Casimir energy & Casimir entropy

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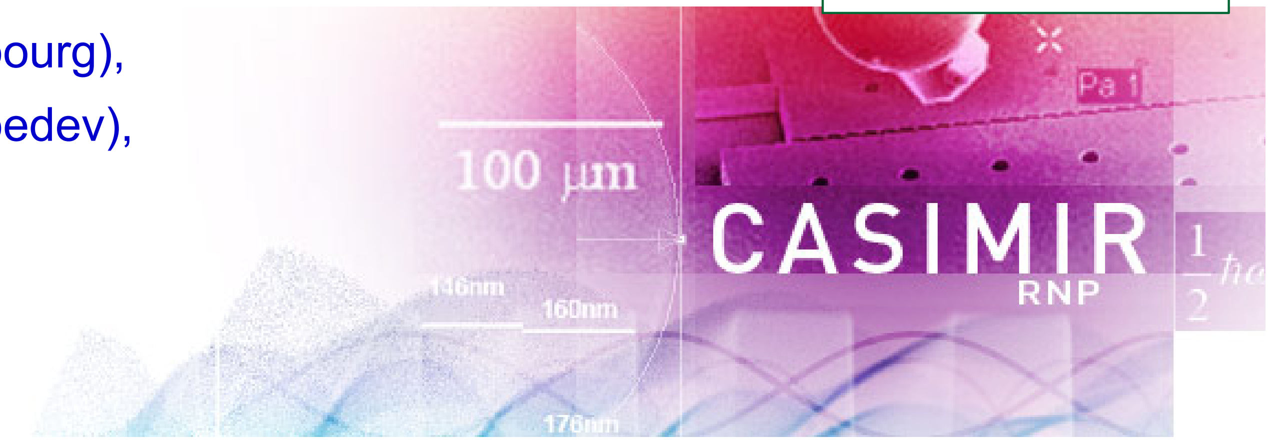
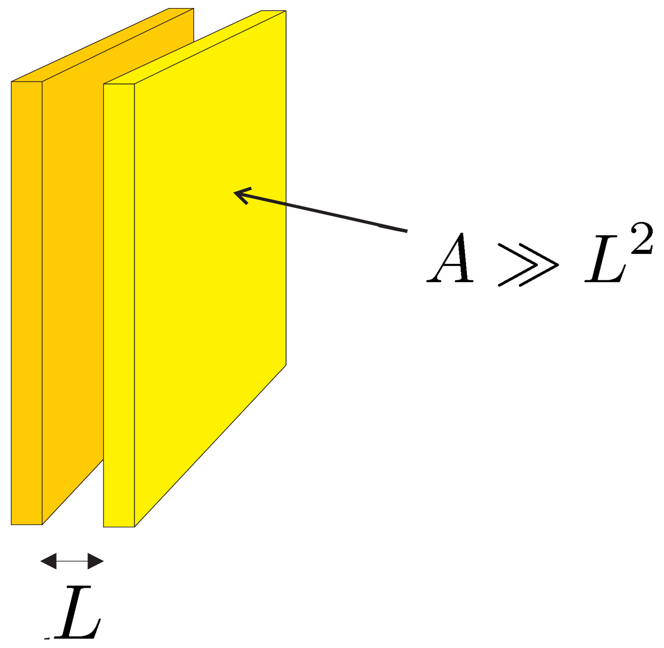


casimir-network.org

The ideal Casimir force

A universal effect from confinement of vacuum energy,
which depends only on *ħ*, *c*, and geometry

$$F_{\mathrm{Cas}} = -\frac{\mathrm{d}E_{\mathrm{Cas}}}{\mathrm{d}L}\ , \quad E_{\mathrm{Cas}} = -\frac{\hbar c \pi^2 A}{720 L^3}$$



- Written here in an idealized case
  - Perfectly parallel plane mirrors
  - Perfectly reflecting mirrors
  - Zero temperature

- Attractive force (negative pressure)

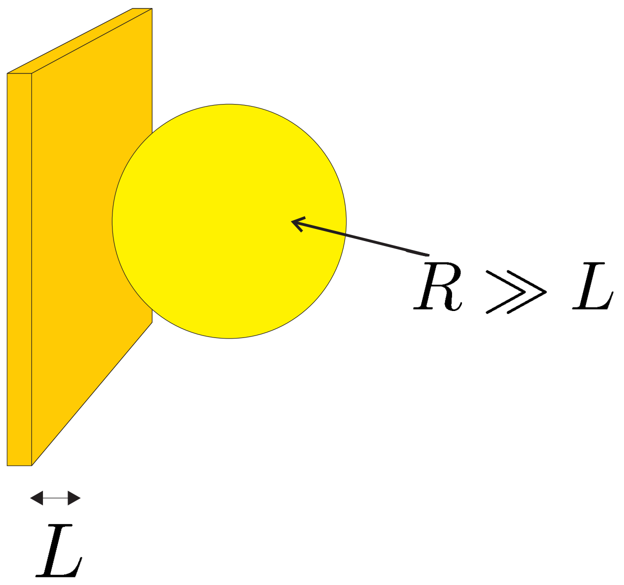
$$F_{\mathrm{Cas}} = P_{\mathrm{Cas}} A\ , \quad P_{\mathrm{Cas}} = -\frac{\hbar c \pi^2}{240 L^4}$$

$$|P_{\mathrm{Cas}}| \sim 1\mathrm{mPa}$$
$$\text{at } L = 1\mu\mathrm{m}$$

H.B.G. Casimir, Proc. K. Ned. Akad. Wet. (Phys.) **51** (1948) 79

The real Casimir force

- Experiments performed with Gold-covered plates
  - Force depends on non universal reflection properties of the metallic plates used in the experiments
- Experiments performed at room temperature
  - Effect of thermal and vacuum field fluctuations have to be taken into account
- Effect of geometry
  - Most precise experiments performed in the plane-sphere geometry

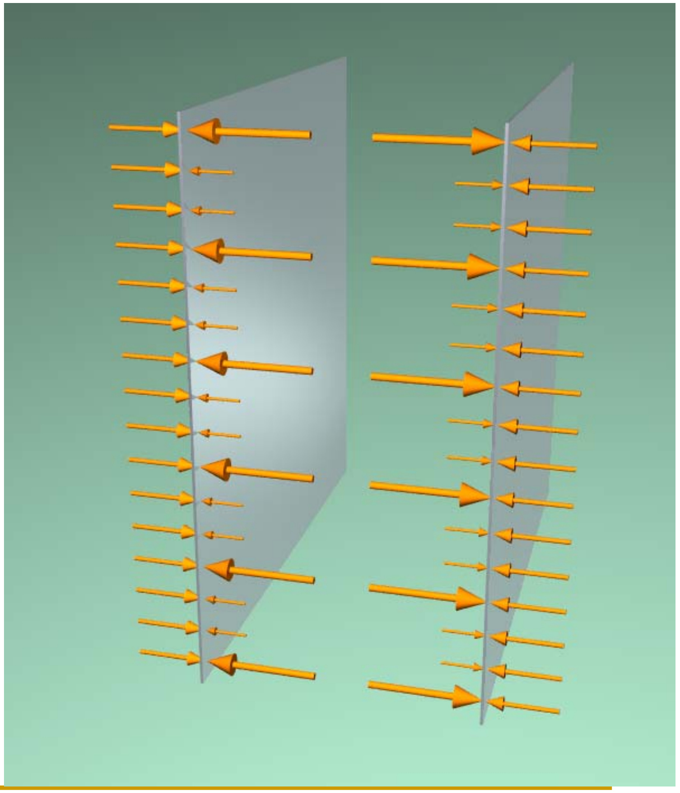


- Non ideality of surfaces
  - Roughness, electrostatic patches, contamination …

"Casimir Physics", Lecture Notes in Physics 834 (Springer-Verlag, 2011)

Radiation pressure of quantum fluctuations

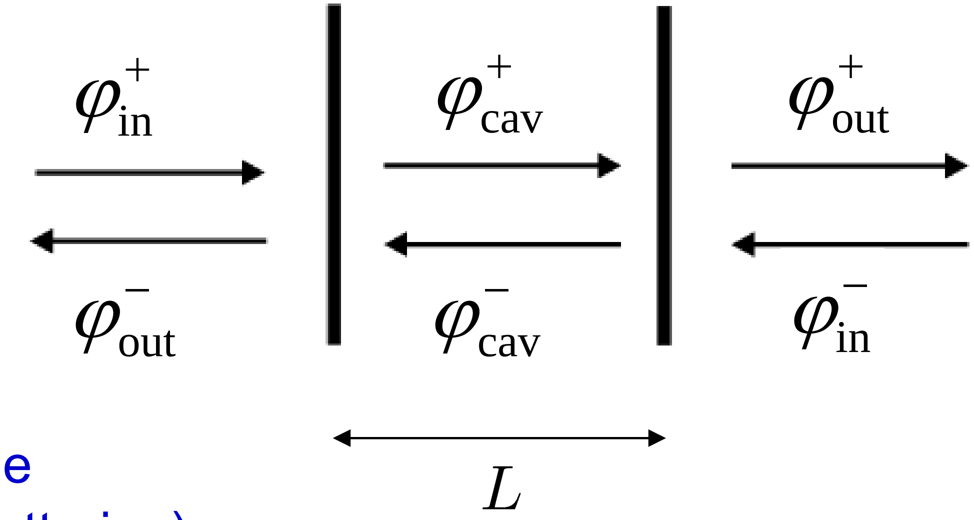
- Many ways to calculate the Casimir effect
- « Quantum Optics » approach
  - Quantum field fluctuations (vacuum and thermal fluctuations) pervade empty space → radiation pressure on mirrors
  - Force = pressure balance between inner and outer sides of the mirrors
- « Scattering theory »
  - Mirrors = scattering amplitudes depending on frequency, incidence, polarization
  - Solves the high-frequency problem
  - Gives results for real mirrors
  - Can be extended to other geometries



A. Lambrecht, P. Maia Neto, S. Reynaud, New J. Physics **8** (2006) 243

A simple derivation of the Casimir effect

- Quantum field theory in 1d space
  - Counterpropagating scalar fields
  - Point-like mirrors

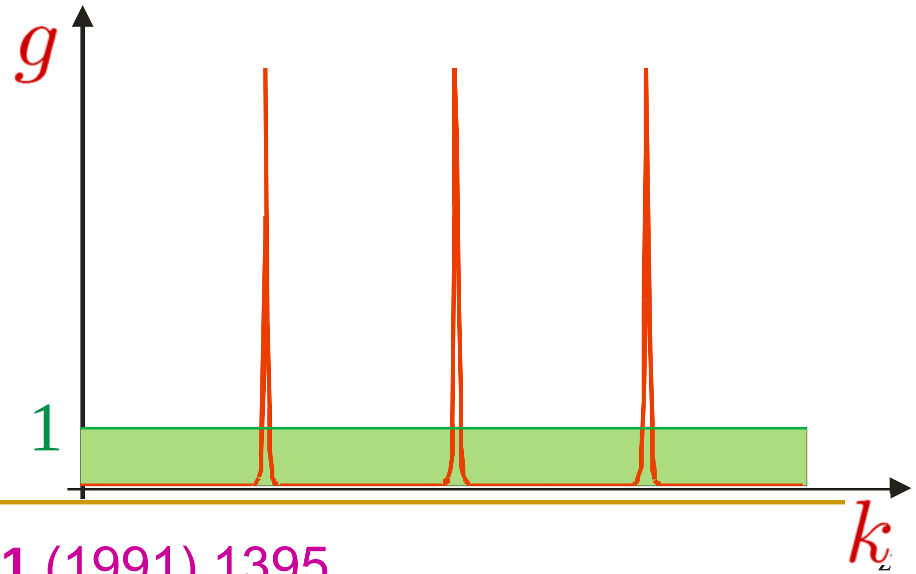


- Fabry-Perot cavity
  - Outer energies are the same as in the absence of the cavity (Unitarity of scattering)
  - Inner energies are enhanced for resonant modes, decreased for non-resonant modes (⬄ Cavity QED)

- Energy enhancement due to cavity confinement

$$g = \frac{1 - \left|re^{2ikL}\right|^2}{\left|1 - re^{2ikL}\right|^2}, \quad r \equiv r_1 r_2$$

r_j : reflection amplitudes on mirrors



M.-T. Jaekel & S. Reynaud, J. Physique **I-1** (1991) 1395

The Casimir force as a radiation pressure

- The Casimir force is the difference between inner and outer radiation pressures summed over all field modes

$$F = \int_0^\infty \frac{d\omega}{2\pi c} 2\hbar\omega N(\omega) \left(g(\omega) - 1\right)$$

Cavity confinement effect

Field fluctuation energy in the counter-propagating modes at frequency ω

$$2\hbar\omega N = 2\hbar\omega\left(\frac{1}{2} + \overline{n}_\omega\right) = \frac{\hbar\omega}{\tanh\frac{\hbar\omega}{2k_B T}}$$

Planck law + vacuum energy

$$\overline{n}_\omega = \frac{1}{e^{\hbar\omega/k_B T} - 1}$$

- The Casimir force can also be written in terms of causal amplitudes

$$F = \mathcal{I}_r^+ + \left(\mathcal{I}_r^+\right)^* \ , \ \mathcal{I}_r^+ = \int_0^\infty \frac{d\omega}{2\pi c} 2\hbar\omega N(\omega) f(\omega) \ , \ f = \frac{re^{2ikL}}{1 - re^{2ikL}}$$

Casimir free energy and phase-shifts

- Casimir force obtained from the free energy through a differentiation wrt *L*

$$F = -\frac{\partial \mathcal{F}(L,T)}{\partial L}$$

$$\mathcal{F} = \imath\hbar \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \left(\frac{1}{2} + \overline{n}_\omega\right) \left(\ln\left(1 - re^{2ikL}\right) - \ln\left(1 - r^* e^{-2ikL}\right)\right)$$

- with

$$\ln \frac{\left(1 - re^{2ikL}\right)^*}{1 - re^{2ikL}} = \ln\left(\det S_{12}\right) - \ln\left(\det S_1\right) - \ln\left(\det S_2\right)$$

Cavity (mirrors 1 and 2)

Mirror 1 or 2 alone

- Casimir free energy can be written as a difference between changes of free energies calculated for different configurations

$$\mathcal{F} = \Delta\mathcal{F}_{12} - \Delta\mathcal{F}_1 - \Delta\mathcal{F}_2$$

M. Jaekel & S. Reynaud, J. Physique **I-1** (1991) 1395 *quant-ph/0101067*

The phase-shift interpretation

- Each of these free energies is given by the phase-shifts for the S-matrix associated with the scattering configuration

$$\Delta\mathcal{F}_{12} = \frac{\hbar}{\imath}\int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\left(\frac{1}{2}+\overline{n}_\omega\right)\ln\left(\det S_{12}\right)$$

Similar expression for configurations with mirrors 1 and 2 alone

- In fact, each such quantity is itself a difference of free energies calculated in the presence and in the absence of the scatterer

$$\Delta\mathcal{F}_{12} \equiv \mathcal{F}_{12} - \mathcal{F}_{\mathrm{vac}}$$

- In the end, the Casimir free energy is a “double difference” involving four different configurations

$$\mathcal{F} = \left(\mathcal{F}_{12} - \mathcal{F}_{\mathrm{vac}}\right) - \left(\mathcal{F}_{1} - \mathcal{F}_{\mathrm{vac}}\right) - \left(\mathcal{F}_{2} - \mathcal{F}_{\mathrm{vac}}\right)$$

$$\mathcal{F} = \mathcal{F}_{12} - \mathcal{F}_{1} - \mathcal{F}_{2} + \mathcal{F}_{\mathrm{vac}}$$

Casimir effect between two planes

- Derivation similar to that in the 1-d case

specular reflection depending also on **k**, *p*

$$\mathcal{F} = \left\{ \imath\hbar \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \left(\frac{1}{2} + \overline{n}_\omega \right) \mathrm{Tr} \ln d \right\} + \{\}^*$$

$$\mathrm{Tr} \ln d \equiv \sum_p \int \frac{A\, \mathrm{d}^2\mathbf{k}}{(2\pi)^2} \ln d_{\mathrm{k}}^p$$

$$r \equiv r_1 r_2$$

$$d_{\mathbf{k}}^p[\omega] = 1 - r_{\mathbf{k}}^p[\omega] e^{2ik_z L}$$

$$k_z = \sqrt{\mathbf{k}^2 - \frac{\omega^2}{c^2}}$$

- Casimir pressure obtained as

$$P = -\frac{\partial \mathcal{F}(L,T)}{A \partial L}$$

Casimir effect and thermodynamics

- From the free energy, one derives the force

$$F = -\frac{\partial \mathcal{F}(L,T)}{\partial L}$$

… as well as an entropy

$$\mathcal{S} = -\frac{\partial \mathcal{F}(L,T)}{\partial T}$$

… and an "internal energy"

$$\mathcal{E} = \mathcal{F} + T\mathcal{S} = \mathcal{F} - T\frac{\partial \mathcal{F}(L,T)}{\partial T}$$

- Usual thermodynamical relations are valid

$$\mathrm{d}\mathcal{F} = -P\mathrm{d}V - \mathcal{S}\mathrm{d}T$$

$$\mathrm{d}\mathcal{E} = -P\mathrm{d}V + T\mathrm{d}\mathcal{S}$$

$$\mathrm{d}V \equiv A\mathrm{d}L$$
$$F \equiv AP$$

Model for reflection amplitudes

- Lifshitz model (1956)
  - bulk mirror (very thick slab)
  - local dielectric response function $\varepsilon[\omega]$

- Reflection amplitudes on each mirror given by Fresnel laws

$$r_1^{\mathrm{TE}}[\omega] = \frac{k_z - K_z}{k_z + K_z} \quad , \quad r_1^{\mathrm{TM}}[\omega] = \frac{K_z - \varepsilon k_z}{K_z + \varepsilon k_z}$$

$$K_z = \sqrt{\varepsilon \frac{\omega^2}{c^2} - k^2} \quad , \quad k_z = \sqrt{\frac{\omega^2}{c^2} - k^2}$$

K_z and k_z
longitudinal
wavevectors
in matter
and vacuum

E.M. Lifshitz, Sov. Phys. JETP **2** (1956) 73

I.E. Dzyaloshinskii, E.M. Lifshitz, L.P. Pitaevskii, Sov. Phys. Uspekhi **4** (1961) 153

M. Jaekel & S. Reynaud, J. Physique **I-1** (1991) 1395 *quant-ph/0101067*

Models for metallic bulk plates

- Simple models for the (reduced) dielectric function for metals
  - bound electrons (inter-band transitions, tables of optical data)
  - conduction electrons
    - determined by (reduced) conductivity σ
  - Drude model for conductivity
    - plasma frequency ω_P
    - relaxation parameter γ

$$\varepsilon[\omega] = \bar{\varepsilon}[\omega] + \frac{\sigma[\omega]}{-i\omega}$$

$$\sigma[\omega] = \frac{\omega_P^2}{\gamma - i\omega}$$

- Drude parameters related to the density of conduction electrons and to the static conductivity
  - finite conductivity σ_0 ⇔ non null γ

$$\omega_P^2 = \frac{nq^2}{\varepsilon_0 m^*}$$

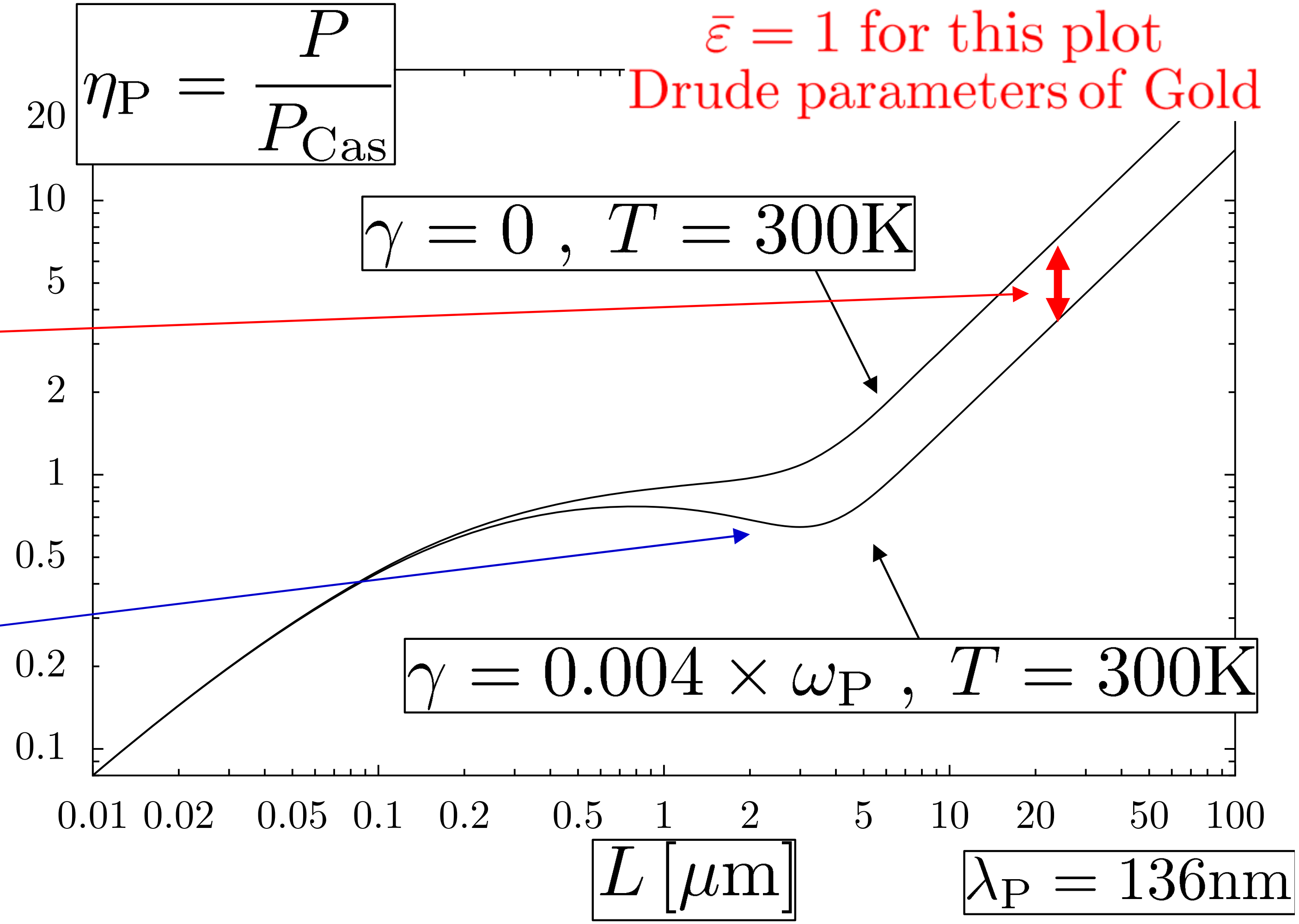
$$\sigma_0 = \frac{\omega_P^2}{\gamma}$$

A. Lambrecht & S. Reynaud, Eur. Phys. J. **D8** 309 (2000)

Pressure between metallic mirrors (room T)

- Pressure different from the ideal Casimir formula
  - Imperfect reflection
  - Non zero temperature

M. Boström and B.E. Sernelius, Phys. Rev. Lett. **84** (2000) 4757



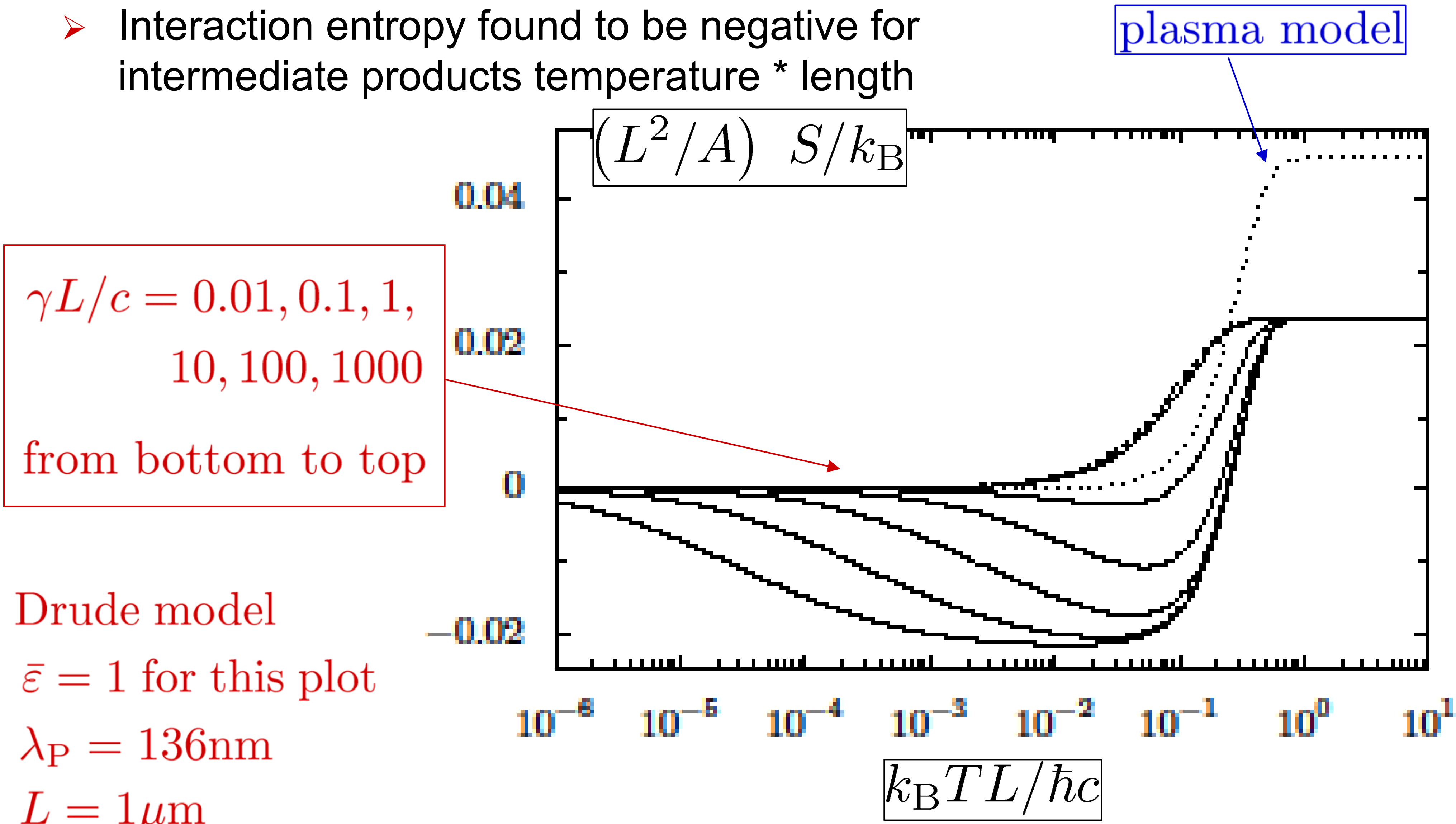
Small losses lead to a large factor 2 at large distances

Negative contribution of thermal photons at intermediate distances for the Drude model

G. Ingold, A. Lambrecht & S. Reynaud, Phys. Rev. **E80** (2009) 041113

Interaction entropy for metallic mirrors

- Interaction entropy found to be negative for intermediate products temperature * length



Drude model

$\bar{\varepsilon} = 1$ for this plot

$\lambda_P = 136\text{nm}$

$L = 1\mu\text{m}$

G. Ingold, A. Lambrecht & S. Reynaud, Phys. Rev. **E80** (2009) 041113

Measurements on micro-torsion resonators

Purdue measurements agree with predictions from the plasma model
but deviate from predictions with dissipation accounted for !

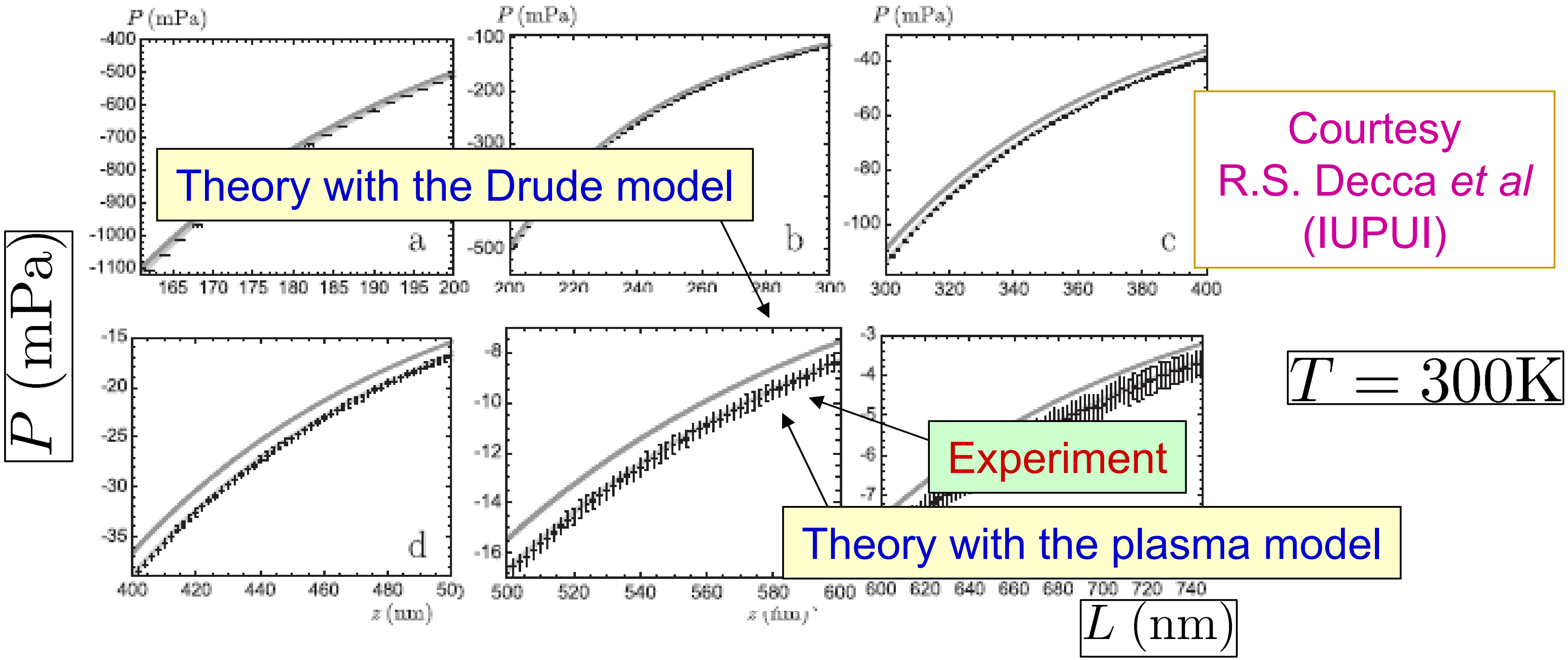
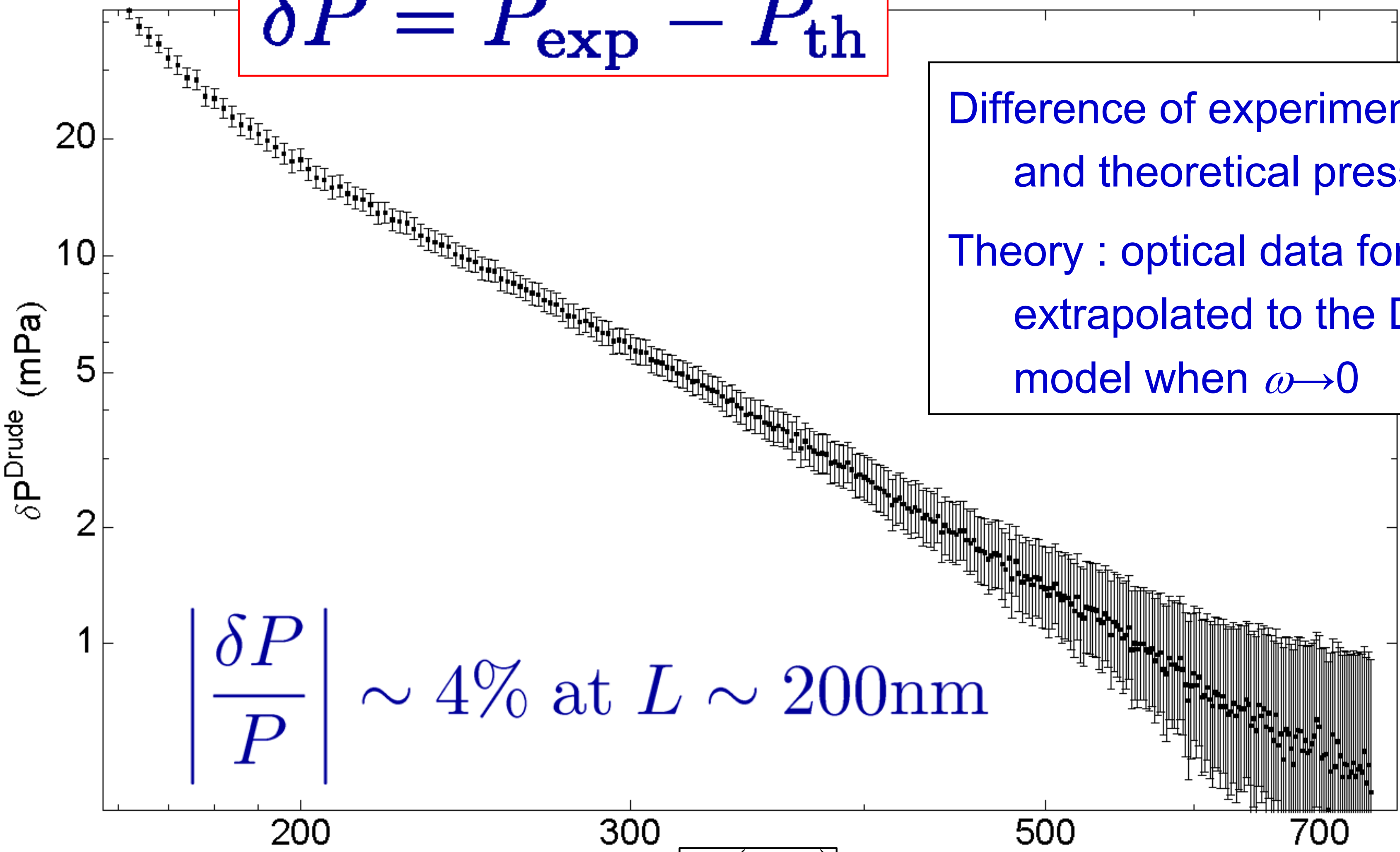


FIG. 1. Experimental data for the Casimir pressure as a function of separation z. Absolute errors are shown by black crosses in different separation regions (a–f). The light- and dark-gray bands represent the theoretical predictions of the impedance and Drude model approaches, respectively. The vertical width of the bands is equal to the theoretical error, and all crosses are shown in true scale.

R.S. Decca, D. Lopez, E. Fischbach *et al*, Phys. Rev. **D75** (2007) 077101

Measurements versus theory

$$\delta P = P_{\mathbf{exp}} - P_{\mathbf{th}}$$



$$L\ (\mathrm{nm})$$

Difference of experimental and theoretical pressures

Theory : optical data for Gold, extrapolated to the Drude model when $\omega \rightarrow 0$

$$\left|\frac{\delta P}{P}\right| \sim 4\% \text{ at } L \sim 200\text{nm}$$

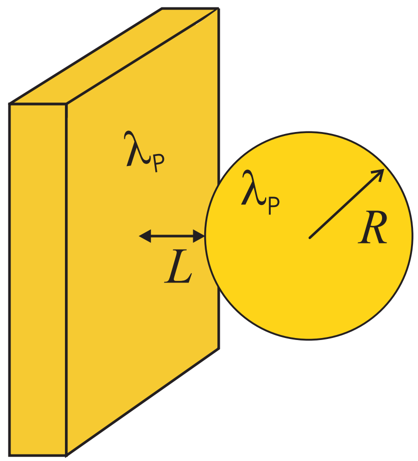
Experimental data kindly provided by R. Decca (IUPUI)

Theoretical pressure calculated by R. Behunin et al PRA **85** (2012) 012504

Plane-sphere geometry

- General scattering formula with big matrices mixing wavevectors and polarizations

$$\mathcal{D} = 1 - \mathcal{R}_{\mathrm{P}} e^{-i\mathcal{K}L} \mathcal{R}_{\mathrm{S}} e^{-i\mathcal{K}L}$$



  - Reflection matrices on the plane written as Fresnel amplitudes in the plane waves basis $\rightarrow \mathcal{R}_{\mathrm{P}}$
  - Reflection matrices on the sphere written as Mie amplitudes in the spherical waves basis $\rightarrow \mathcal{R}_{\mathrm{S}}$
  - Transformation from plane to spherical waves (for electromagnetic fields)

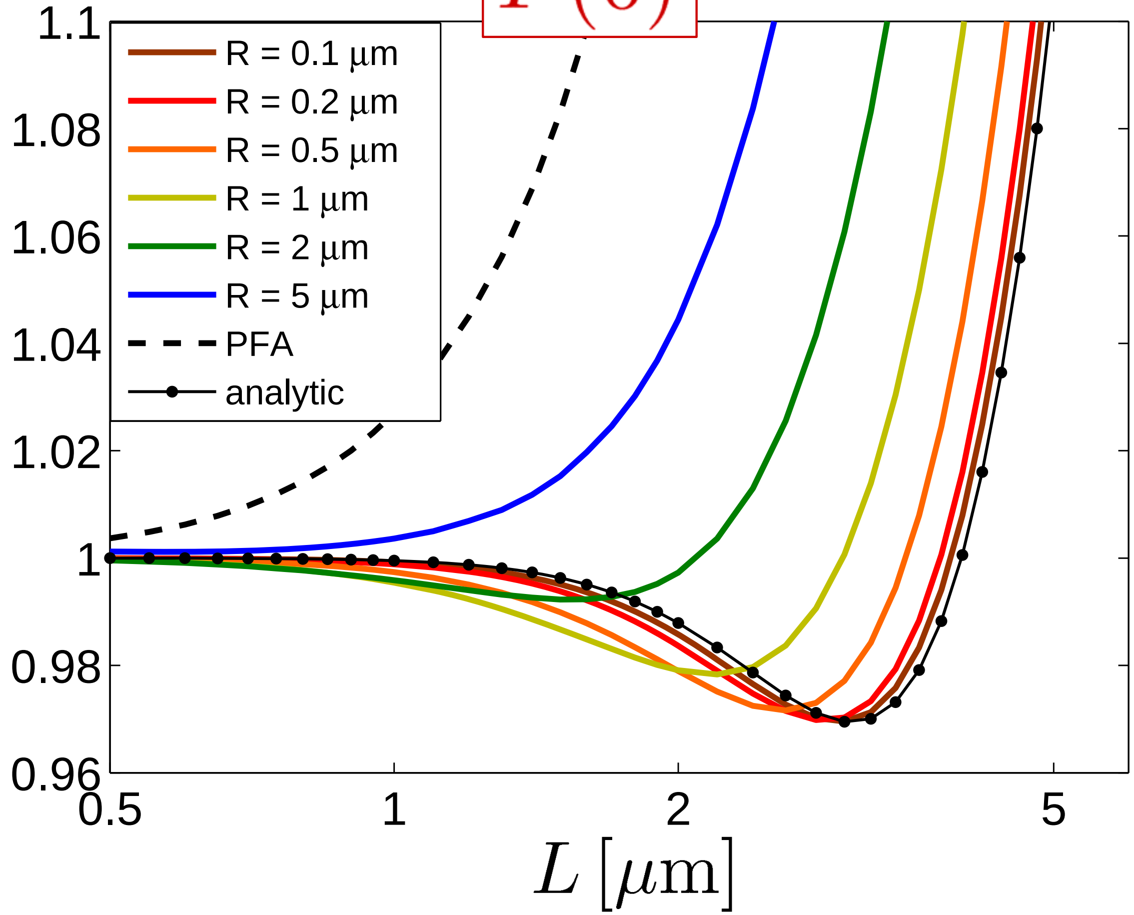
- We obtain an “exact” multipolar expansion of the energy
  - Spherical waves labeling : $(\ell, m) \quad , \quad |m| \leq \ell$
  - Sums truncated for the numerics $\ell \leq \ell_{\mathrm{max}}$
  - Results accurate for $x \equiv \dfrac{L}{R} > x_{\mathrm{min}} \quad , \quad x_{\mathrm{min}} \propto \dfrac{1}{\ell_{\mathrm{max}}}$

A. Canaguier-Durand, P.A. Maia Neto, I. Cavero-Pelaez, A. Lambrecht, S. Reynaud, PRL **102** (2009) 230404

Correlation geometry - temperature

- Force between plane and spherical perfect reflectors at room or zero temperature
- Drawn as the ratio of force at $T \neq 0$ to force at $T=0$

$$\frac{F(T)}{F(0)}$$



- Contribution of thermal photons repulsive at intermediate distances !

$$F(T) < F(0)$$

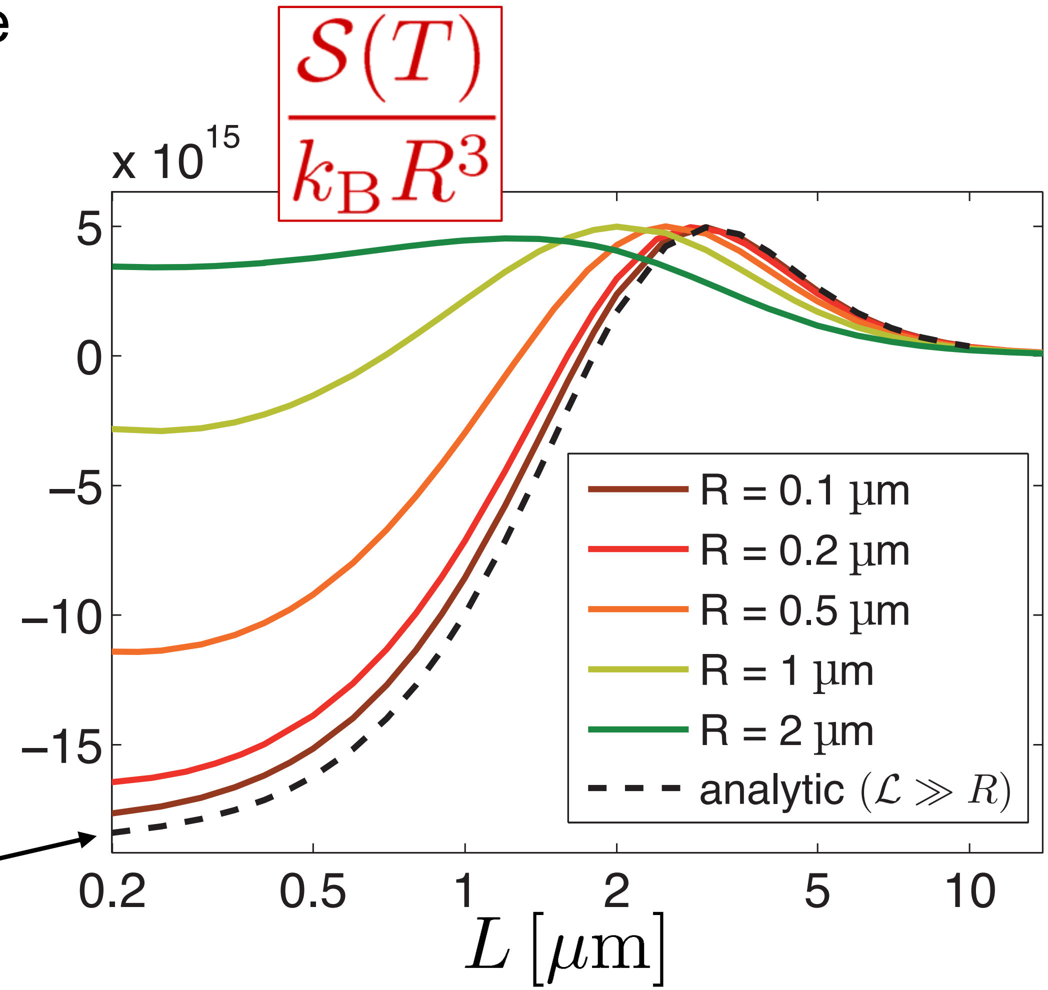
A. Canaguier-Durand, P.A. Maia Neto, A. Lambrecht, S. Reynaud
PRL **104** (2010) 040403

Casimir entropy in the plane-sphere case

- Casimir entropy at room temperature computed between perfectly reflecting sphere and plane, as a function of separation distance
- Drawn after division by the volume of the sphere

- Casimir entropy negative at some distances, for perfect mirrors here
- Features not seen for perfect plane mirrors

- Analytical expressions available for small spheres (dipolar approximation)

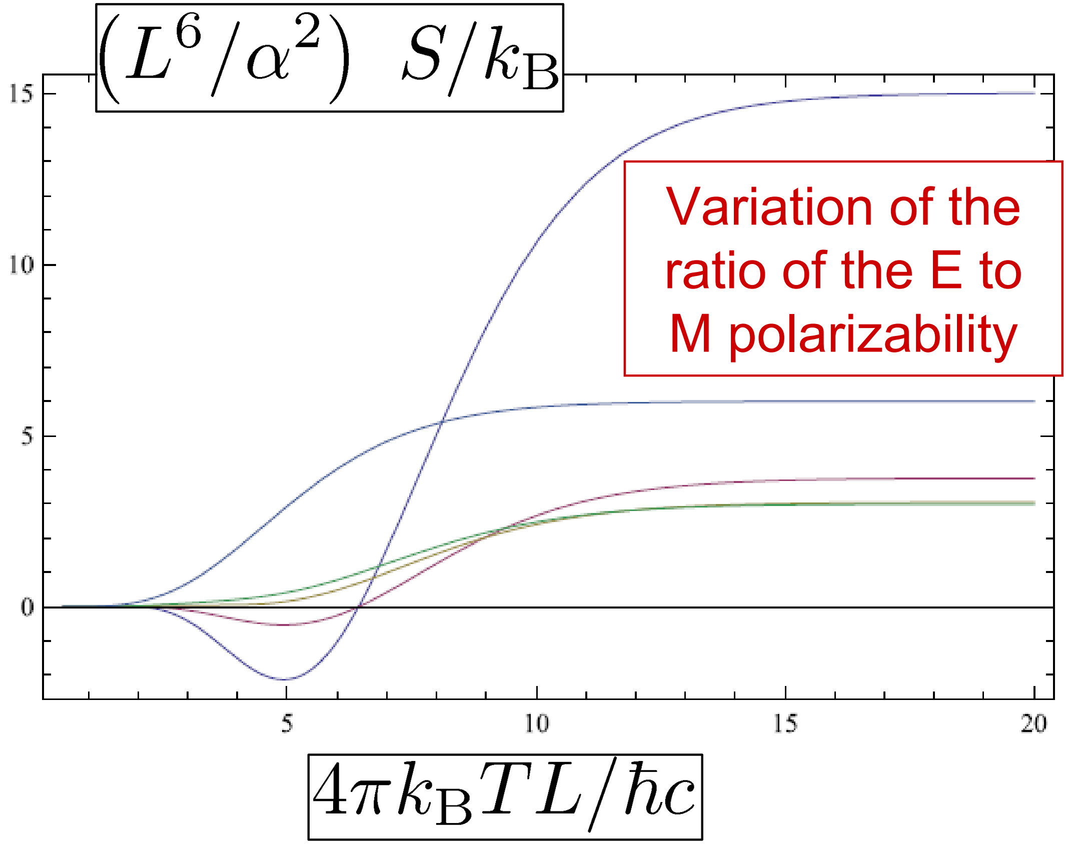


A. Canaguier-Durand, P.A. Maia Neto, A. Lambrecht, S. Reynaud
PRA **82** (2010) 012511

Negative Casimir-Polder entropies in nanoparticle interactions

- Systematic study for the interaction with planes or between them of atoms or nanoparticles, in the limit R→0 (dipolar approximation)

- Negative interaction entropies obtained in many different configurations
  - Example of the interaction between two identical nanoparticles →
- No contradiction with the principles of thermodynamics
- Phenomenon not exceptional
- In fact it is nearly ubiquitous



K. Milton, R. Guérout, G.-L. Ingold, A. Lambrecht, S. Reynaud,
accepted for the special issue on Casimir Forces of JPCM : arXiv/1405.0311