The reabsorption effect with single-electron sources: heat current vs charge current

Géraldine Haack

with Michael Moskalets and Markus Büttiker

Phys. Rev. B **87**, 125429 (2013)

Grenoble, 30.09.2014

Thermodynamics at the quantum level

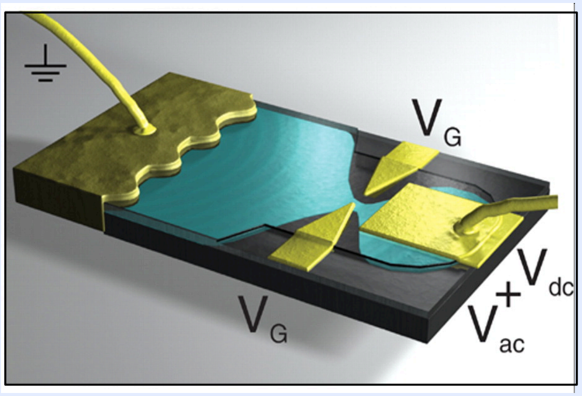
- Defining properly thermodynamical quantities: Efficiency, heat, work, power…

  Efficiency at a given output power (R. S. Whitney)

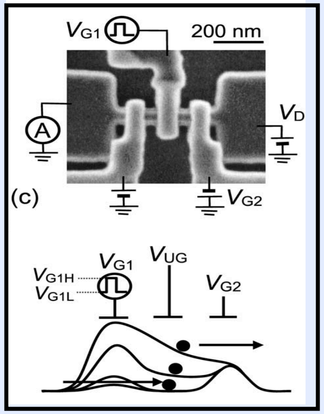
  Entropy generation for out-of-equilibirum systems (M. Esposito)

  Minimal amount of work for 1 qubit erasure (R. Renner, C. Browne)

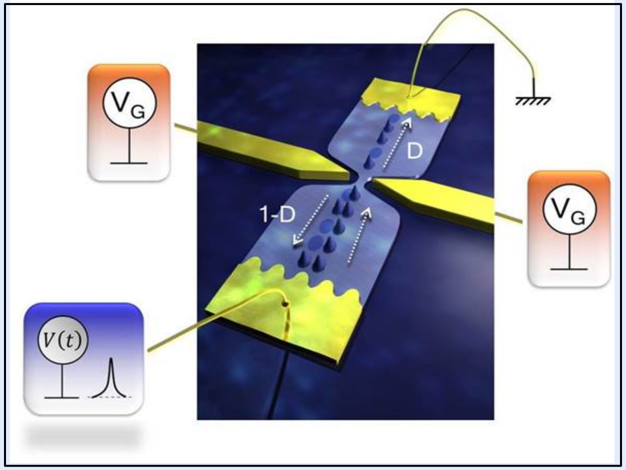
- Out-of-equilibrium systems : driven sources for single-electrons



Gabelli *et al.*, Science **313** (2006)
Fève *et al.*, Science **316** (2007)
Bocquillon et *al.*, Science **339** (2013)



Blumenthal *et al.*, Nat. Phys.**3** (2007)
Fujiwara *et al.*, App. Phys. Lett **92** (2008)



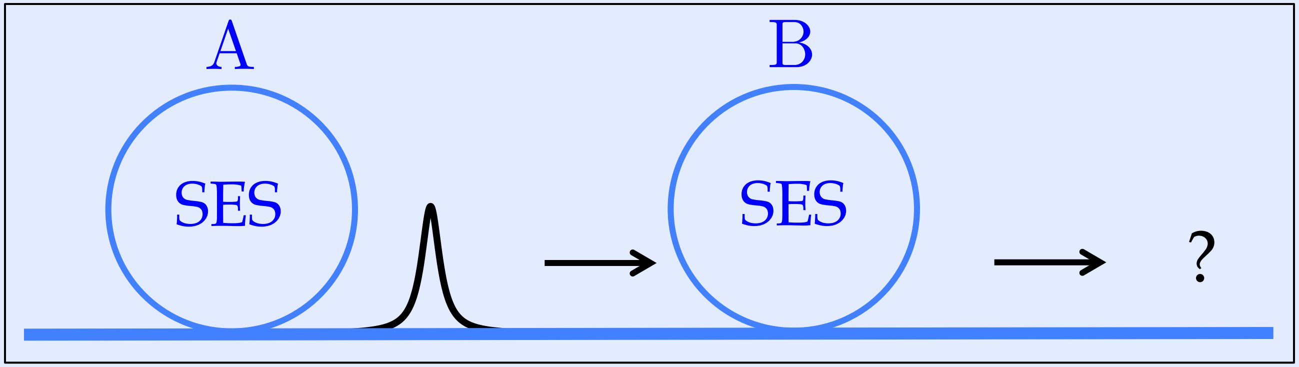
Dubois *et al.*, Nature **502** (2013)

Outline

- Heat current emitted by the periodically driven SES

- Multi-particle emitter

  - Reabsorption effect
  - Emission of energy modes (carry no charge)



Heat current

- Scattering-matrix approach
  Low temperature, low-frequency limit

- Charge current operator

$$\hat{I}_\alpha(t) = \frac{e}{h} \iint dE\, dE'\, e^{i(E-E')t/\hbar} \left[\hat{b}^\dagger_\alpha(E)\hat{b}_\alpha(E') - \hat{a}^\dagger_\alpha(E)\hat{a}_\alpha(E')\right]$$

$$\hat{b}_\alpha(E') = \sum_\beta \sum_{E'_n} S_{F,\beta\alpha}(E', E'_n)\hat{a}_\beta(E'_n)$$

Büttiker, PRB **46** (1992)
Moskalets and Büttiker, PRB **80** (2009)

Heat current

- Scattering-matrix approach
  Low temperature, low-frequency limit

- Heat flow operator

$$\hat{J}_\alpha(t) = \frac{1}{h} \iint dE\, dE' (E-\mu)\, e^{i(E-E')t/\hbar} \left[\hat{b}^\dagger_\alpha(E) \hat{b}_\alpha(E') - \hat{a}^\dagger_\alpha(E) \hat{a}_\alpha(E') \right]$$

$$\hat{b}_\alpha(E') = \sum_\beta \sum_{E'_n} S_{F,\beta\alpha}(E', E'_n) \hat{a}_\beta(E'_n)$$

R. S. Whitney, arXiv:1408.3348
P. N. Butcher, J. Phys. Cond. Matter 2 (1990)

Büttiker, PRB **46** (1992)
Moskalets and Büttiker, PRB **80** (2009)

Heat current

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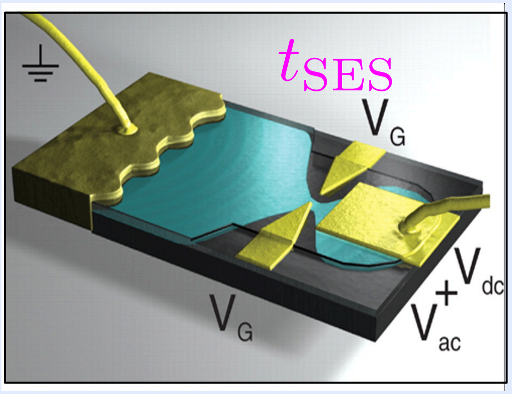
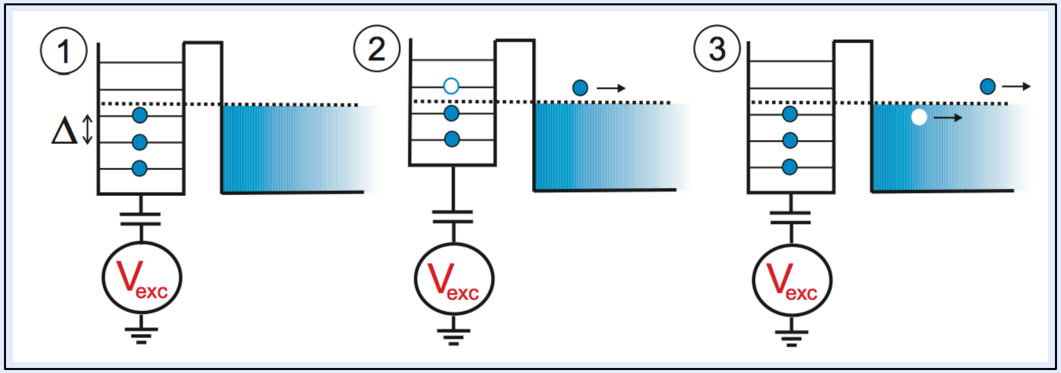




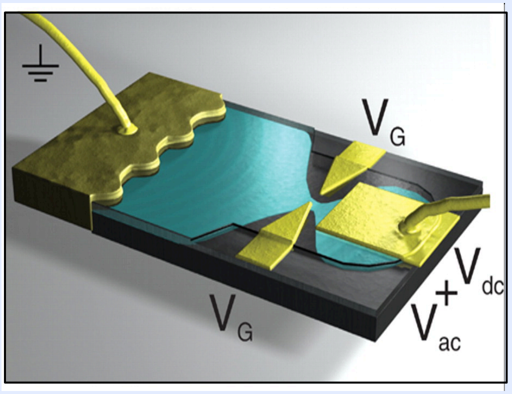
Gabelli *et al.*, Science **313** (2006), Fève *et al.*, Science **316** (2007)

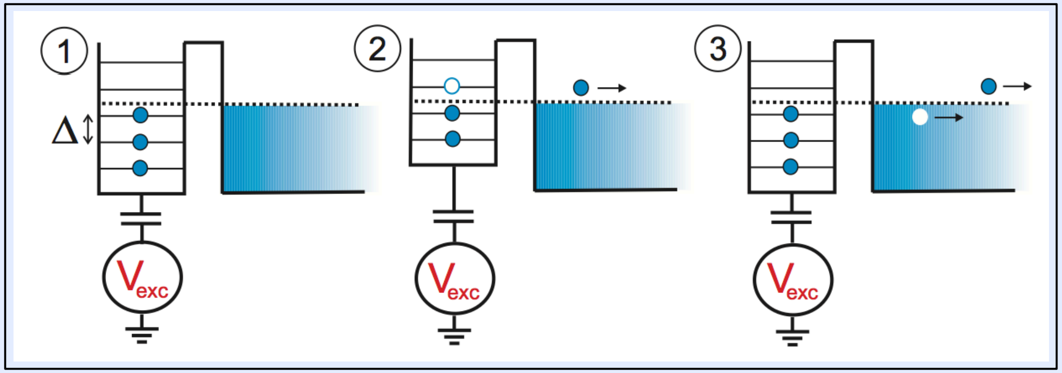
- Fabry-Pérot like expression:

$$S_{\mathrm{SES}}(t,E) = r_{\mathrm{SES}} + \frac{t^2_{\mathrm{SES}}}{r_{\mathrm{SES}}} \sum_{n=1}^{\infty} r^n_{\mathrm{SES}}\, e^{\frac{i}{\hbar}\left(nE\tau - \int_{t-n\tau}^{t} dt'\, eV(t')\right)}$$

Büttiker, PRB **46** (1992)
Moskalets and Büttiker, PRB **80** (2009)

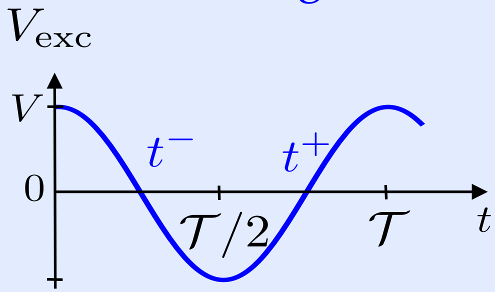
Heat current for the SES



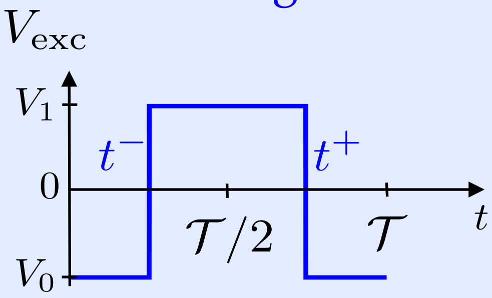


Gabelli *et al.*, Science **313** (2006), Fève *et al.*, Science **316** (2007)

Adiabatic regime



Non-adiabatic regime



Büttiker, PRB **46** (1992)
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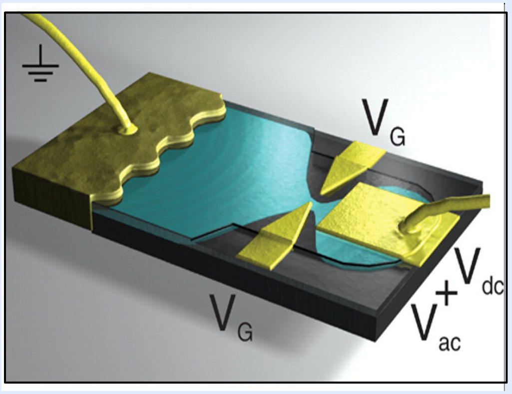
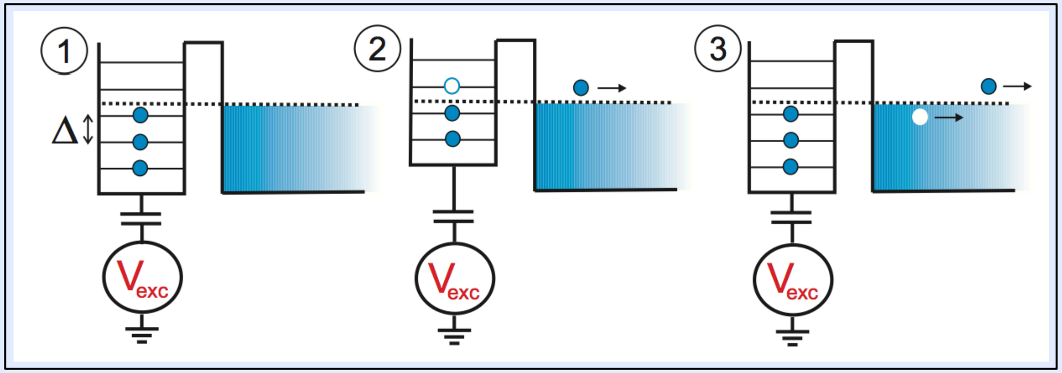
Heat current for the SES





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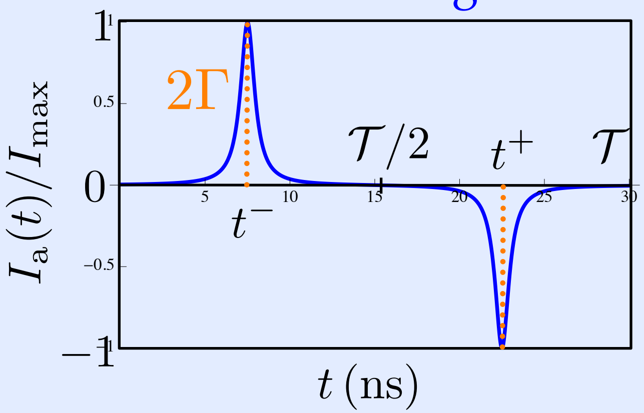
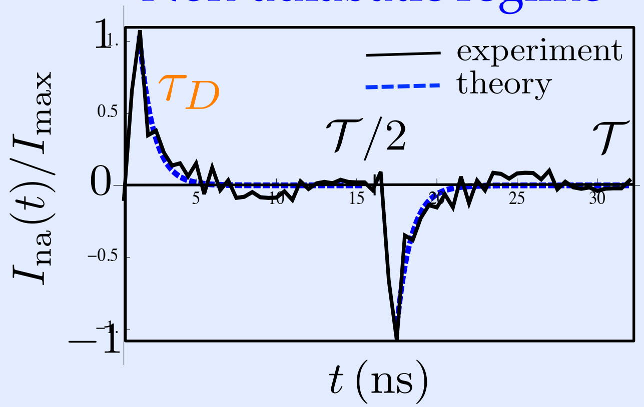


Non-adiabatic regime



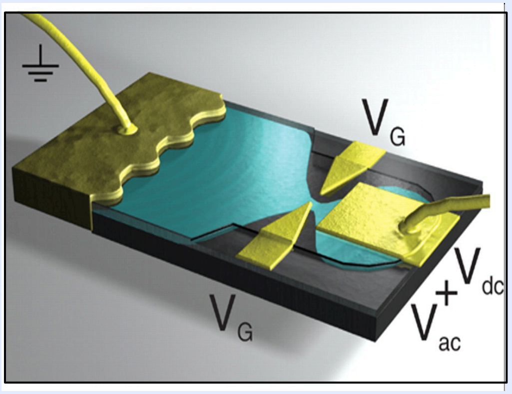
- For a single-electron state :

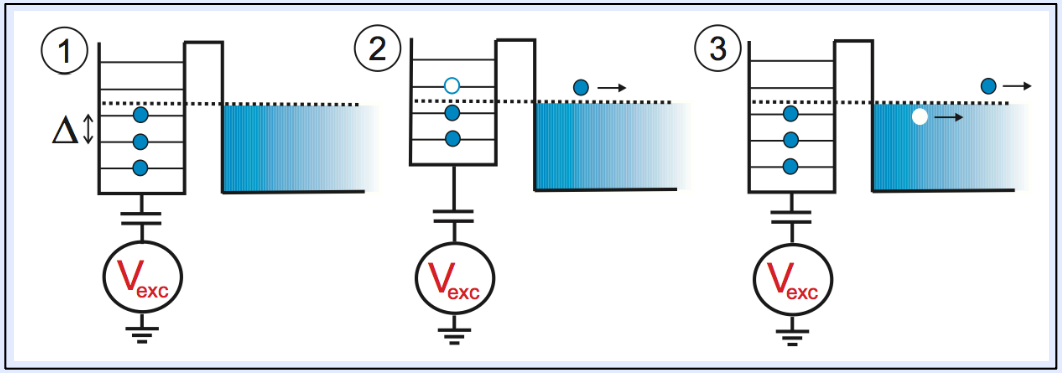
$$I_a(t) = \frac{e\Gamma/\pi}{(t-t^-)^2+\Gamma^2}$$

$$I_{na}(t) = \frac{e}{\tau_D}\exp^{-(t-t^-)/\tau_D}$$

Büttiker, PRB **46** (1992), Splettstoesser *et al.*, PRB **78** (2008), Moskalets and Büttiker, PRB **80** (2009)

Heat current for the SES





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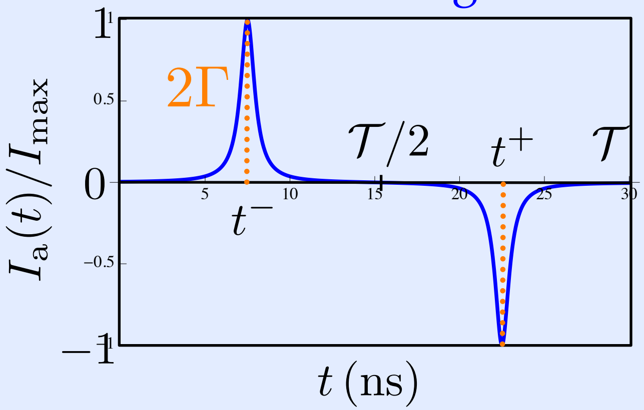
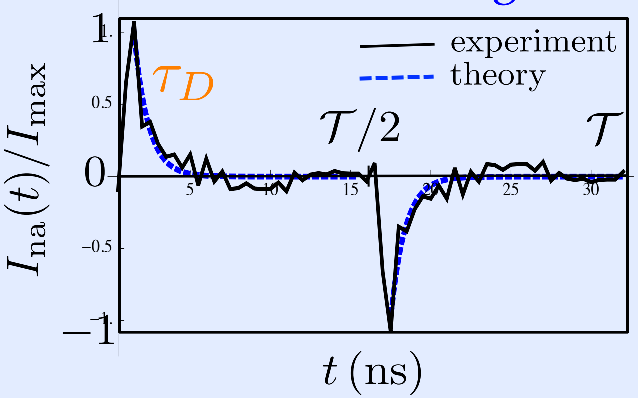
Adiabatic regime



Non-adiabatic regime



- For a single-electron state :

$$I_a(t) = \frac{e\Gamma/\pi}{(t-t^-)^2+\Gamma^2}$$

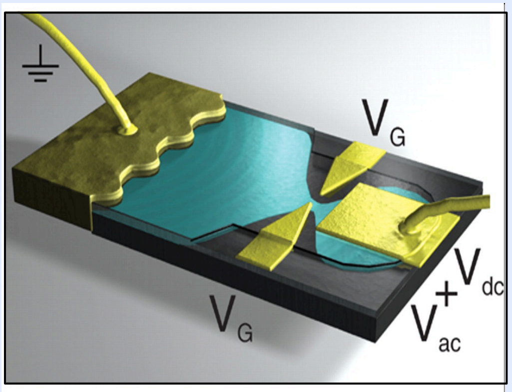
$$J_a(t) = \frac{\hbar\Gamma^2/\pi}{((t-t^-)^2+\Gamma^2)^2}$$

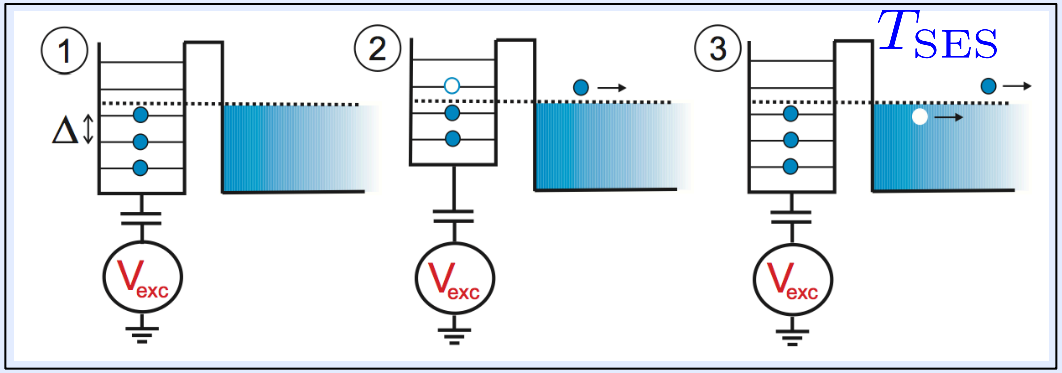
$$I_{na}(t) = \frac{e}{\tau_D}\exp^{-(t-t^-)/\tau_D}$$

$$J_{na}(t) = \frac{\Delta}{2\tau_D}\exp^{-(t-t^-)/\tau_D}$$

Büttiker, PRB **46** (1992), Splettstoesser *et al.*, PRB **78** (2008), Moskalets and Büttiker, PRB **80** (2009)

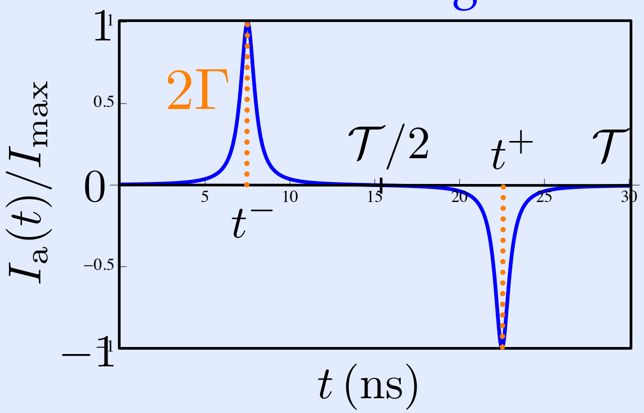
Heat current for the SES



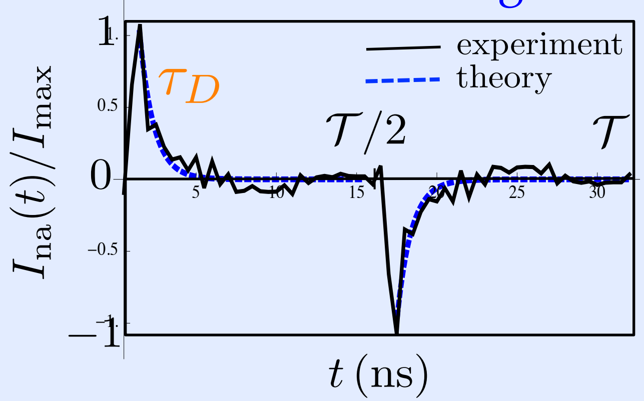


Gabelli *et al.*, Science **313** (2006), Fève *et al.*, Science **316** (2007)

Adiabatic regime



Non-adiabatic regime



- For a single-electron state :

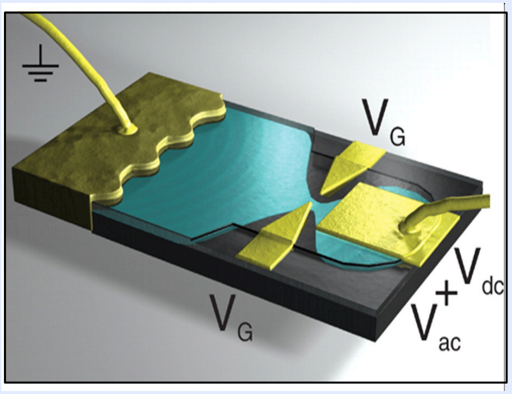
$$\overline{J_a(t)} = \frac{h}{2e^2}\overline{I_a^2(t)}$$

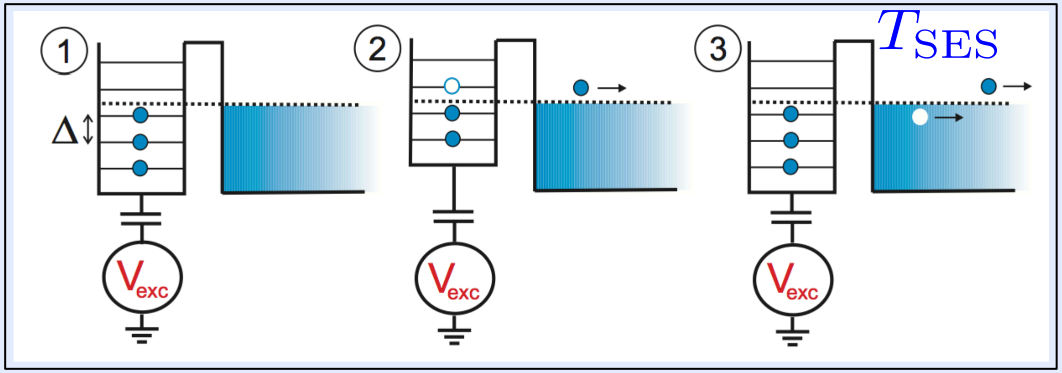
$$\overline{J_{na}(t)} = \frac{h}{e^2 \mathcal{T}_{\rm SES}}\overline{I_{na}^2(t)}$$

Joule-Lenz law holds for single-particle state in both regimes

Moskalets, Büttiker, PRB **80** (2009), Moskalets, Haack, Büttiker, PRB **87** (2013)

Heat current for the SES





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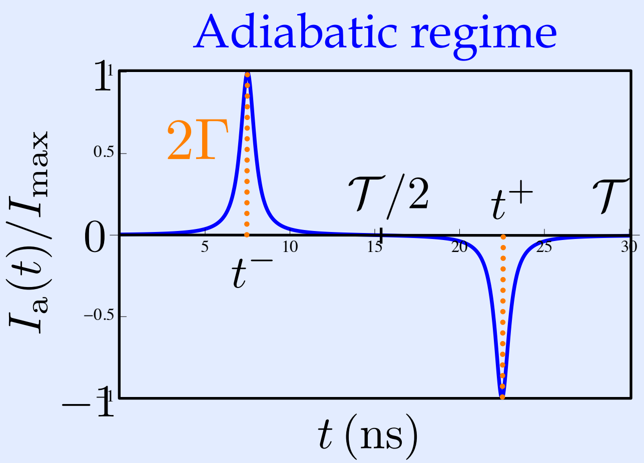
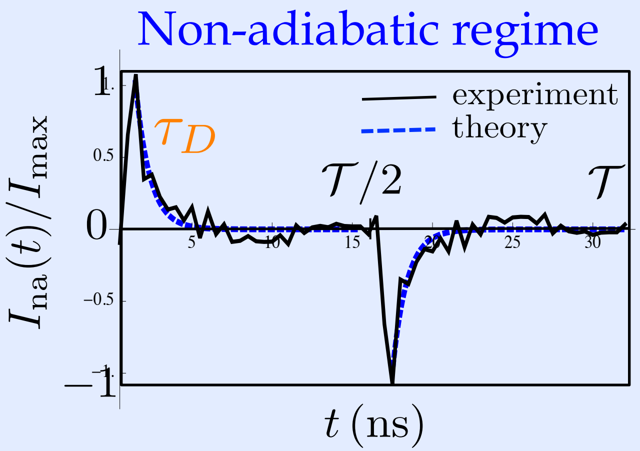




- For a single-electron state :

$$\overline{J_a(t)} = \frac{h}{2e^2}\overline{I_a^2(t)}$$

Joule heat due to
the relaxation resistance quantum

$$\overline{J_{na}(t)} = \frac{h}{e^2 \mathcal{T}_{\rm SES}}\overline{I_{na}^2(t)}$$

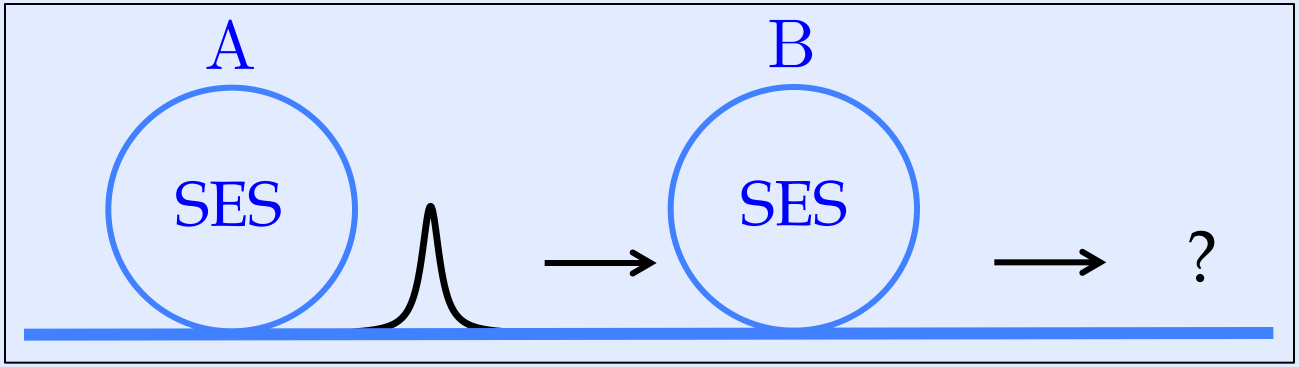
Joule heat due to
the 2-terminal resistance

Moskalets, Büttiker, PRB **80** (2009), Moskalets, Haack, Büttiker, PRB **87** (2013)

Outline

- Heat current emitted by the periodically driven SES

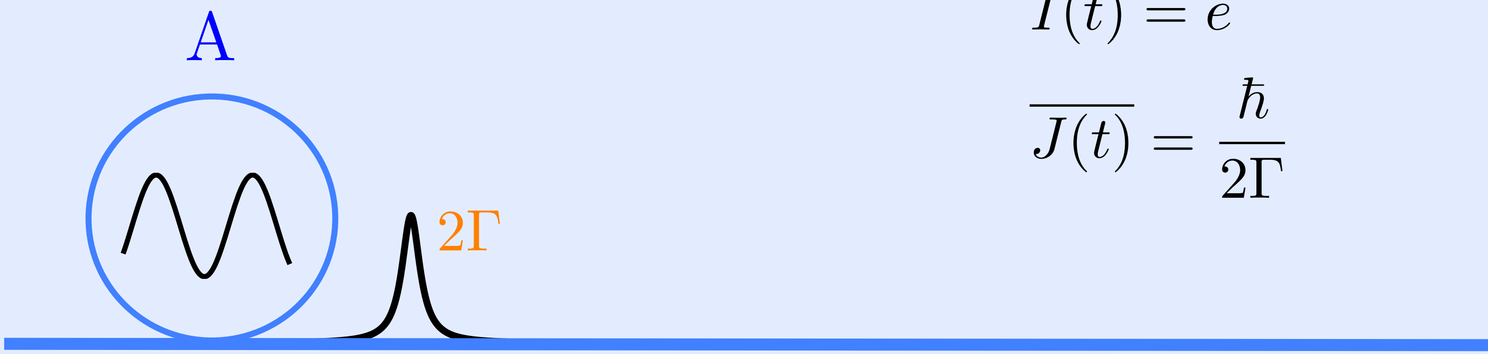
- Multi-particle emitter
    - Reabsorption effect
    - Emission of energy modes (carry no charge)



The reabsorption effect

Single-particle states

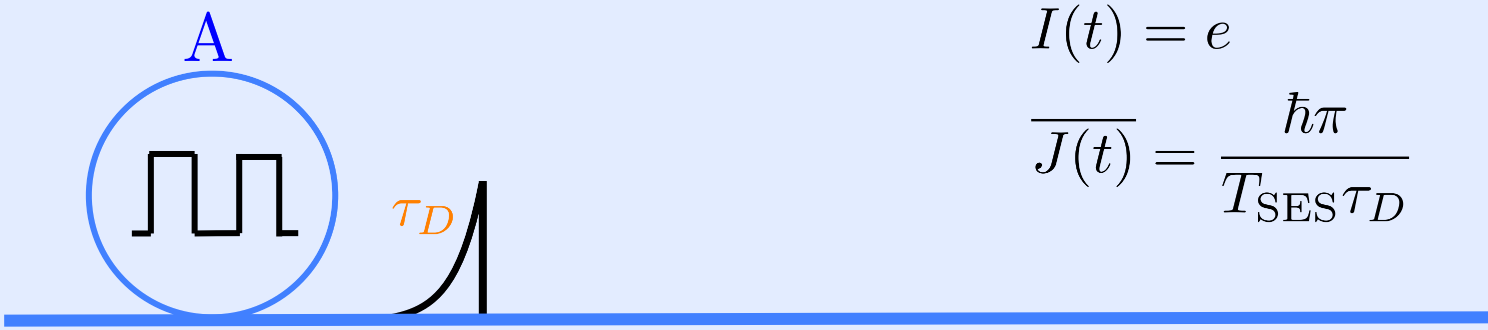
a) Adiabatic emission



$$\overline{I(t)} = e$$

$$\overline{J(t)} = \frac{\hbar}{2\Gamma}$$

b) Non-adiabatic emission

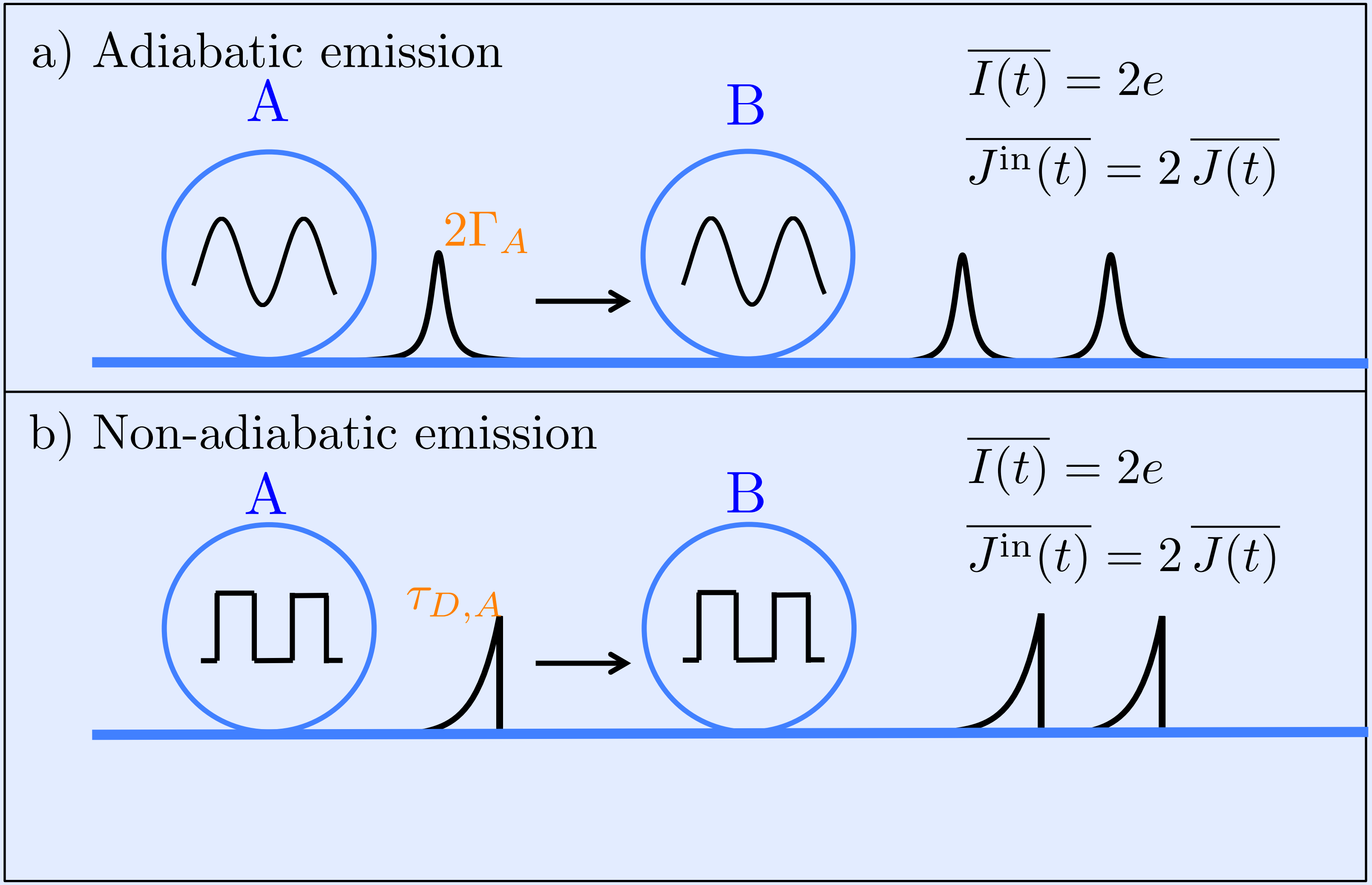


$$\overline{I(t)} = e$$

$$\overline{J(t)} = \frac{\hbar\pi}{T_{\mathrm{SES}}\tau_D}$$

The reabsorption effect

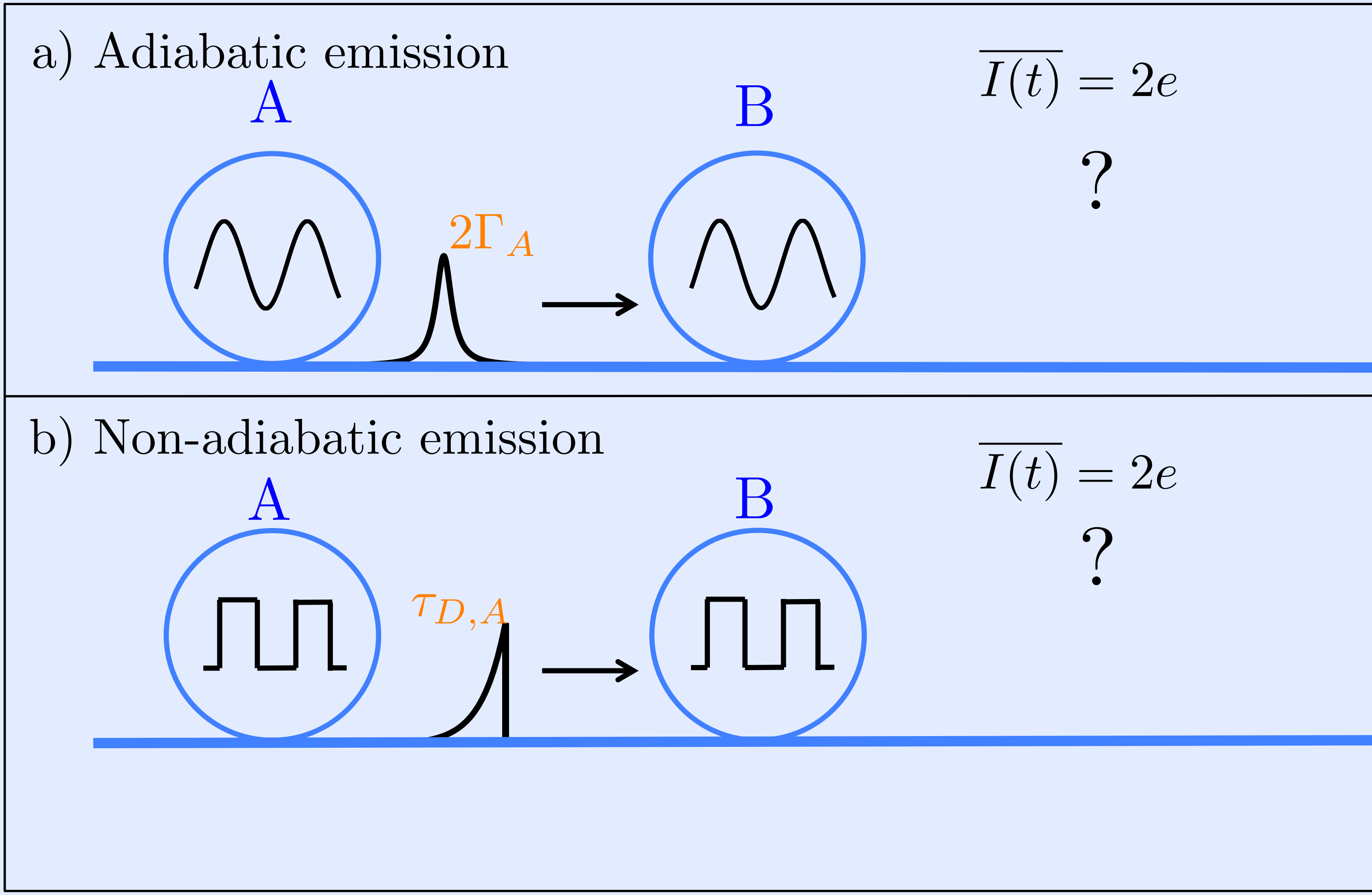
Independent single-particle states



Floquet S-matrix: $S_F^{(2)}(E, E_n) = \sum_{l=-\infty}^{\infty} S_F^B(E, E_l) e^{i\phi_L} e^{il\Omega\tau_L} S_F^A(E_l, E_n)$

The reabsorption effect

Electron-electron pair emission



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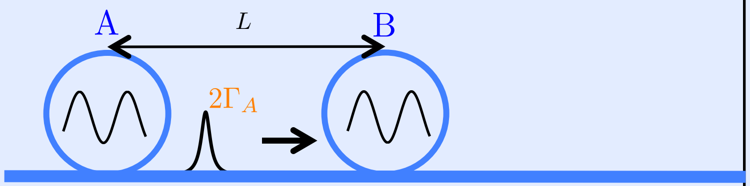
The reabsorption effect



$$S_F^{(2)}(E,E_n) = \sum_{l=-\infty}^{\infty} S_F^B(E,E_l) e^{i\phi_L} e^{il\Omega\tau_L} S_F^A(E_l,E_n)$$

$$\Delta t_{AB}^{++} \equiv t_A^+ - t_B^+$$

$$\Gamma \equiv \Gamma_A + \Gamma_B$$

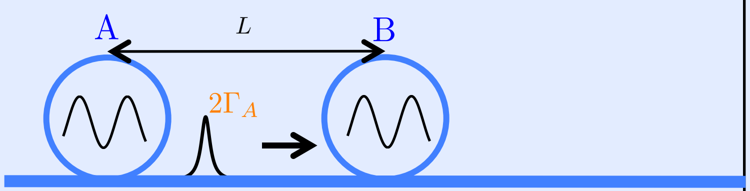
- Adiabatic emission:

$$\overline{I^2(t)} \propto \frac{e^2}{\pi}\left(1 + \frac{2\Gamma_A\Gamma_B}{(\Delta t_{AB}^{++})^2+\Gamma^2} + \frac{2\Gamma_A\Gamma_B}{(\Delta t_{AB}^{--})^2+\Gamma^2} - \frac{2\Gamma_A\Gamma_B}{(\Delta t_{AB}^{+-})^2+\Gamma^2} - \frac{2\Gamma_A\Gamma_B}{(\Delta t_{AB}^{-+})^2+\Gamma^2}\right)$$

$$\overline{J(t)} \propto \frac{h}{2\pi}\left(1 + \frac{2\Gamma_A\Gamma_B}{(\Delta t_{AB}^{++})^2+\Gamma^2} + \frac{2\Gamma_A\Gamma_B}{(\Delta t_{AB}^{--})^2+\Gamma^2} - \frac{2\Gamma_A\Gamma_B}{(\Delta t_{AB}^{+-})^2+\Gamma^2} - \frac{2\Gamma_A\Gamma_B}{(\Delta t_{AB}^{-+})^2+\Gamma^2}\right)$$

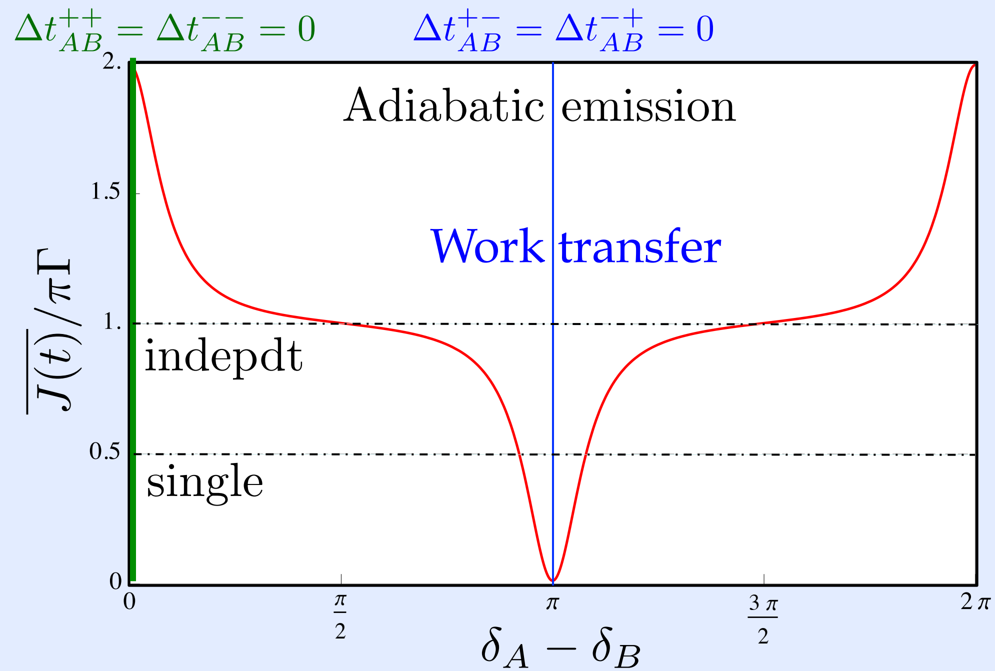
Splettstoesser *et al.*, PRB **78** (2008), Moskalets and Büttiker, PRB **80** (2009)

The reabsorption effect



$$V_{A/B}(t) = V_{A/B}\cos(\Omega t + \delta_{A/B})$$

$$S_F^{(2)}(E, E_n) = \sum_{l=-\infty}^{\infty} S_F^B(E, E_l) e^{i\phi_L} e^{il\Omega\tau_L} S_F^A(E_l, E_n)$$



- Adiabatic emission:

$$\overline{I^2(t)} \propto \frac{e^2}{\pi}\left(1 + \frac{2\Gamma_A\Gamma_B}{(\Delta t_{AB}^{++})^2 + \Gamma^2} + \frac{2\Gamma_A\Gamma_B}{(\Delta t_{AB}^{--})^2 + \Gamma^2} - \frac{2\Gamma_A\Gamma_B}{(\Delta t_{AB}^{+-})^2 + \Gamma^2} - \frac{2\Gamma_A\Gamma_B}{(\Delta t_{AB}^{-+})^2 + \Gamma^2}\right)$$

$$\overline{J(t)} \propto \frac{h}{2\pi}\left(1 + \frac{2\Gamma_A\Gamma_B}{(\Delta t_{AB}^{++})^2 + \Gamma^2} + \frac{2\Gamma_A\Gamma_B}{(\Delta t_{AB}^{--})^2 + \Gamma^2} - \frac{2\Gamma_A\Gamma_B}{(\Delta t_{AB}^{+-})^2 + \Gamma^2} - \frac{2\Gamma_A\Gamma_B}{(\Delta t_{AB}^{-+})^2 + \Gamma^2}\right)$$

Joule-Lenz law valid: $\overline{J(t)} = \frac{h}{2e^2}\overline{I^2(t)}$

Splettstoesser *et al.*, PRB **78** (2008), Moskalets and Büttiker, PRB **80** (2009)

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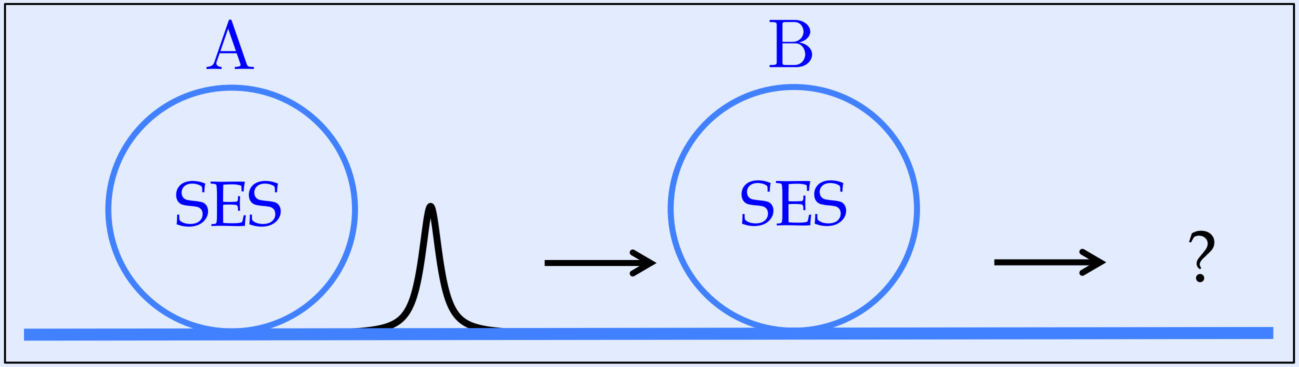
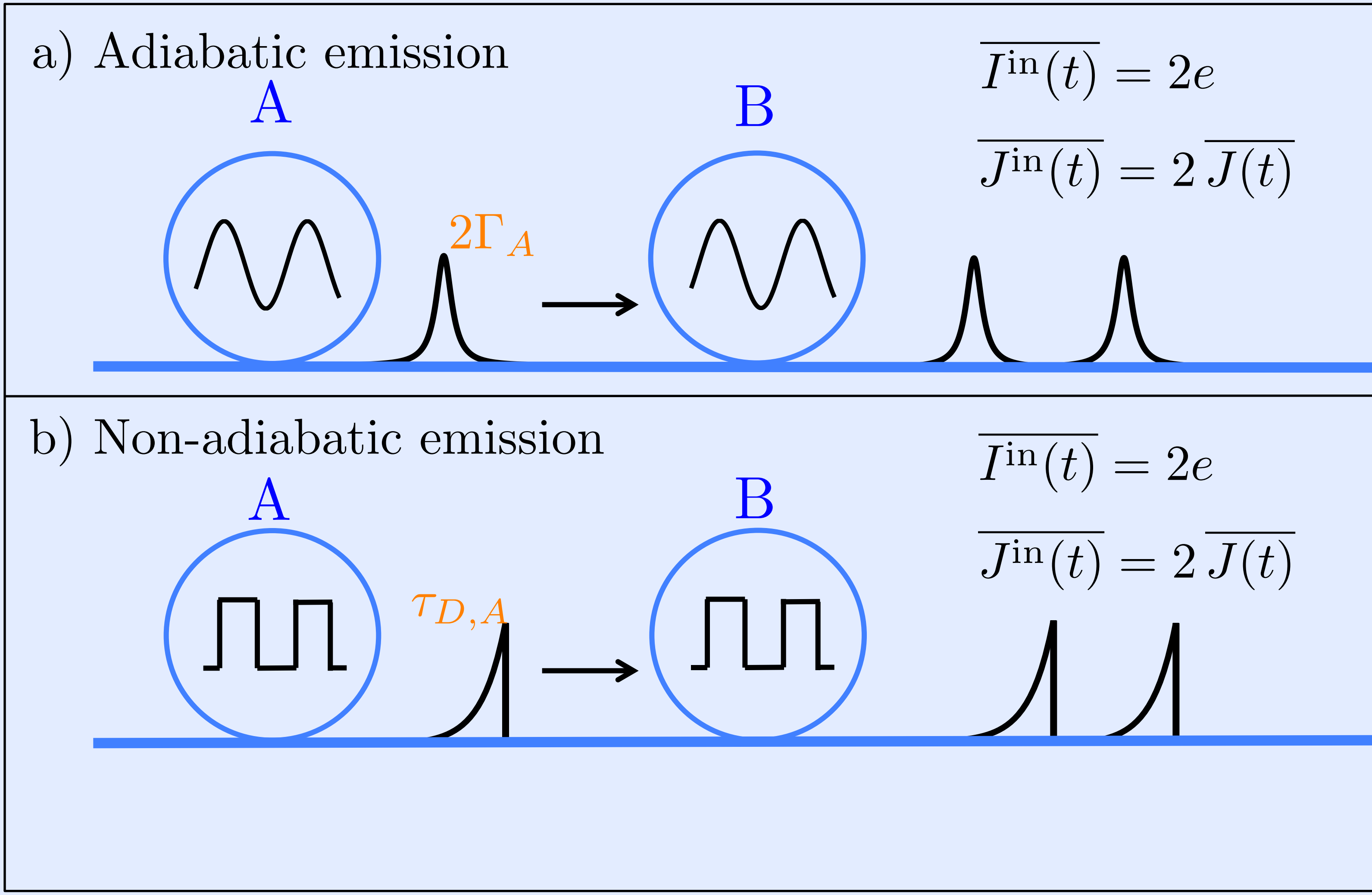


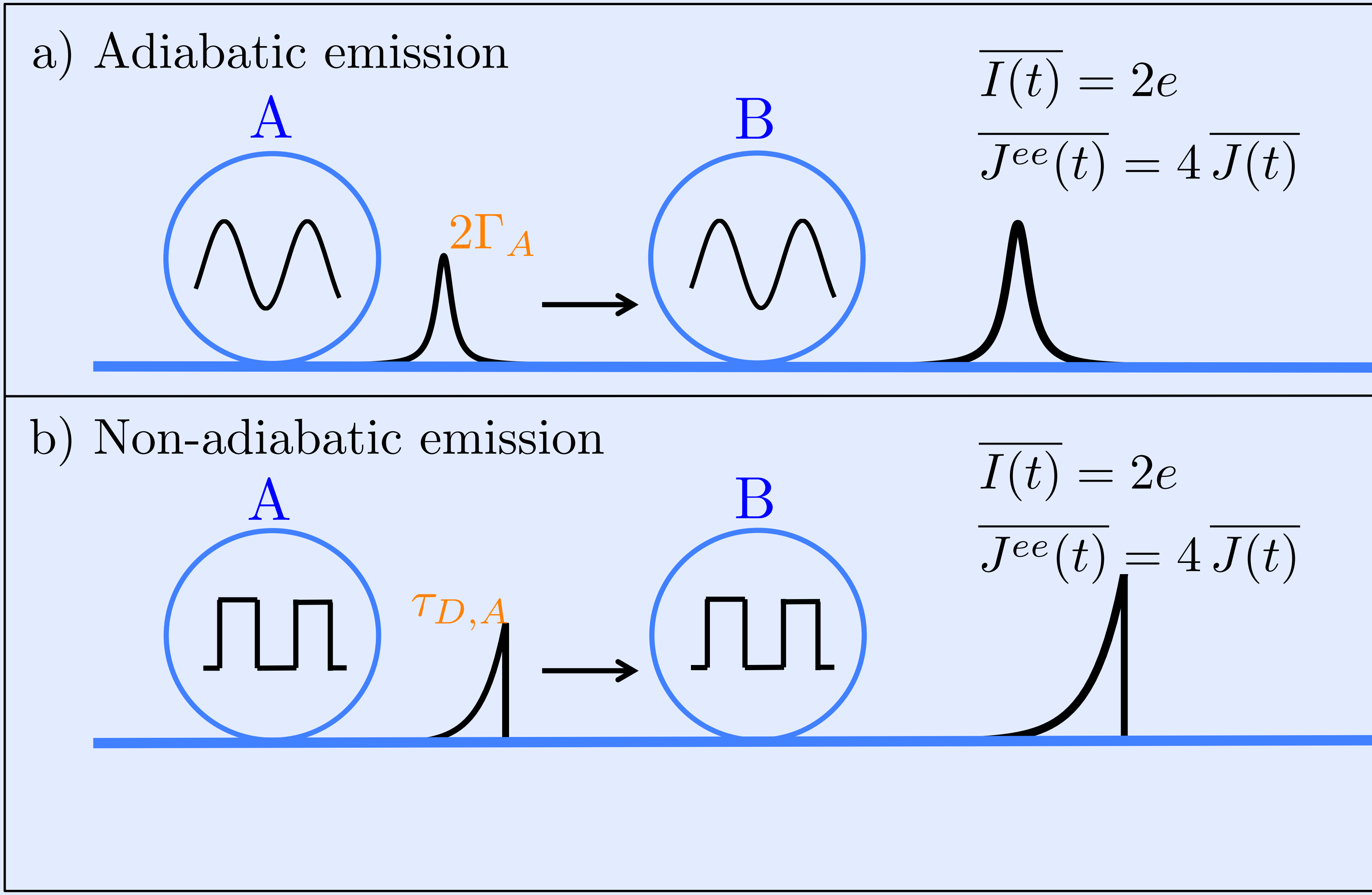
The reabsorption effect

Independent single-particle states



The reabsorption effect

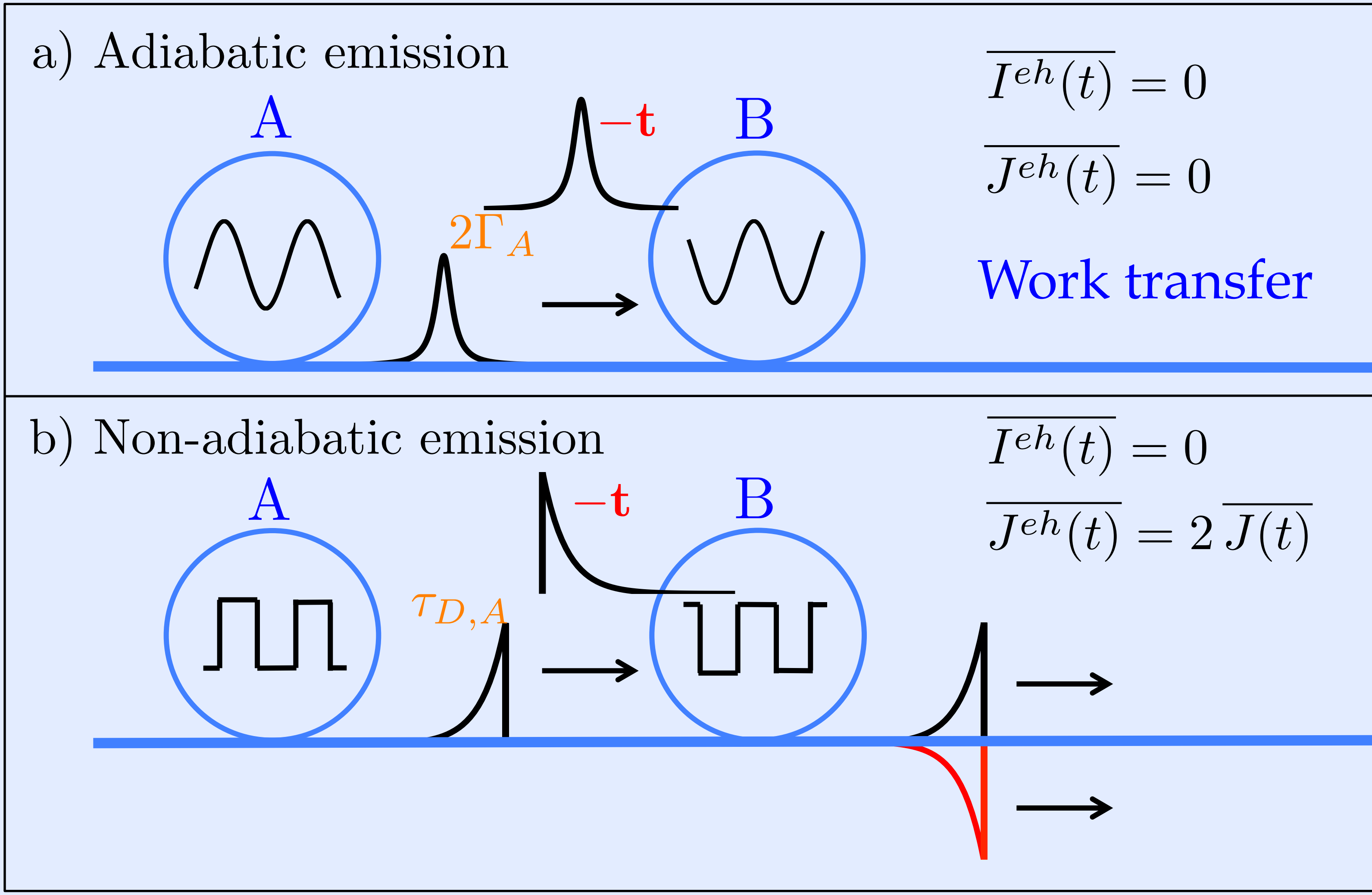
Electron-electron pair emission



Factor 2 due to the Pauli principle

The reabsorption effect

Electron-hole pair emission

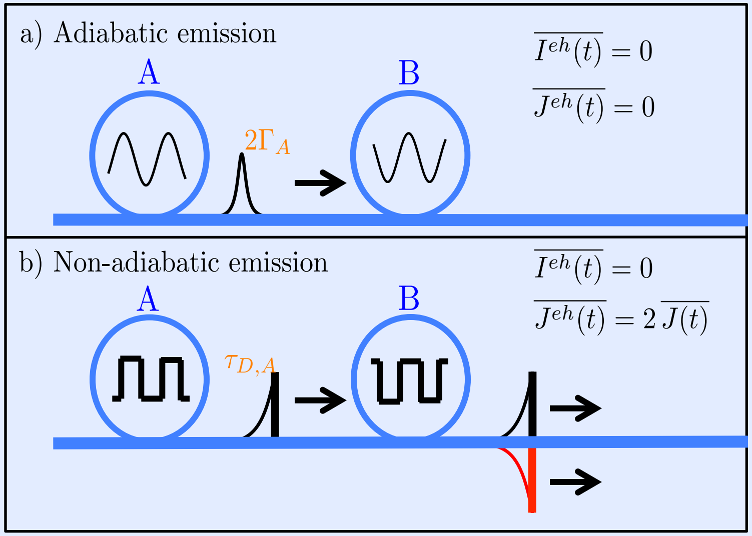


Reabsorption in the adiabatic regime

No-charge but energy mode in the non-adiabatic regime

Main results

- Time-dependent heat current and charge current
- Reabsorption effect, work transfer
- Joule-Lenz law



M. Moskalets, G. Haack, and M. Büttiker, Phys. Rev. B **87**, 125429 (2013)

Outlook:

- Adiabatic vs non-adiabatic: linear vs non-linear response ?
- Validity of the Fluctuation-dissipation theorem ?
- Generation of energy modes