropy change $$\langle \Sigma \rangle = \beta(\underline{\langle W \rangle - \Delta F})$$

$$\langle W \rangle_{diss}$$

**Crooks Relation**

$$\frac{P_F(W)}{P_B(-W)} = \exp(\beta(W - \Delta F)) \longrightarrow \beta(\langle W \rangle - \Delta F) = \int P_F(W) \log \frac{P_F(W)}{P_B(-W)} dW$$

$$\langle \Sigma \rangle = \beta \langle W \rangle_{diss} = K(P_F(W) || P_B(-W))$$

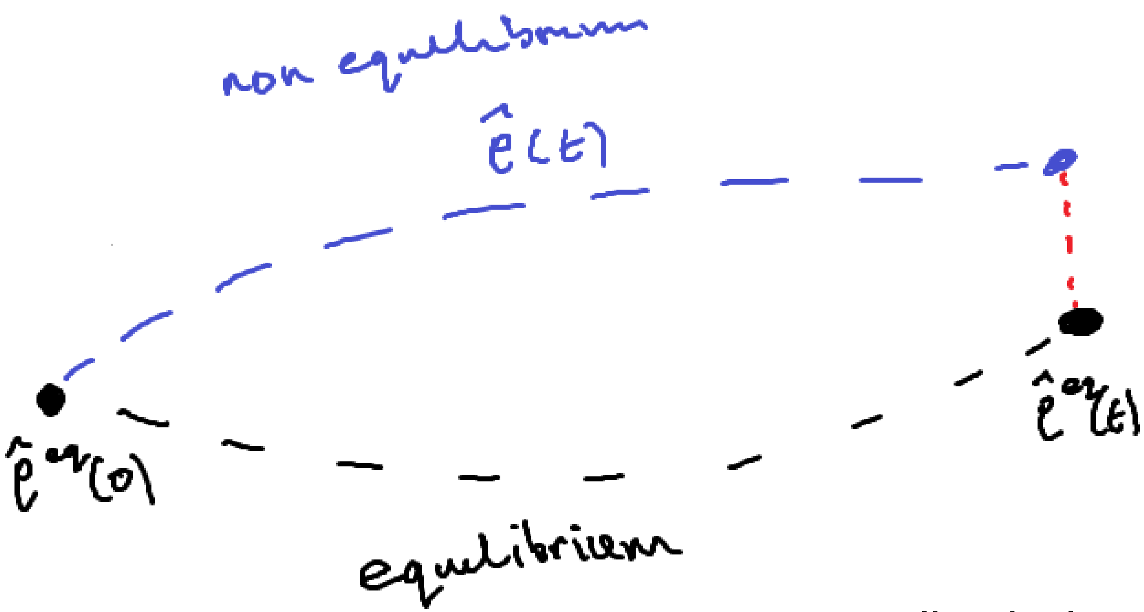
**Relative Entropy** between forward and backward distributions

Second law $$\langle \Sigma \rangle \geq 0$$ by Klien's inequality

# Irreversibility

$$\langle\Sigma\rangle = \beta\langle W\rangle_{diss} = K(P_F(W)||P_B(-W))$$

$$= tr[\rho(t) ln \rho(t) - \rho(t) ln \rho^{eq}(t)] = S(\rho(t)||\rho^{eq}(t))$$




Use mathematical properties of relative entropy to look at bounds on entropy production:

$$\langle\Sigma\rangle \geq \frac{8}{\pi^2}\mathcal{L}(\rho(t), \rho^{eq}(t))$$

Generalised Claussius Inequality

S. Deffner and E. Lutz, Phys. Rev. Lett. 105, 170402 (2010)

# Probe the characteristic function of work

$$P_{\mathrm{F}}(W) := \sum_{n,m} p_n^0 \, p_{m|n}^\tau \delta\left[W - (E'_m - E_n)\right],$$

↓

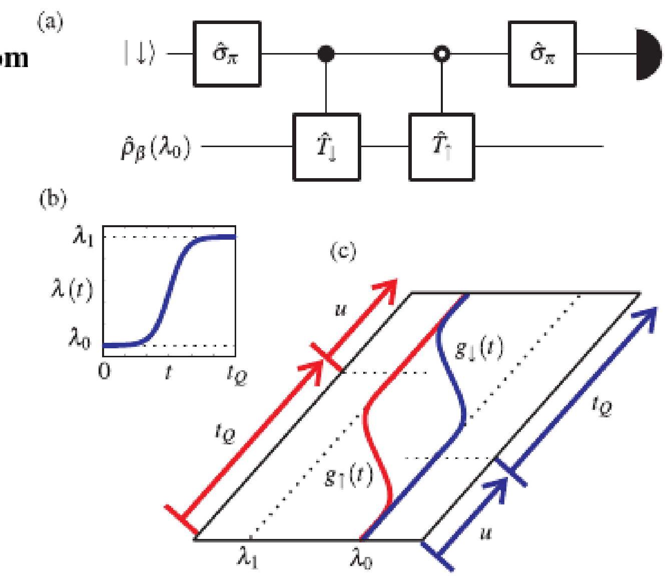
$$\chi_F(u) = \int dW e^{iuW} P_F(W) = \mathrm{tr}[\hat{U}^\dagger(\tau,0) e^{iu\hat{H}(\lambda_1)} \hat{U}(\tau,0) e^{-iu\hat{H}(\lambda_0)} \hat{\rho}_\beta(\lambda_0)]$$

Phys. Rev. E. 75, 050102, (2007)
Talkner, Lutz, Hanggi

# The papers

PRL **110,** 230601 (2013) PHYSICAL REVIEW LETTERS week ending 7 JUNE 2013

## Extracting Quantum Work Statistics and Fluctuation Theorems by Single-Qubit Interferom




R. Dorner,[1,2,*] S. R. Clark,[2,3] L. Heaney,[3] R. Fazio,[3,4] J. Goold,[2,5] and V. Vedral[2,3]

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[2] *Clarendon Laboratory, University of Oxford, Parks Road, Oxford OX1 3PU, United Kingdom*
[3] *Centre for Quantum Technologies, National University of Singapore, 3 Science Drive 2, Singapore 117543*
[4] *NEST, Scuola Normale Superiore and Istituto Nanoscienze-CNR, I-56126 Pisa, Italy*
[5] *Department of Physics, University College Cork, Cork, Ireland*

(Received 11 February 2013; revised manuscript received 29 April 2013; published 7 June 2013)

We propose an experimental scheme to verify the quantum nonequilibrium fluctuation relations using current technology. Specifically, we show that the characteristic function of the work distribution for a nonequilibrium quench of a general quantum system can be extracted by Ramsey interferometry of a single probe qubit. Our scheme paves the way for the full characterization of nonequilibrium processes in a variety of quantum systems, ranging from single particles to many-body atomic systems and spin chains. We demonstrate our idea using a time-dependent quench of the motional state of a trapped ion, where the internal pseudospin provides a convenient probe qubit.

DOI: 10.1103/PhysRevLett.110.230601 PACS numbers: 05.70.Ln

PRL **110,** 230602 (2013) PHYSICAL REVIEW LETTERS week ending 7 JUNE 2013

## Measuring the Characteristic Function of the Work Distribution

L. Mazzola, G. De Chiara, and M. Paternostro

*Centre for Theoretical Atomic, Molecular and Optical Physics, School of Mathematics and Physics, Queen's University, Belfast BT7 1NN, United Kingdom*

(Received 11 February 2013; revised manuscript received 25 March 2013; published 7 June 2013)

We propose an interferometric setting for the ancilla-assisted measurement of the characteristic function of the work distribution following a time-dependent process experienced by a quantum system. We identify how the configuration of the effective interferometer is linked to the symmetries enjoyed by the Hamiltonian ruling the process and provide the explicit form of the operations to implement in order to accomplish our task. We finally discuss two physical settings, based on hybrid optomechanical-electromechanical devices, where the theoretical proposals discussed in our work could find an experimental demonstration.

DOI: 10.1103/PhysRevLett.110.230602 PACS numbers: 05.70.Ln, 05.30.Rt, 05.40.−a, 64.60.Ht

# NMR

**Experimental reconstruction of work distribution and verification of fluctuation relations at the full quantum level**

Tiago Batalhão,[1] Alexandre M. Souza,[2] Laura Mazzola,[3] Ruben Auccaise,[2] Roberto S. Sarthour,[2] Ivan S. Oliveira,[2] John Goold,[4,5,6] Gabriele De Chiara,[3] Mauro Paternostro,[3,7] and Roberto M. Serra[1]

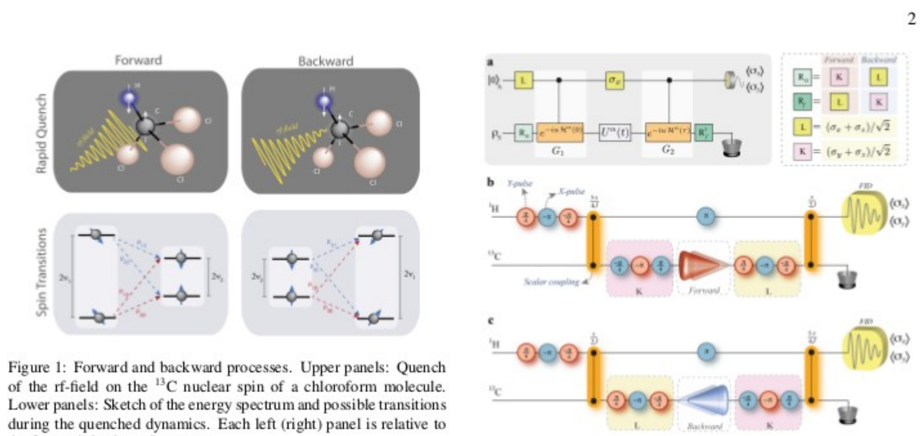


Figure 1: Forward and backward processes. Upper panels: Quench of the rf-field on the $^{13}$C nuclear spin of a chloroform molecule. Lower panels: Sketch of the energy spectrum and possible transitions during the quenched dynamics. Each left (right) panel is relative to the forward (backward) process.

To appear in Physical Review Letters – this week

CBPF
Centro Brasileiro de Pesquisas Físicas

# Open systems and heat

“Energetic fluctuation in an open quantum process”
John Goold and Kavan Modi
ArXiv:1407.4618 (2014)

## Probability distribution of energy changes

$$\mathcal{H}(\lambda_i) \longrightarrow \mathcal{H}(\lambda_f)$$

$$\Gamma(\rho_i) = \sum_k A_k \rho_i A_k^\dagger = \sigma_f$$

$$\sum_k A_k^\dagger A_k = 1$$

$$\rho_i = \frac{\exp(-\beta \mathcal{H}_i)}{Z_i}$$

$$\rho_f = \frac{\exp(-\beta \mathcal{H}_f)}{Z_f}$$

Probability of changing internal energy in a single run

$$P_F(\Delta U) = \sum_{nm} < m| \sum_k A_k |n >< n|\rho_i|n >< n|A^\dagger|m > \delta(\Delta U - (E'_m - E_n))$$

$$P_F(\Delta U) = \sum_{nm} P_n P^k(n|m) \delta(\Delta U - (E'_m - E_n))$$

**1. Prepare → 2. Measure → 3. Evolve → 4. Measure – 5. Build statistics**

# Proposal

**RAPID COMMUNICATIONS**

PHYSICAL REVIEW E **90**, 020101(R) (2014)

## Measuring the heat exchange of a quantum process

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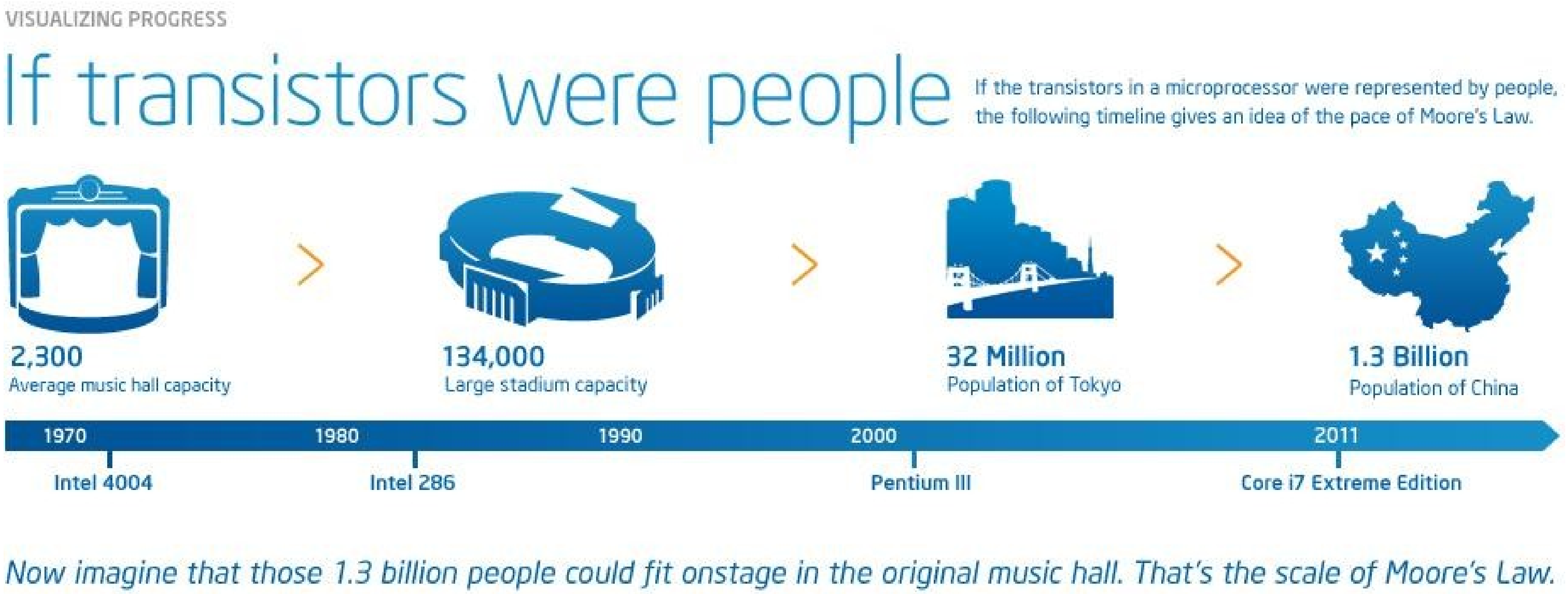
(Received 31 January 2014; revised manuscript received 26 April 2014; published 12 August 2014)

# Motivation to understand information to energy conversion in quantum systems




"Removal of the heat generated by an integrated circuit has become perhaps the crucial constraint on the performance of modern electronics"*

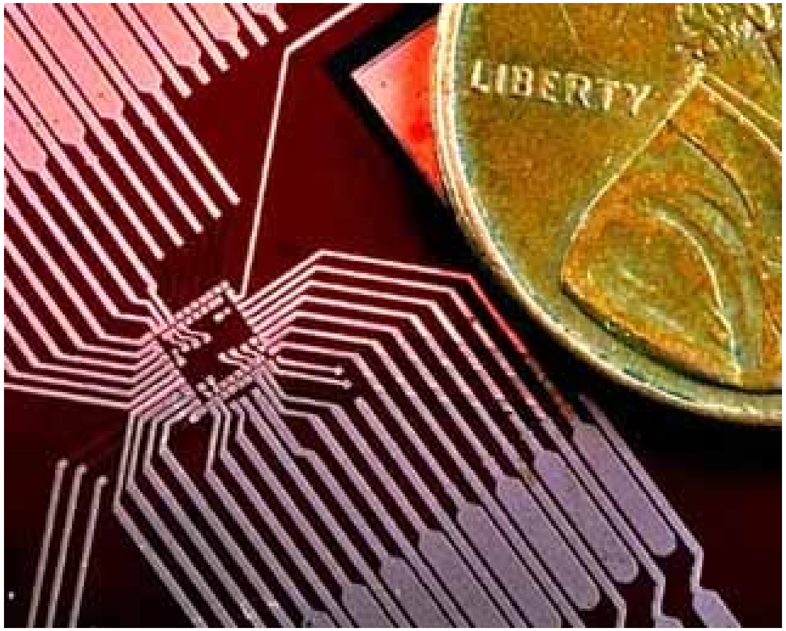
* MIT course on Nano-electronics

# Motivation to understand information to energy conversion in quantum systems

$$\Delta Q \geq \Delta S$$

"From a technological perspective, energy dissipation per logic operation in present-day silicon-based digital circuits is about a factor of 1,000 greater than the ultimate Landauer limit, but is predicted to quickly attain it within the next couple of decades" **

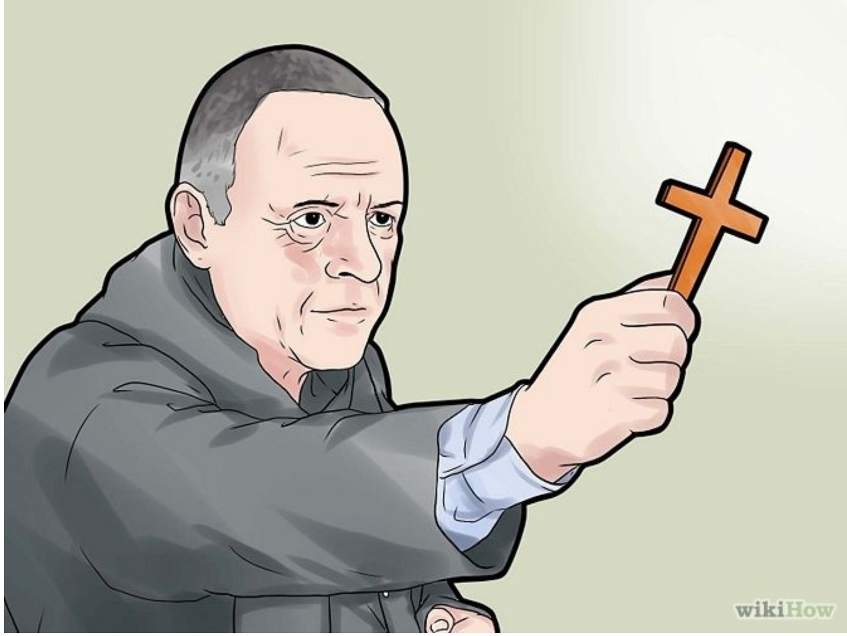
***Energy dissipation and transport in nanoscale devices. Nano Res. 3, 147–169 (2010)*




# Motivation to understand information to energy conversion in quantum systems

Perhaps help settle some foundational issues ?

Exorcist XIV: The Wrath of Maxwell's Demon. Part I and Part II
From Maxwell to Szilard.
Earman and Norton, Stud. Hist. Phil. Mod. Phys. 1998 1999

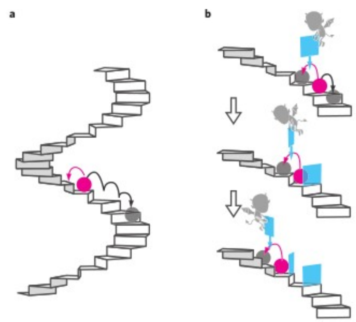


# Classical experiments

LETTERS
PUBLISHED ONLINE: 14 NOVEMBER 2010 | DOI: 10.1038/NPHYS1821
nature physics

## Experimental demonstration of information-to-energy conversion and validation of the generalized Jarzynski equality

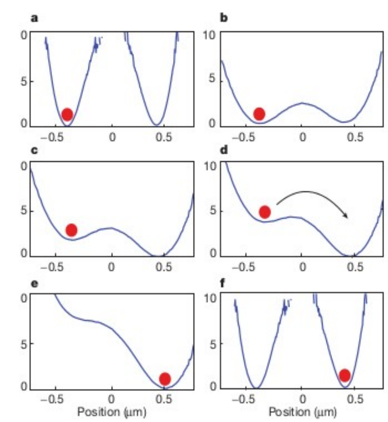
Shoichi Toyabe[1], Takahiro Sagawa[2], Masahito Ueda[2,3], Eiro Muneyuki[1]* and Masaki Sano[2]*




LETTER
doi:10.1038/nature10872

## Experimental verification of Landauer's principle linking information and thermodynamics

Antoine Bérut[1], Artak Arakelyan[1], Artyom Petrosyan[1], Sergio Ciliberto[1], Raoul Dillenschneider[2] & Eric Lutz[3]†



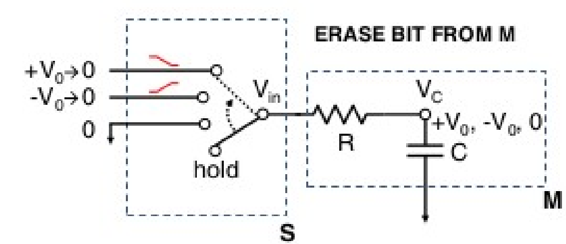

Japanese Journal of Applied Physics **51** (2012) 06FE10
**REGULAR PAPER**
DOI: 10.1143/JJAP.51.06FE10

## Experimental Test of Landauer's Principle at the Sub-$k_BT$ Level

Alexei O. Orlov, Craig S. Lent, Cameron C. Thorpe, Graham P. Boechler, and Gregory L. Snider*

*Department of Electrical Engineering, University of Notre Dame, Notre Dame, IN 46556, U.S.A.*

Received November 21, 2011; accepted January 6, 2012; published online June 20, 2012



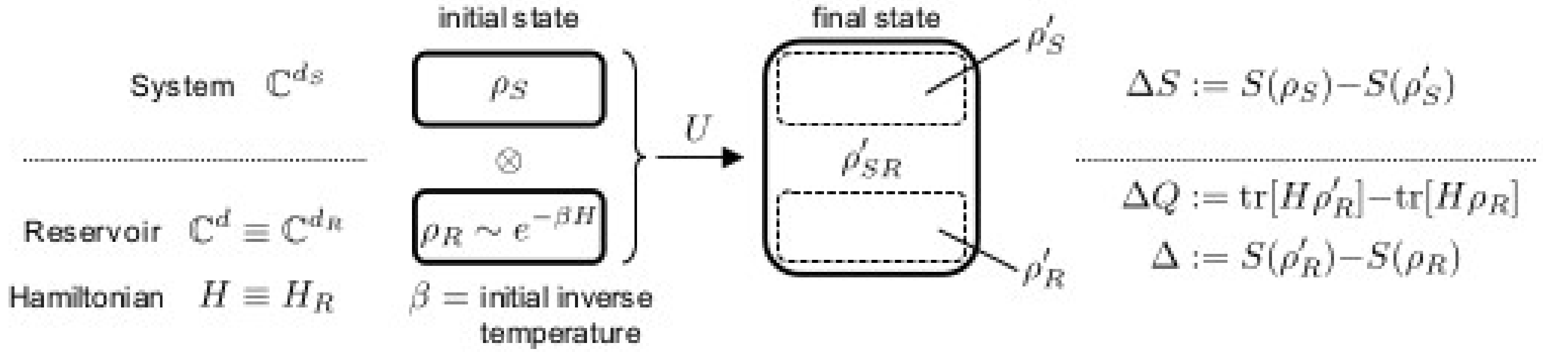
$$U = \mathrm{tr}[\mathcal{H}\rho]$$

$$\frac{dU}{dt} = \mathrm{tr}\left[\mathcal{H}\frac{d\rho}{dt}\right] + \mathrm{tr}\left[\rho\frac{d\mathcal{H}}{dt}\right]$$

$$\Delta U = \int_{t_i}^{t_f} dt \left\{ \mathrm{tr}\left[\mathcal{H}\frac{d\rho}{dt}\right] + \mathrm{tr}\left[\rho\frac{d\mathcal{H}}{dt}\right] \right\}$$

Heat

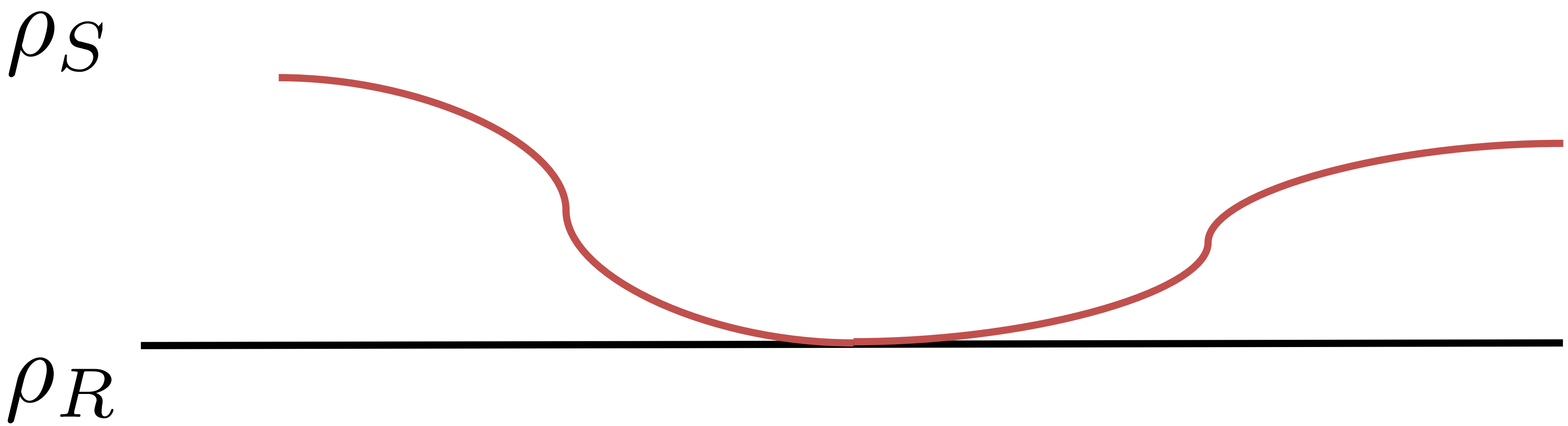
# An operational quantum Landauer principle




We now discuss each of these four assumptions in more detail, arguing that this setup is minimal.

***Entropy production as correlation between system and reservoir***
***Esposito et al***
***New Journal of Physics 12 (2010) 013013***

***(Im-)Proving Landauer's Principle***
**David Reeb and Michael M. Wolf**
**arXiv:1306.4352 (2014)**




$$\mathcal{H}_R \longrightarrow \mathcal{H}_R \longrightarrow \mathcal{H}_R \longrightarrow \mathcal{H}_R \longrightarrow \mathcal{H}_R \longrightarrow \mathcal{H}_R$$

$$[U, \mathcal{H}_R \otimes \mathbb{I}] = 0$$

## The distribution of dissipated heat

Consider the distibution of dissipated heat to the **bath**

$$P(\mathbf{Q}) = \sum_{\mathbf{l,m,n}} \langle \mathbf{r_n}|\hat{\mathbf{A}}_\mathbf{l}|\mathbf{r_m}\rangle\langle \mathbf{r_m}|\hat{\rho}_\mathcal{E}|\mathbf{r_m}\rangle\langle \mathbf{r_m}|\hat{\mathbf{A}}^\dagger|\mathbf{r_n}\rangle\delta(\mathbf{Q}-(\mathbf{E_n}-\mathbf{E_m}))$$

$$\hat{\rho}_s = \sum_j \lambda_j |s_j\rangle\langle s_j|$$

Now have non unitary dynamics on bath described by CPTP map: $\hat{A}_{l=jk} = \sqrt{\lambda_j}\langle s_k|\hat{U}|s_j\rangle$

average heat dissipated to bath $< \mathbf{Q} >= \mathbf{tr}[\hat{\mathcal{H}}_\mathcal{E}\hat{\rho}'_\mathcal{E}] - \mathbf{tr}[\hat{\mathcal{H}}_\mathcal{E}\hat{\rho}_\mathcal{E}]$

***A non-equilibrium quantum Landauer principle***
**John Goold, Mauro Paternostro and Kavan Modi**
**arXiv:1402.4499 (2014)**

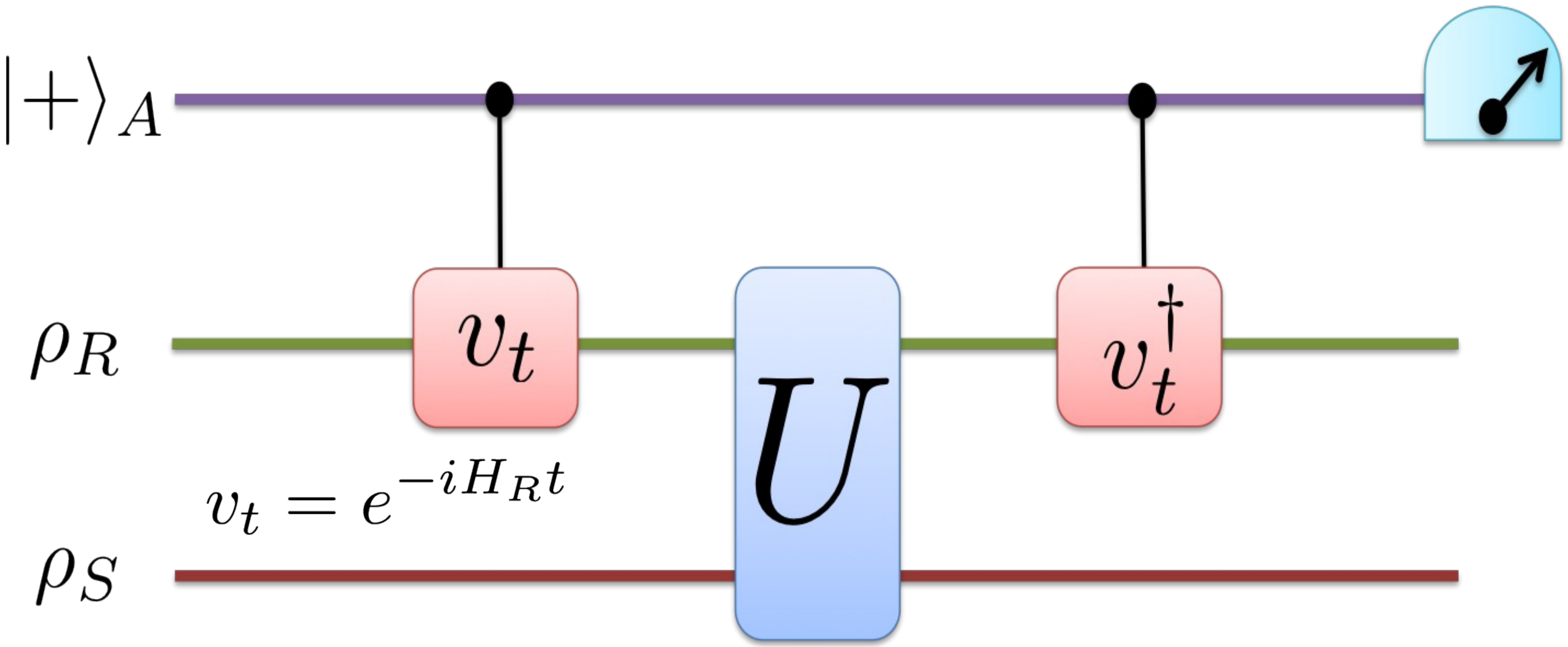
# Characteristic Function of heat distribution

$$P(\mathbf{Q}) = \sum_{\mathbf{lmn}} \langle \mathbf{r_n} | \mathbf{A_l} | \mathbf{r_m} \rangle \langle \mathbf{r_m} | \rho_{\mathbf{R}} | \mathbf{r_m} \rangle \langle \mathbf{r_m} | \mathbf{A_l^\dagger} | \mathbf{r_n} \rangle \delta(\mathbf{Q} - (\mathbf{E_n} - \mathbf{E_m}))$$

$$\Theta(t) = \int P(\mathbf{Q}) \mathbf{e^{itQ}} \mathbf{dQ} = \sum_{\mathbf{mn}} \mathbf{p_m} \mathbf{p_{n|m}} \mathbf{e}^{-\mathbf{i}(\mathbf{E_n} - \mathbf{E_m})\mathbf{t}}$$

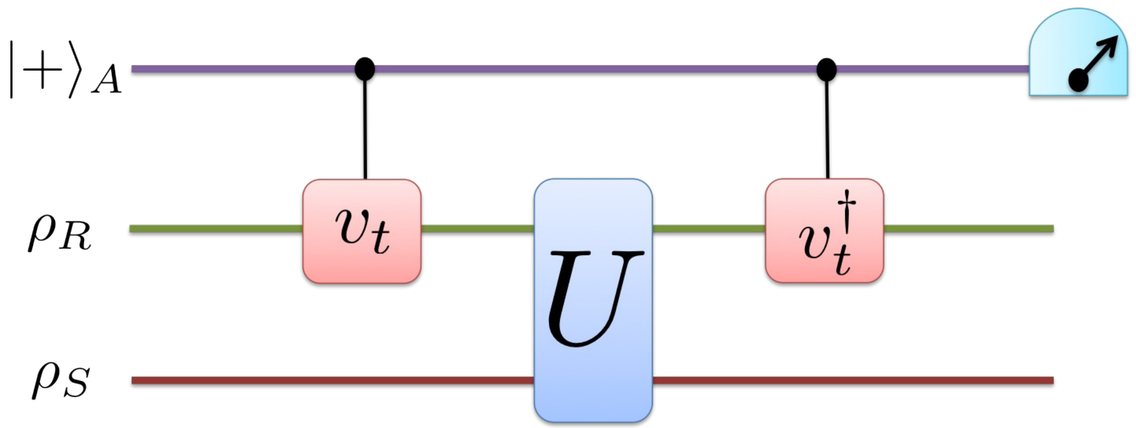
$$\Theta(t) = tr[U \rho_R v^\dagger \otimes \rho_S U^\dagger v]$$

# Modified Ramsey Interferometry




$$\rho'_{ARS} = \frac{1}{2}\begin{pmatrix} U\rho_R \otimes \rho_S U^\dagger & U\rho_R v_t^\dagger \otimes \rho_S U^\dagger v_t \\ v_t^\dagger U \rho_R v_t \otimes \rho_S U^\dagger v_t & v_t^\dagger U v_t \rho_R v_t^\dagger \otimes \rho_S U^\dagger v_t \end{pmatrix}$$

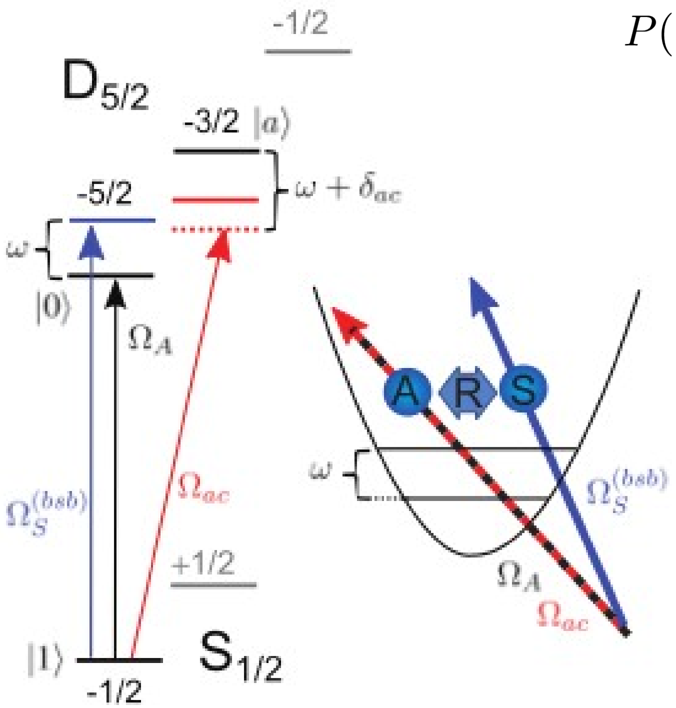
# Measuring the heat exchange of a quantum process




Can probe again characteristic function:

$$\Theta(t) = \int P(\mathbf{Q})\mathbf{e}^{\mathbf{itQ}}\mathbf{dQ} = \sum_{\mathbf{mn}} \mathbf{p_m p_{n|m} e^{-i(E_n - E_m)t}}$$

$$P(\mathbf{Q}) = \sum_{\mathbf{l,m,n}} \langle \mathbf{r_n}|\hat{\mathbf{A}}_\mathbf{l}|\mathbf{r_m}\rangle\langle \mathbf{r_m}|\hat{\rho}_\mathcal{E}|\mathbf{r_m}\rangle\langle \mathbf{r_m}|\hat{\mathbf{A}}^\dagger|\mathbf{r_n}\rangle\delta(\mathbf{Q} - (\mathbf{E_n} - \mathbf{E_m}))$$




Use hyperfine states as qubit and ancilla and normal mode as resivoir

**Exploring quantum Landauer experimentally?**

***Measuring the heat exchange of a quantum process***
**John Goold, U. Poschinger and Kavan Modi**
**arXiv:1401.4088 (2014)**

## The role of non-unital channels and fluctuation "like" relations

Non unital channels on the bath

$$\int e^{-\beta \mathbf{Q}} P(\mathbf{Q}) \mathbf{dQ} = \mathbf{tr}[\hat{\rho}_{\mathcal{E}} \sum_{\mathbf{l}} \hat{\mathbf{A}}_{\mathbf{l}} \hat{\mathbf{A}}_{\mathbf{l}}^{\dagger}] \qquad \sum_{l} \hat{A}_l \hat{A}_l^{\dagger} \neq 1$$

Apply Jensen inequality

$$\langle f(x) \rangle \geq f(\langle x \rangle)$$

Get non trivial bound: $\beta \langle \mathbf{Q} \rangle \geq \mathcal{B}_{\mathbf{Q}}$

$$\mathcal{B}_{\mathbf{Q}} = -\ln(tr[\hat{U} \hat{\rho}_S \otimes 1_E \hat{U}^{\dagger} 1_S \otimes \hat{\rho}_E])$$

***A non-equilibrium quantum Landauer principle***
**John Goold, Mauro Paternostro and Kavan Modi**
**arXiv:1402.4499 (2014)**

# A non-equillibrium quantum Landauer principle



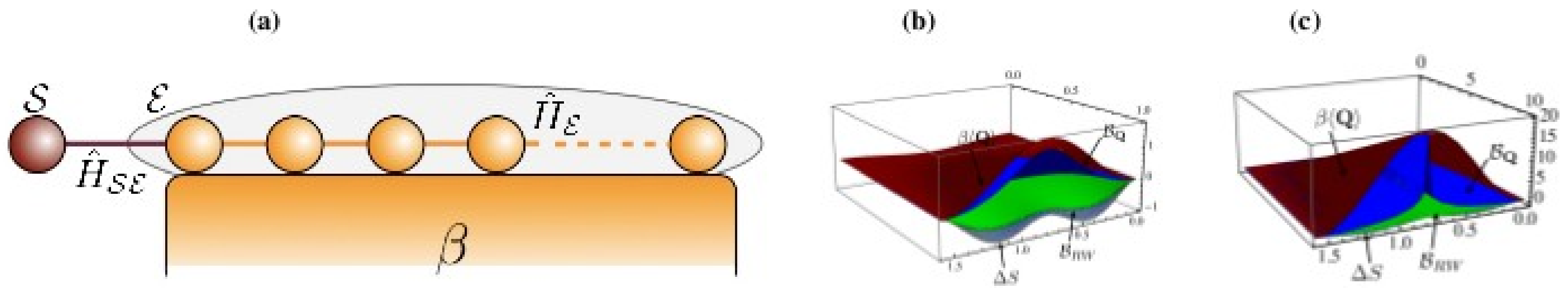



FIG. 1. (**a**) Schematic representing the system under consideration. (**b**) Comparison between $\beta\langle\mathbf{Q}\rangle$, the bound $\mathcal{B}_\mathbf{Q}$ derived in Eq. (6), and the one found in Ref. [4] for a spin-1/2 particle interacting for a dimensionless time $Jt$ with a single-spin environment at inverse temperature $\beta = 1$. We also plot the change in entropy $\Delta S$. All the quantities are studied against the initial preparation $\alpha|1\rangle_\mathcal{S} + \sqrt{1-\alpha^2}|0\rangle_\mathcal{S}$ ($\alpha \in \mathbb{R}$) of the system state. (**c**) Analogous comparison as in panel (**b**), but performed against the environmental temperature and for the system being prepared in the pure state $|1\rangle_\mathcal{S}$.

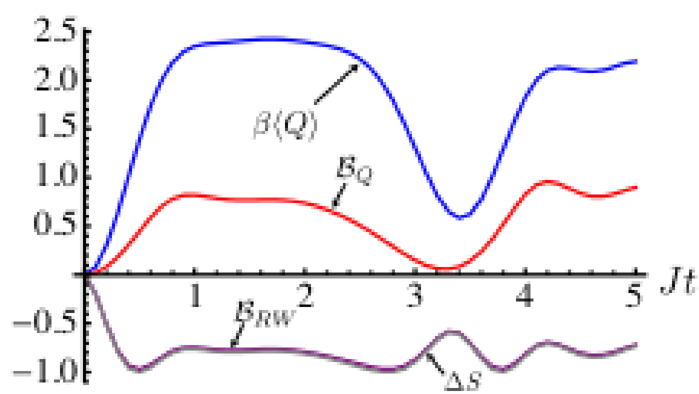



FIG. 3. Similarly to Fig. 2 (**a**) and (**b**), we plot the key quantities of our study for $J_0/J = B_0/J = B/J = 1$, $\beta = 1$, $\alpha = 1$ and an environment of $N = 4$ elements. The curve showing the behaviour of $\Delta S$ is basically indistinguishable from the one for $\mathcal{B}_{RW}$.

# Conclusion

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Go raibh mile maith agaibh!
(Thank you all!)
30
SLOW
THROUGH
VILLAGE
30


**“When you can measure what you are speaking about, and express it in numbers, you know something about it, when you cannot, your knowledge is of a meager and unsatisfactory kind; it may be the beginning of knowledge, but you have scarely, in your thoughts advanced to the stage of science.”**

William Thomson, 1st Baron Kelvin, born in Belfast 1824

# Fluctuation relation?

$$\int P(Q)e^{-\beta Q}dQ = \langle e^{-\beta Q}\rangle = \gamma e^{-\beta \Delta F}$$

$$\langle Q\rangle \geq \Delta F + \beta^{-1}\ln(\gamma) = \beta^{-1}\ln(\gamma)$$

$$\gamma = \mathrm{tr}\left[\sum_l A_l A_l^\dagger \rho'_{eq}\right]$$

***A non-equilibrium quantum Landauer principle***
**John Goold, Mauro Paternostro and Kavan Modi**
**arXiv:1402.4499 (2014)**