Information thermodynamics in a hybrid opto-mechanical system

Cyril Elouard, Maxime Richard, Alexia Auffèves

Team NPSC, Neel Institute – CNRS, Grenoble, France

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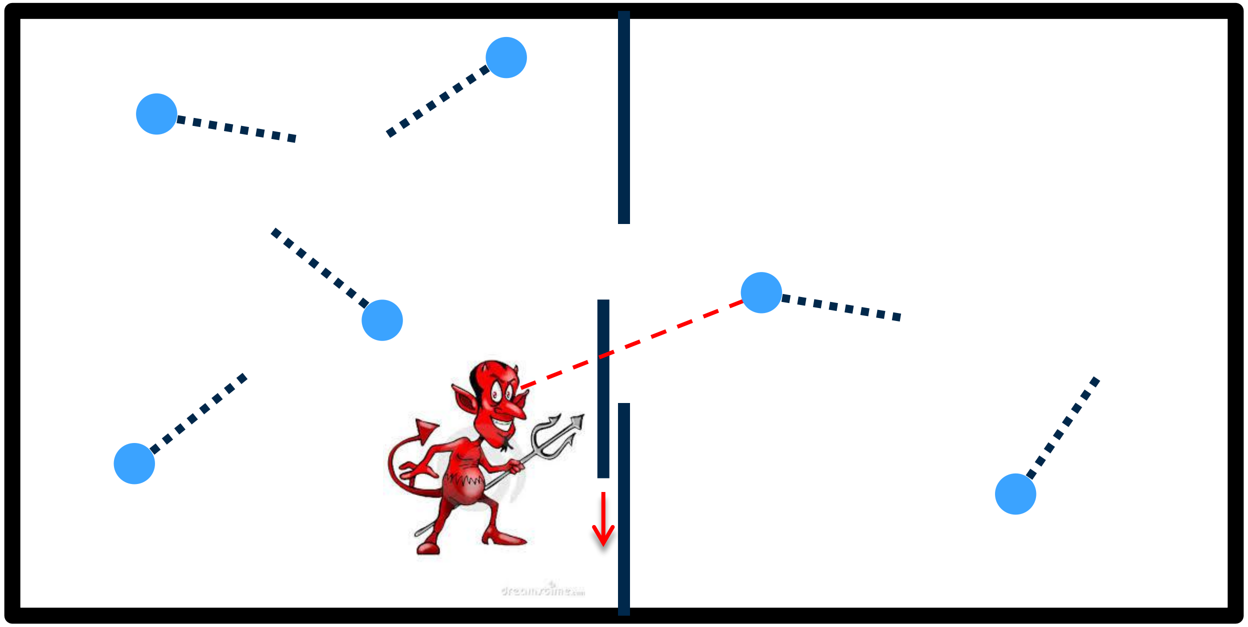
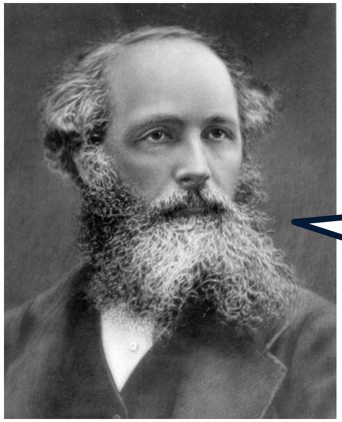
Maxwell's demon paradox

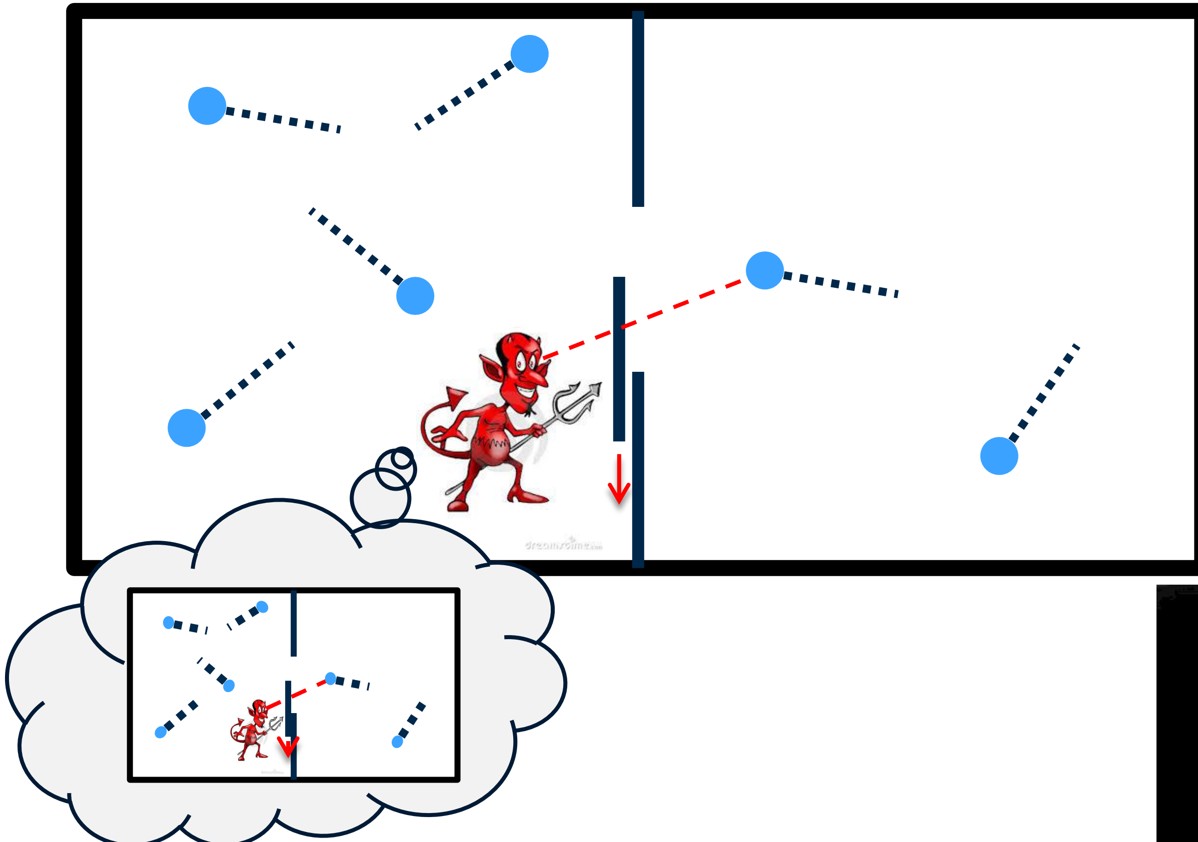




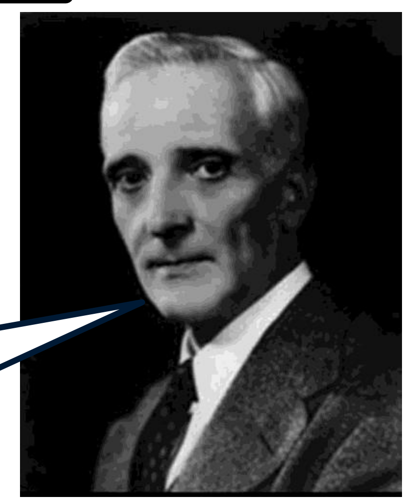
The demon can decrease the entropy of the gas without paying work !

J.C. Maxwell - 1871

The demon's memory

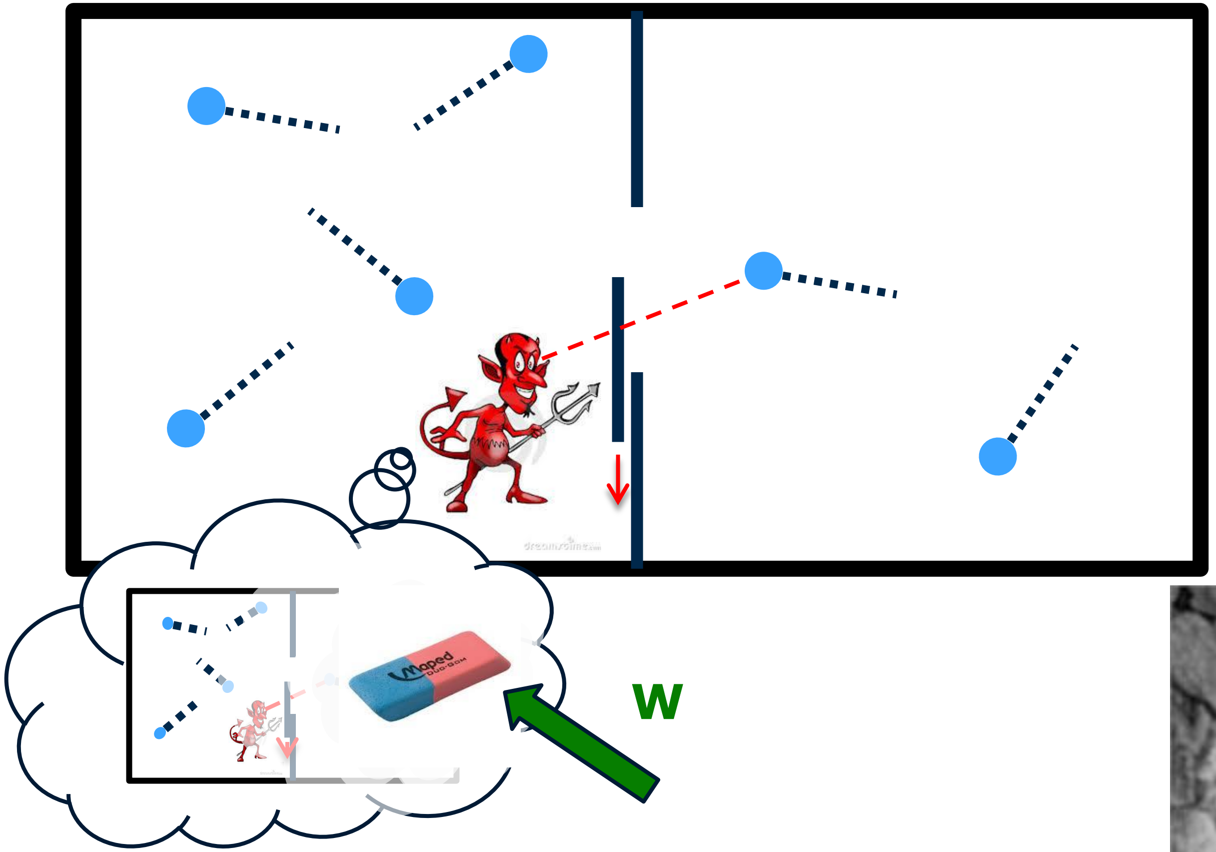


Actually, the demon stores the information about the particles in its memory…



L. Brillouin - 1949

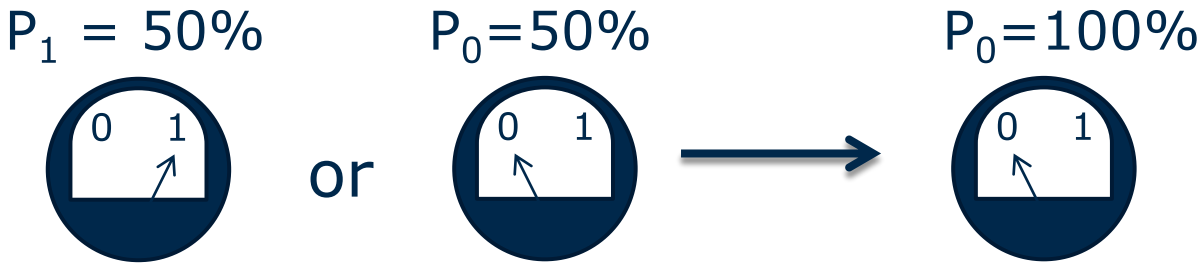
The demon's memory



…and work is required to erase the memory and thus to perform cycles !

R. Landauer - 1961

Landauer's Erasure of 1 bit



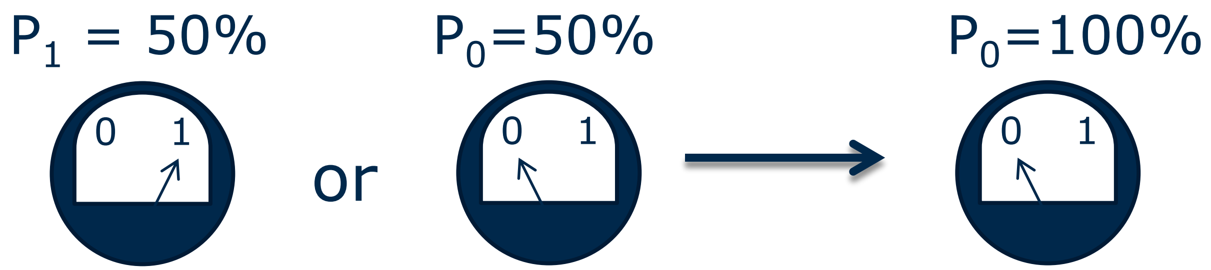
$$H_i = 1 \longrightarrow H_f = 0$$

Where Shannon's entropy of the bit is:

$$H = -P_1 \log_2 P_1 - P_0 \log_2 P_0$$

(in bits)

Landauer's Erasure of a bit



$$H_i = 1 \longrightarrow H_f = 0$$

Work required W

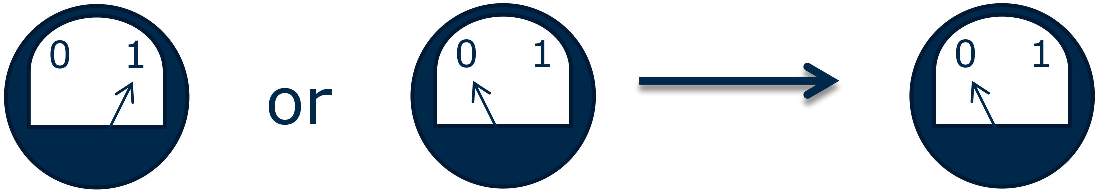
Rolf Landauer

Landauer's principle

$$W \geq W_0 = kT \ln 2$$

Landauer's Erasure of a bit

$P_1 = 50\%$ or $P_0=50\%$ → $P_0=100\%$



$$H_i = 1 \longrightarrow H_f = 0$$

Work required W

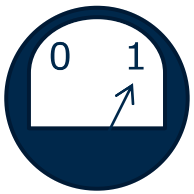
Rolf Landauer

If the erasure is a *reversible* (very slow) transformation:

$$W = kT \ln 2$$

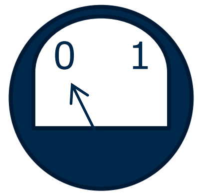
Szilard 's engine

$P_1 = 50\%$

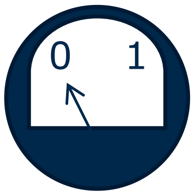


or

$P_0 = 50\%$



$P_0 = 100\%$



$$H_i = 1 \leftarrow H_f = 0$$

Work *extracted* W

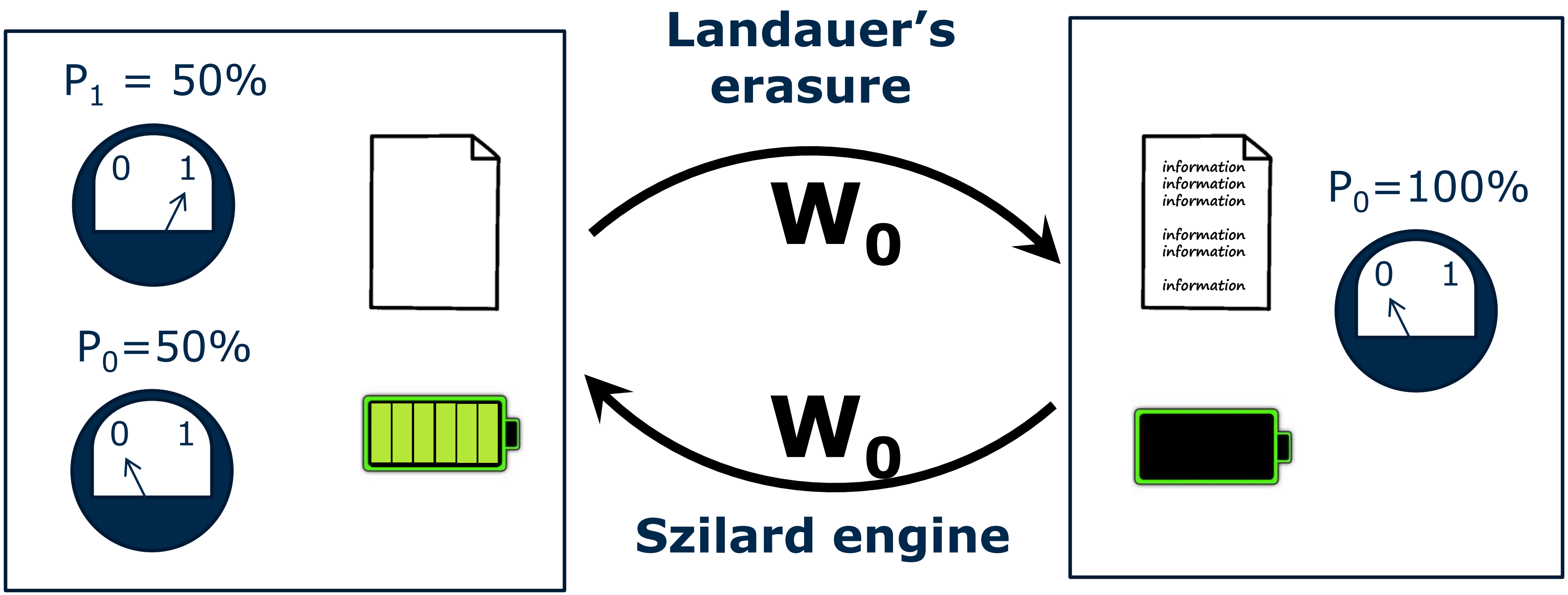
Rolf Landauer

If the erasure is a *reversible* (very slow) transformation:

$$W = kT \ln 2$$

Leo Szilard

Information - energy conversions



W_0 = kT ln 2 is the elementary work corresponding to 1 bit of information

If information becomes quantum…

<u>EPR state</u>

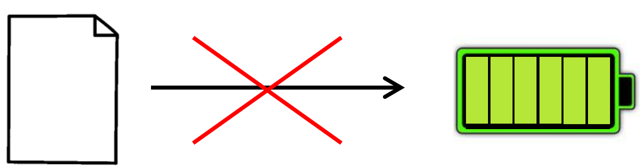
$$\rho_{AB} = |\Psi\rangle\langle\Psi|$$

$$\Psi = (|0_A 1_B\rangle + |1_A 0_B\rangle)/\sqrt{2}$$

<u>Alice’s point of view</u>

$$\mathrm{Tr}_B\, \rho_{AB} = \mathbb{I}/2$$

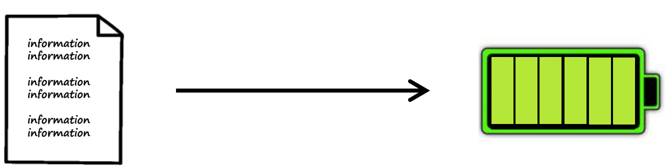
Maximally mixed state,
no work extraction possible



<u>Global point of view</u>

Pure state, H = 0

Can perform a Szilard engine and convert the information into work



EPR state

$\rho_{AB} = |\Psi\rangle\langle\Psi|$

The peculiarities of quantum information let their imprint on the work we can extract / have to pay

→ Many theoretical results linking work to quantum correlations, discord, entanglement…

L. del Rio et al., Nature 474, 61--63 (2011)
Oppenheim, Horodecki, PRL 89 (2002)
Zurek, PRA 67 (2003) .....

Alice'

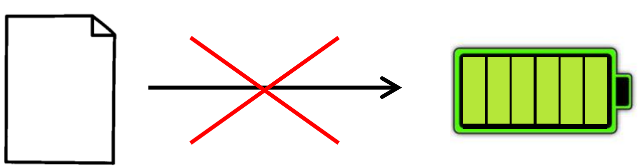
$\mathrm{Tr}_B\, \rho_A$

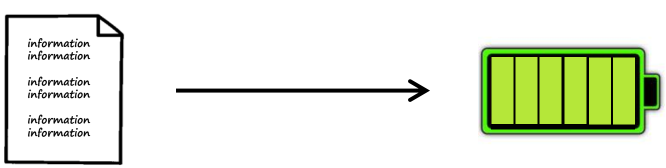
f view

$= 0$

Maximally
no work ex

zilard engine
information
into work





Motivations

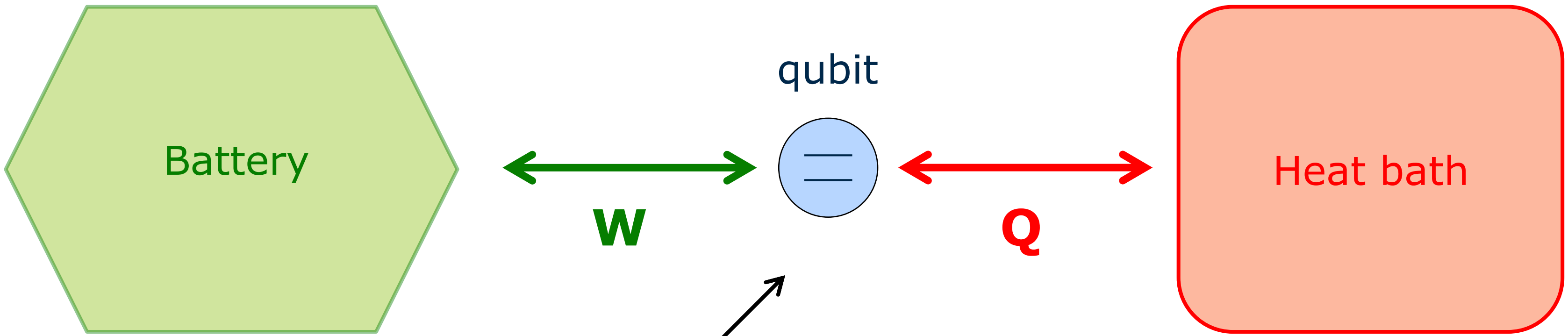
→ Many theoretical results linking work to quantum correlations, discord, entanglement…

L. del Rio et al., Nature 474, 61--63 (2011)
Oppenheim, Horodecki, PRL 89 (2002)
Zurek, PRA 67 (2003) .....

Experimental verification of this theorems remains elusive…

Need of a proper setup in which:
- Some qubits exchange work with a battery, heat with a bath.
- Work exchanges can be measured.

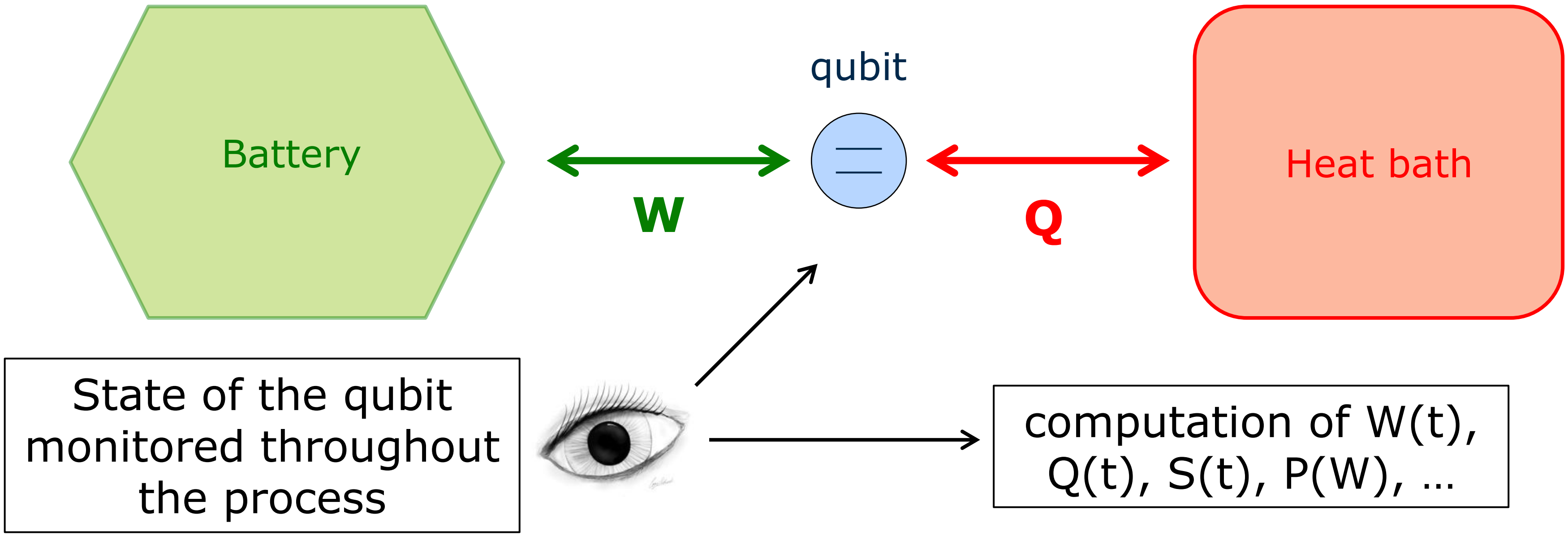
Strategy 1



State of the qubit monitored throughout the process:
→ trajectory in phase space

computation of W(t), Q(t), S(t), P(W), …

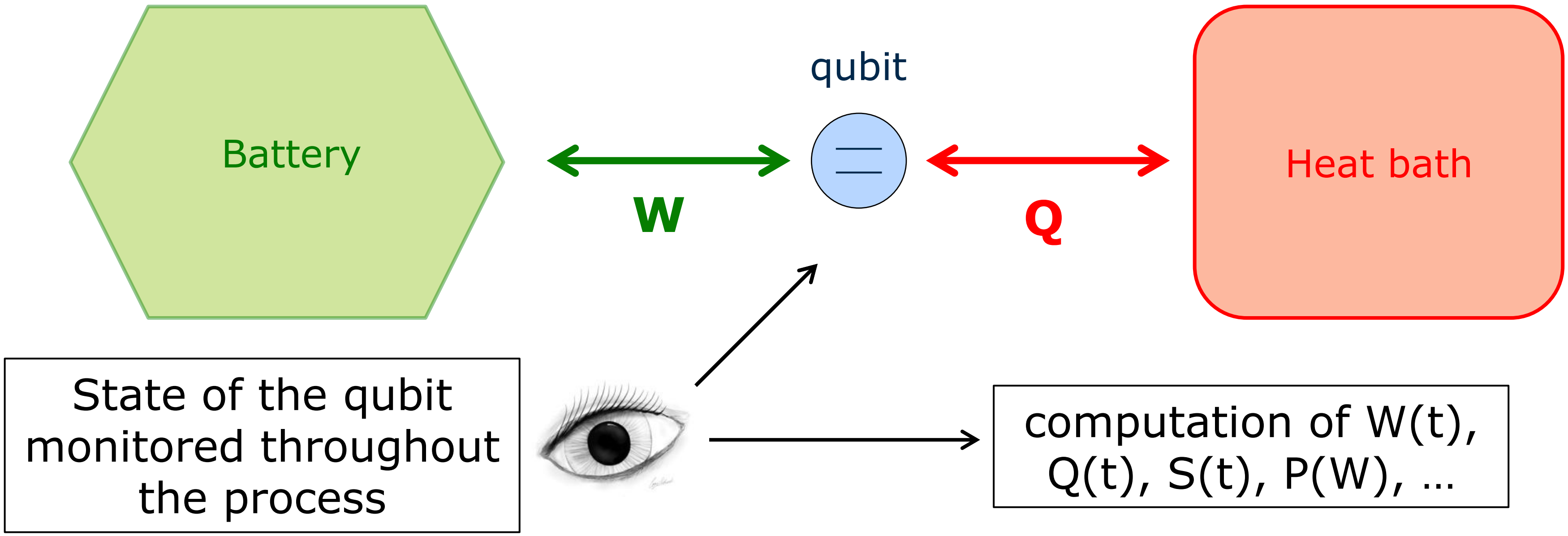
Strategy 1



Experimental difficulties:

- Reaching reversibility
- Ultrafast QND measurement to get the qubit trajectory

Strategy 1

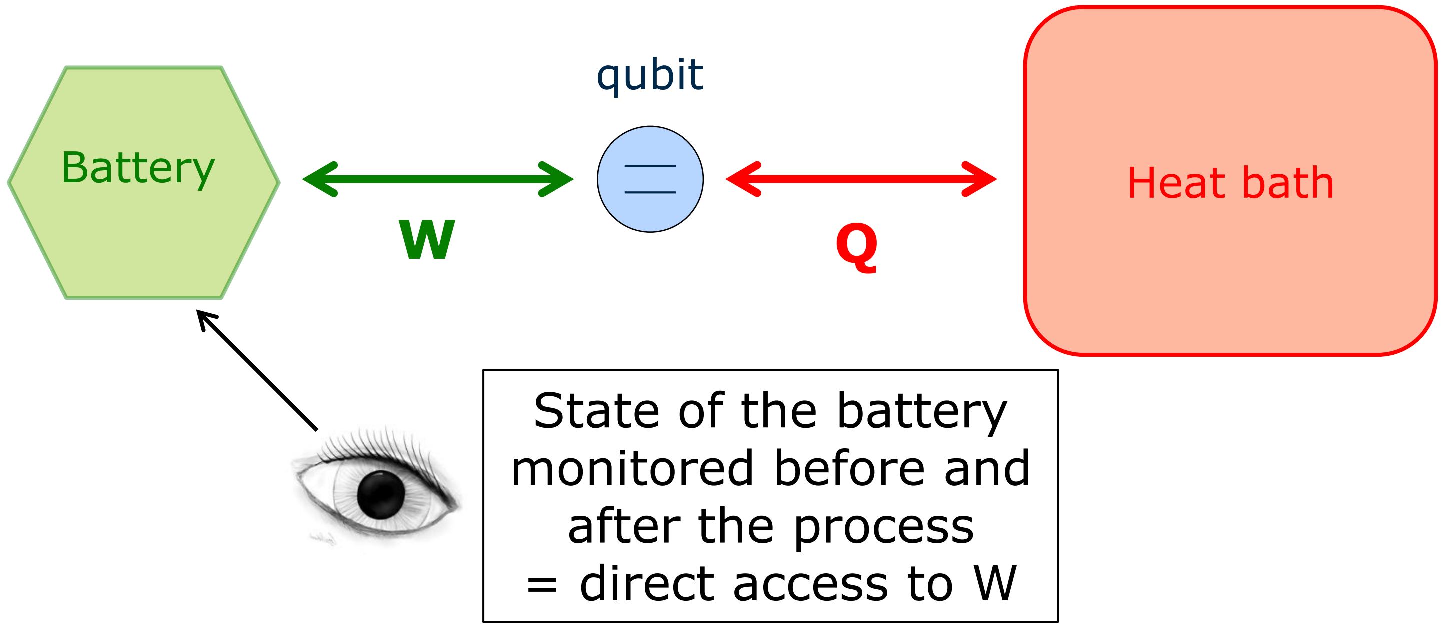


Implementations only with a *classical bit*

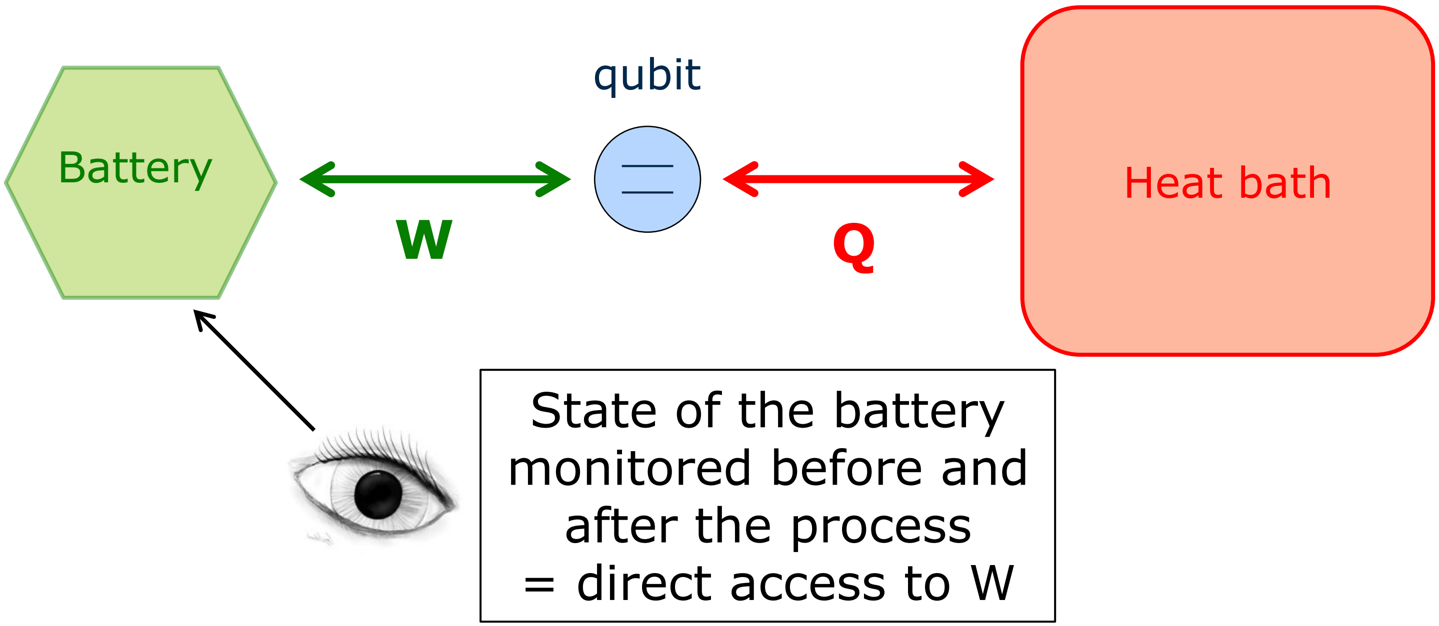
Toyabe, S. et al., Nature Physics 6, 988992 (2010) → (Irreversible) Szilard engine

Bérut et al., Nature 483, 187-189 (2012) → Reversible Landauer’s erasure

Strategy 2


qubit
Battery
W
Q
Heat bath
State of the battery
monitored before and
after the process
= direct access to W

Strategy 2

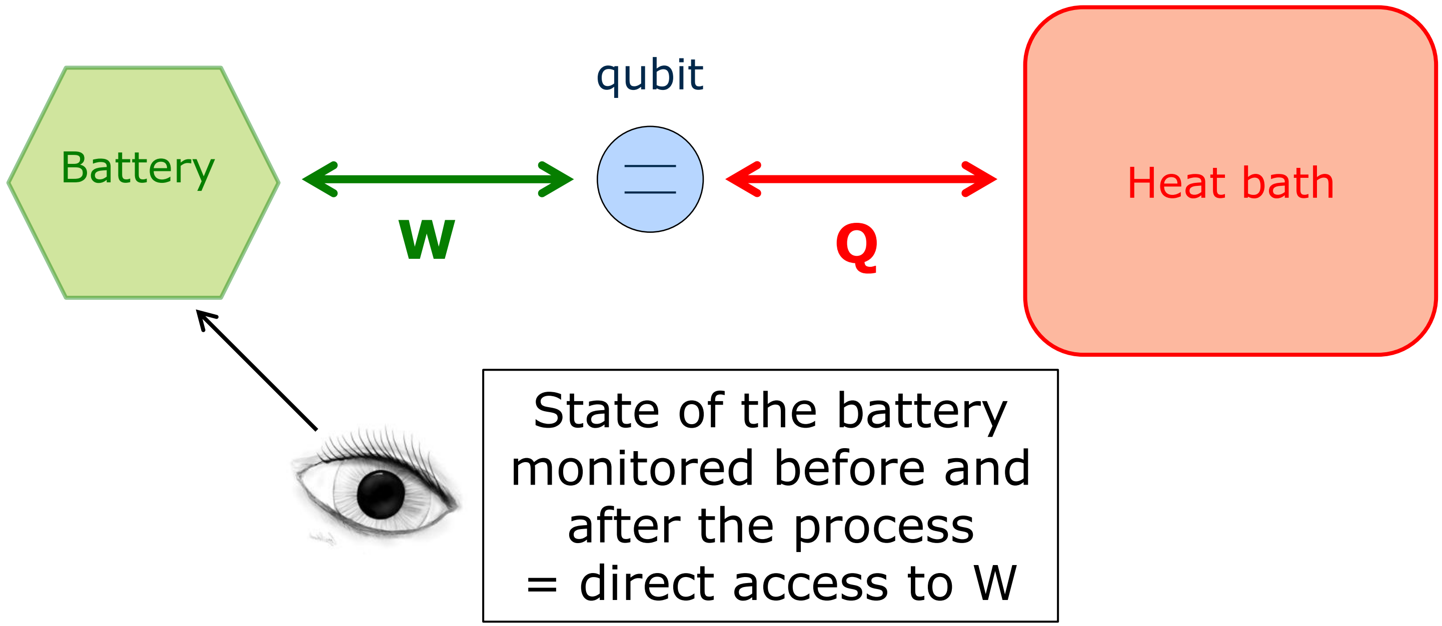


<u>Experimental difficulties:</u>

- Reaching reversibility
- Finding a battery which can be easily monitored

Experimental test of these theorems

Strategy 2



<u>Experimental difficulties:</u>

- Reaching reversibility
- Finding a battery which can be easily monitored

→ Idea: using a mechanical oscillator coupled to the qubit!

Outline

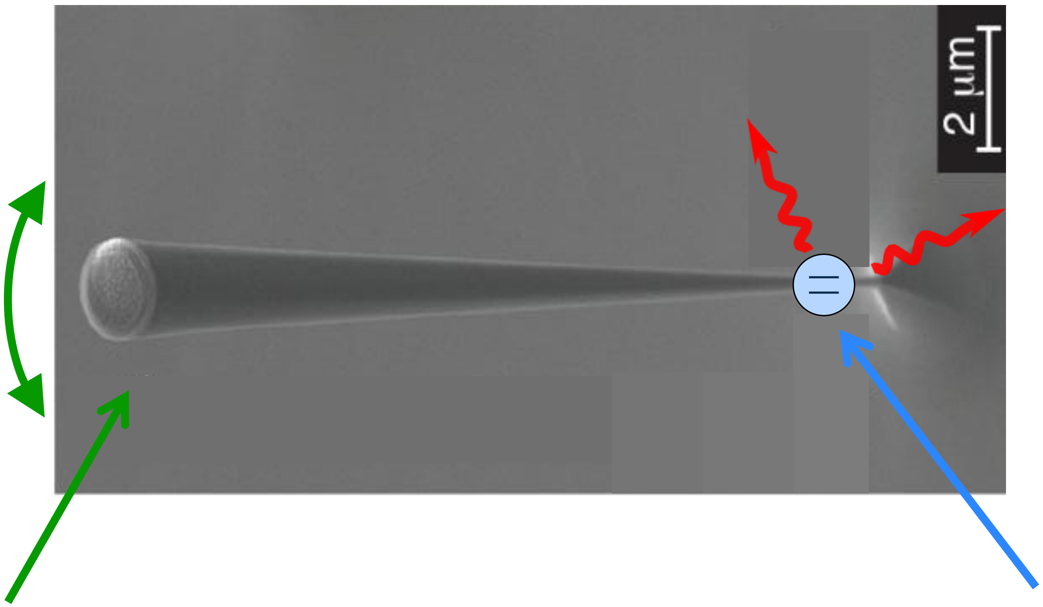
- I) Measuring work in a hybrid opto-mechanical system

- II) Information to energy conversions in a thermal bath

- III) Information to energy conversions in a driven system

Set up : nano 'trumpets'

I.Yeo et al., Nature Nanotechnology 9, 106–110 (2014)

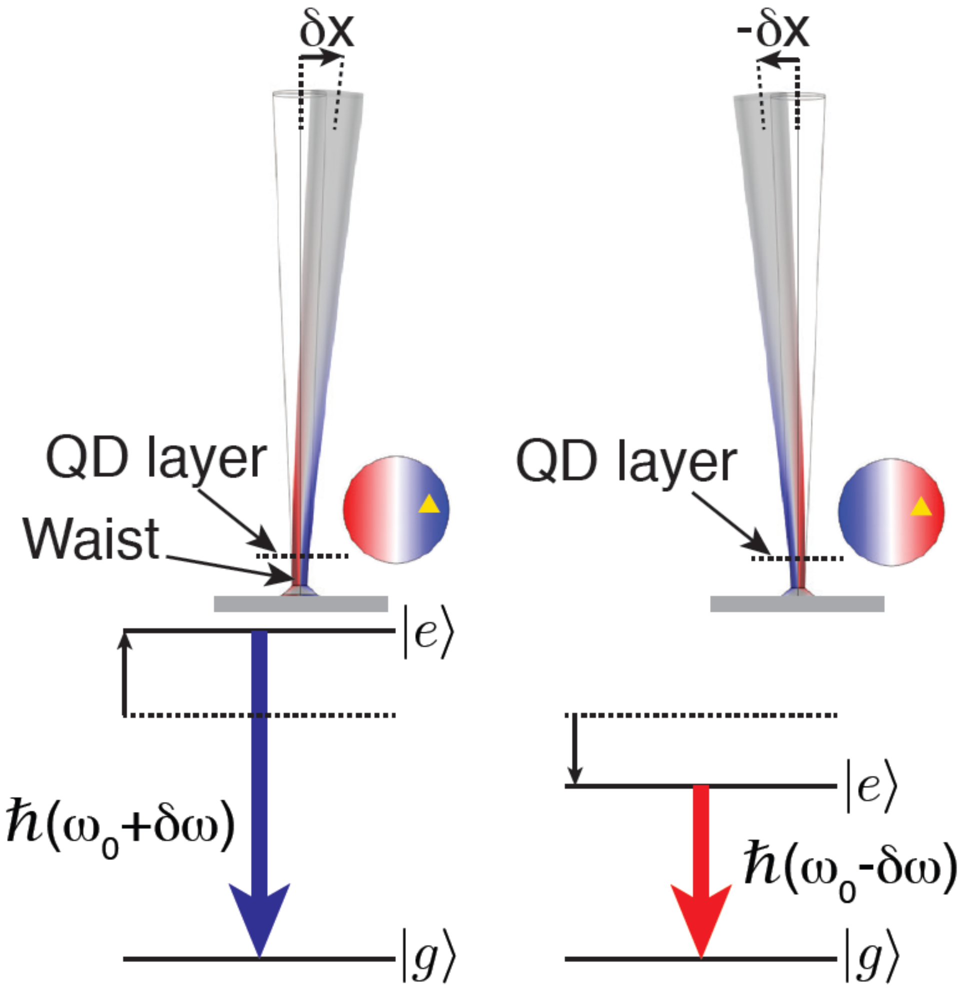


Thermal bath :
Electromagnetic
reservoir

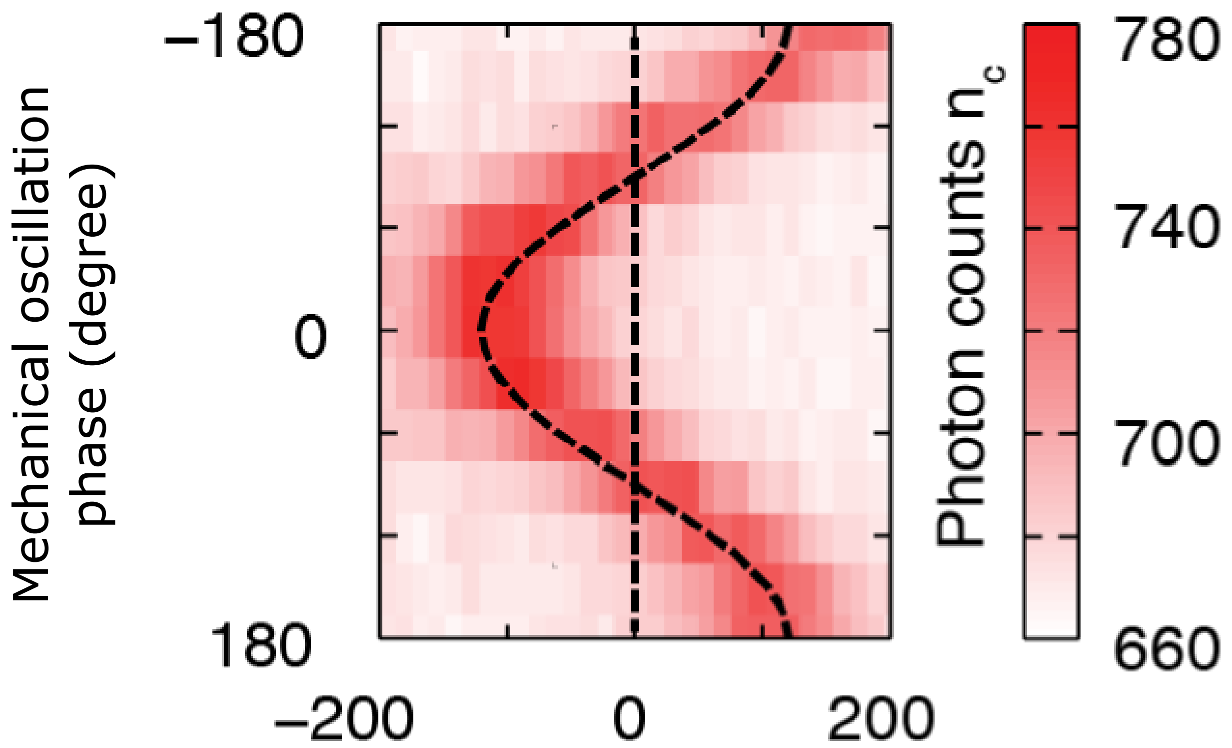
Qubit:
Artificial atom
(Quantum dot).

Battery:
Oscillating nanowire.

Strain-mediated coupling



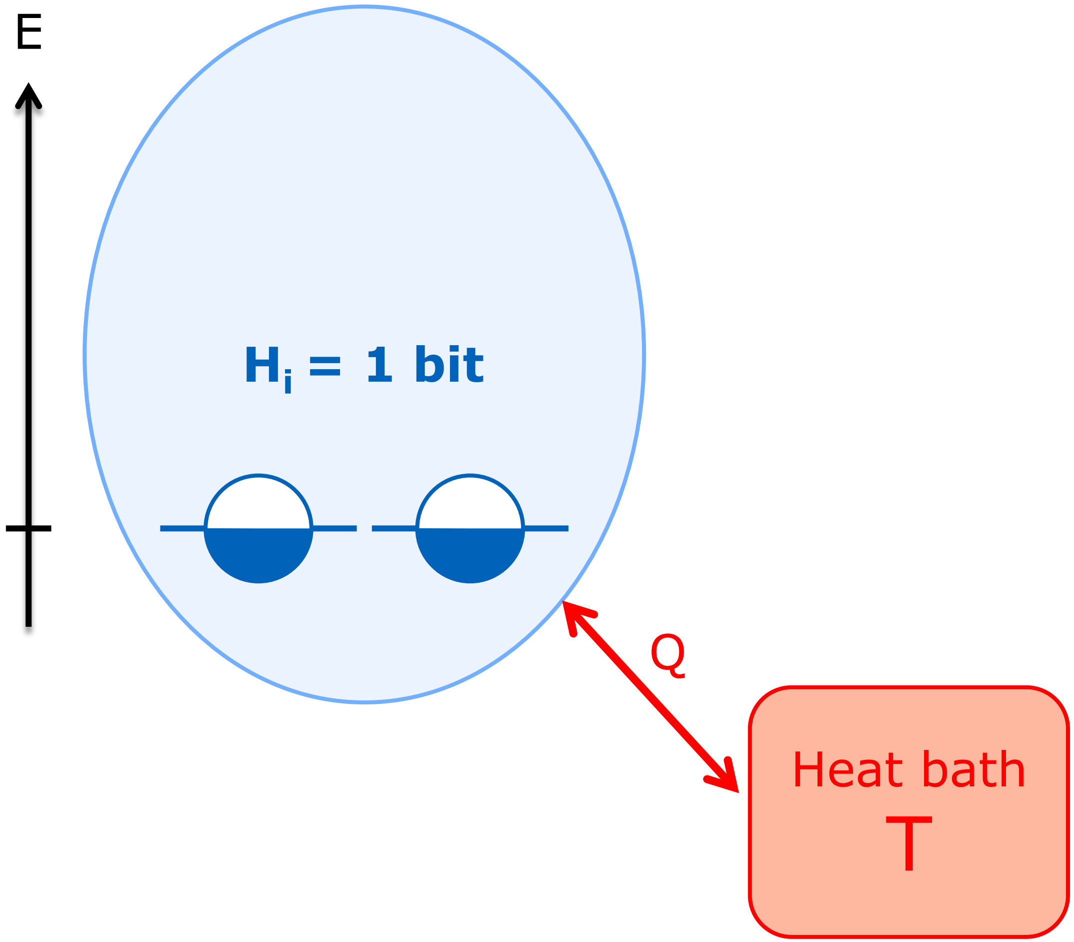
Fluorescence spectroscopy of the embedded atom



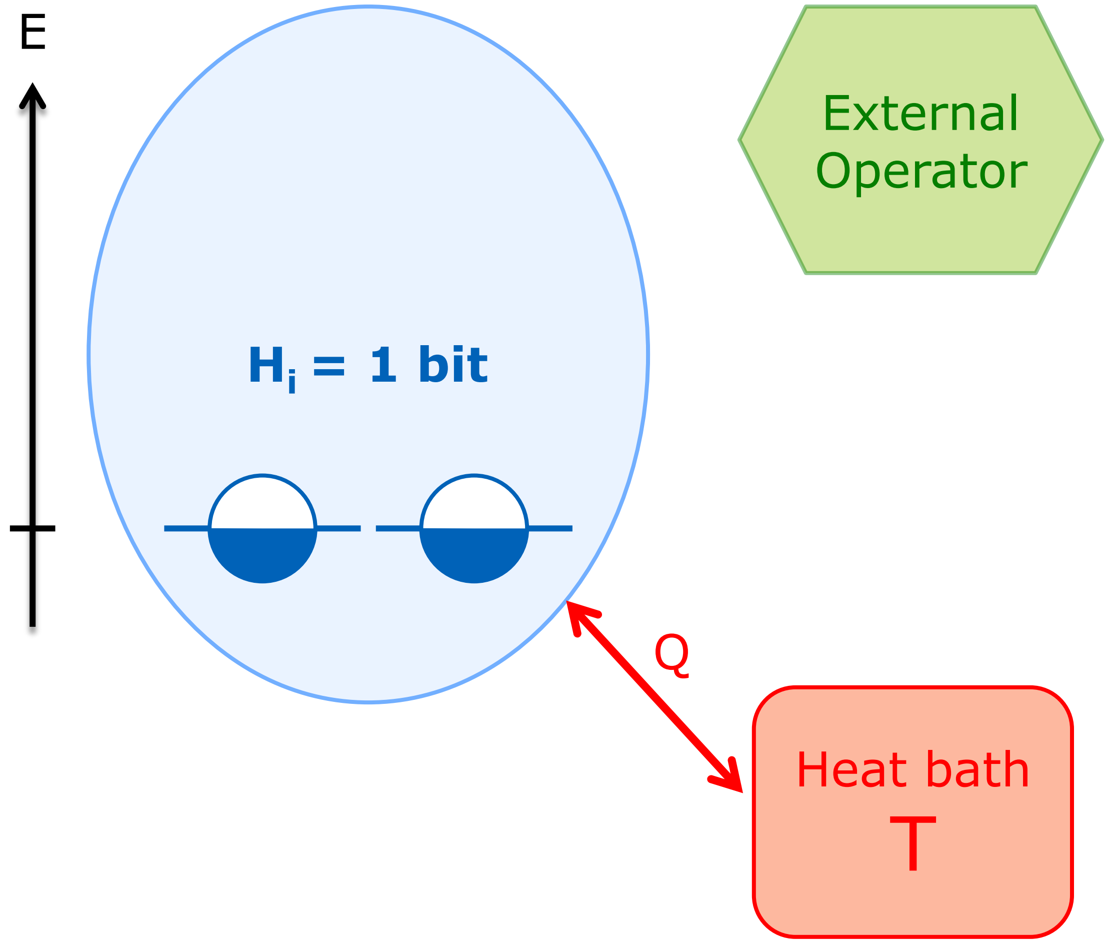
Atomic frequency variation $\omega_q(t) - \omega_0$ (μeV)

Source: I.Yeo et al., Nature Nanotechnology 9, 106–110 (2014)

Paradigmatic erasure protocol

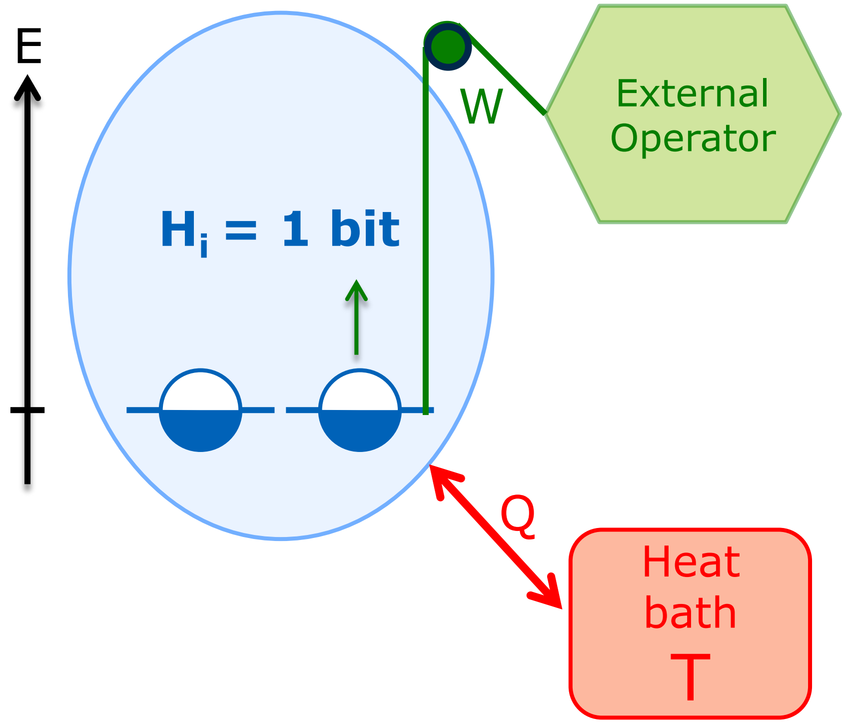


t = 0



t = 0

Paradigmatic erasure protocol



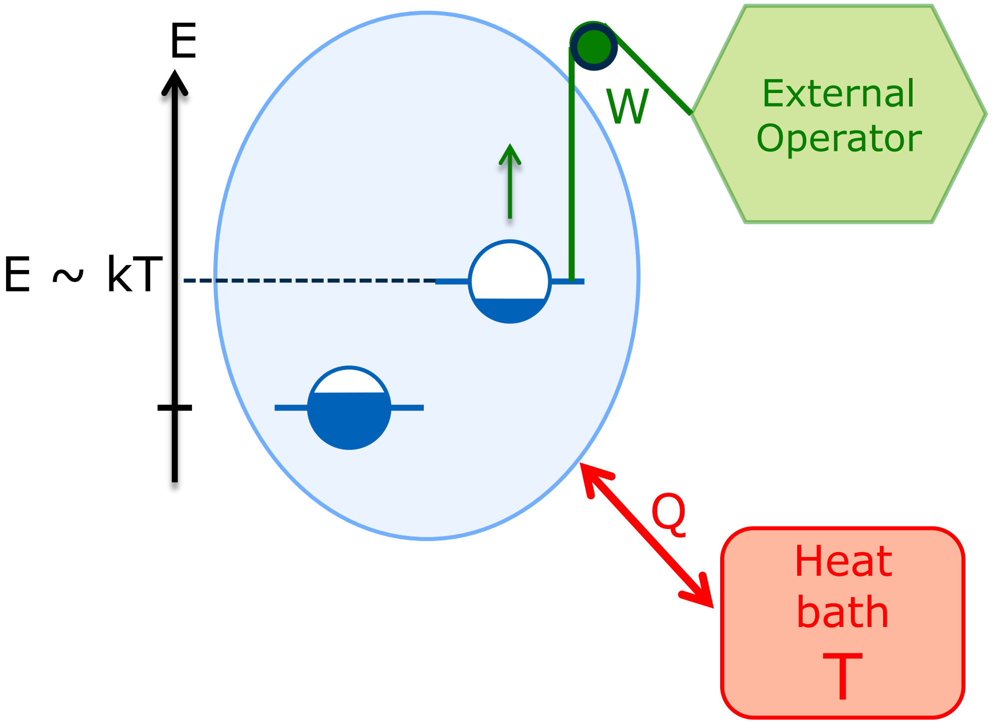
Work performed by the operator while raising one of the states

$$W(t) = \int_0^t P(E)\, dE$$

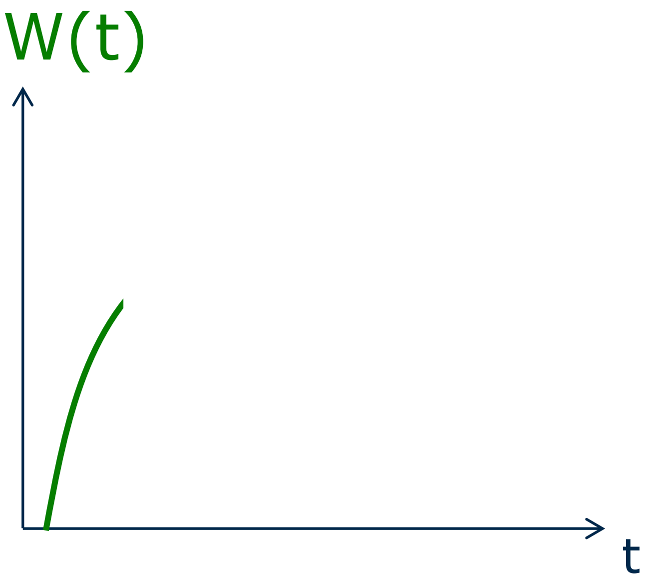
Population of the state

t = 0

Paradigmatic erasure protocol



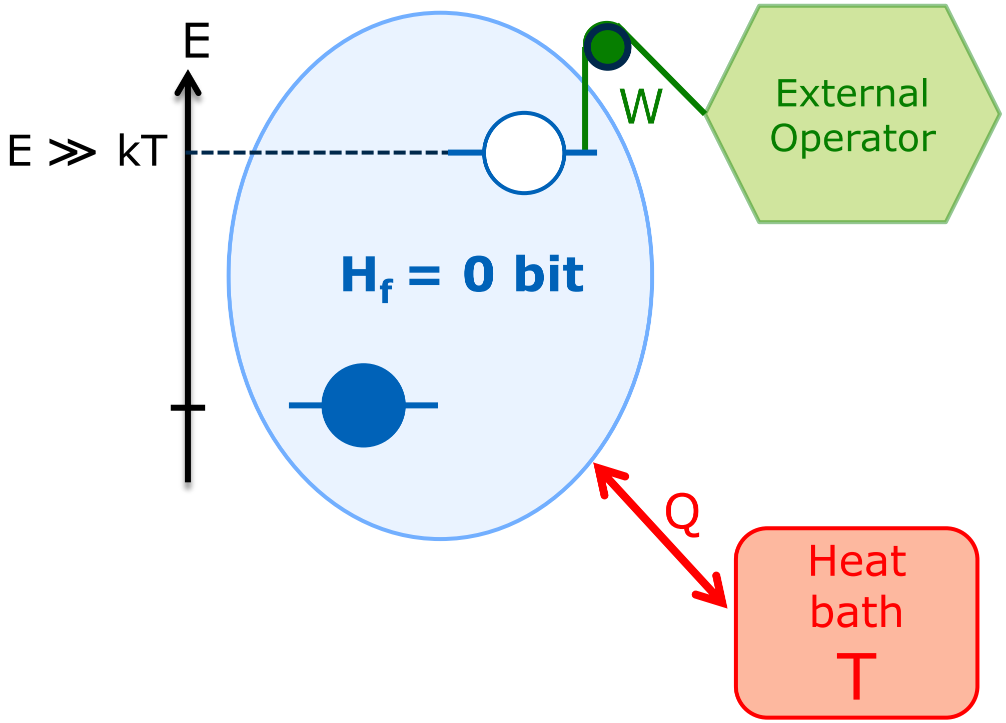
Work performed by the operator

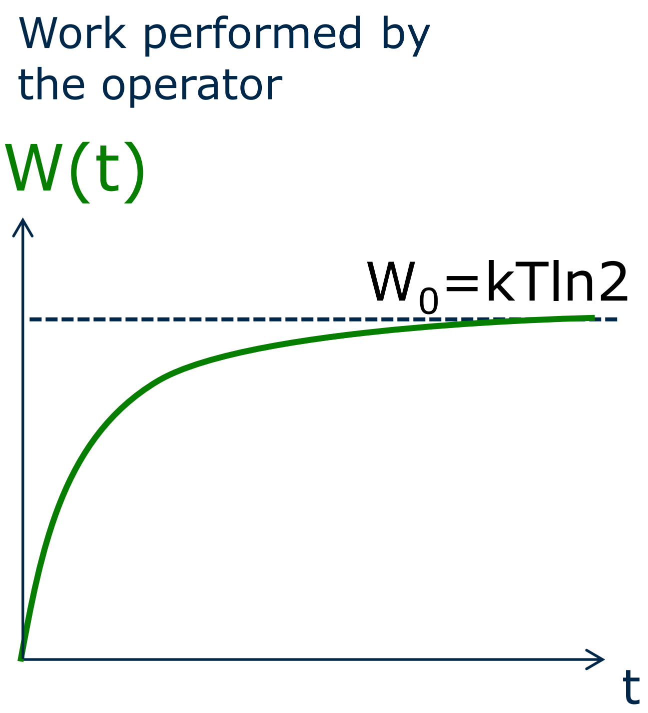


The qubit is at any time in equilibrium with the bath

$$P(E) = e^{-E/kT}/Z$$

$$0 < t < t_f$$



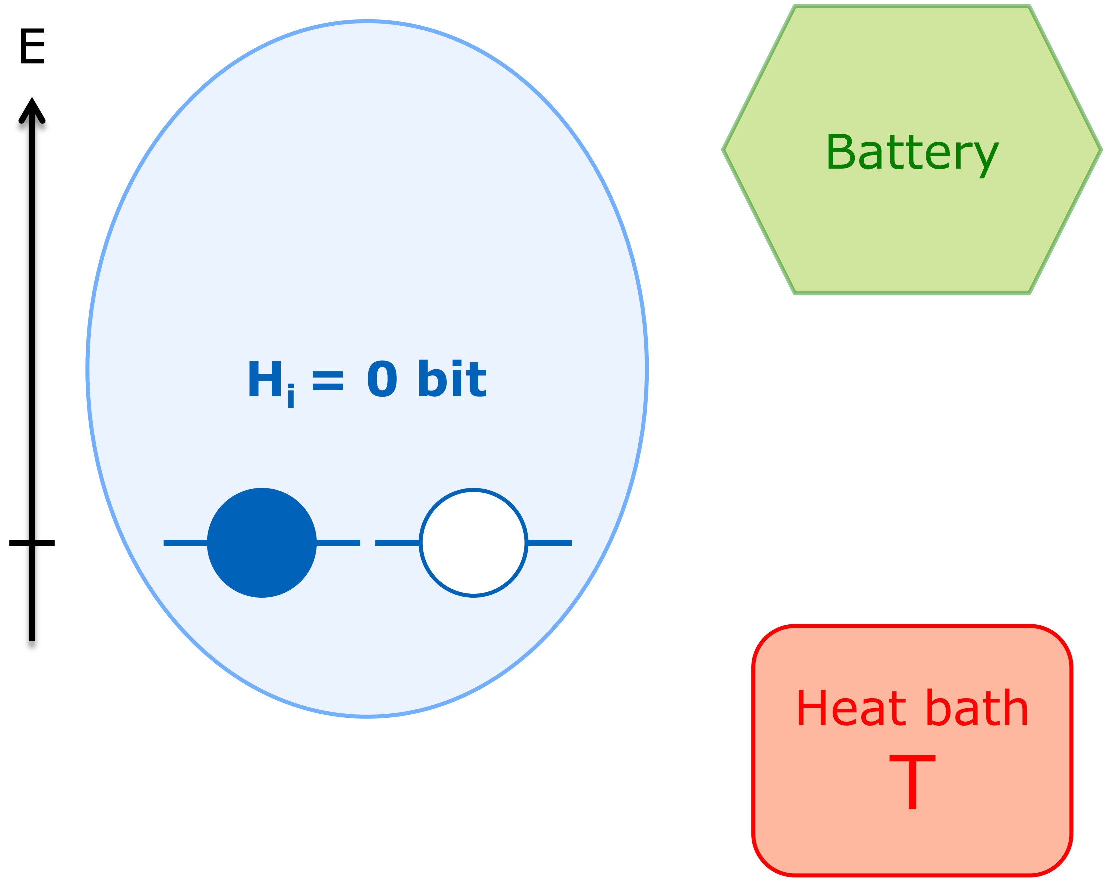


The qubit is at any time in equilibrium with the bath

$$P(E) = e^{-E/kT}/Z$$

$$t = t_f$$

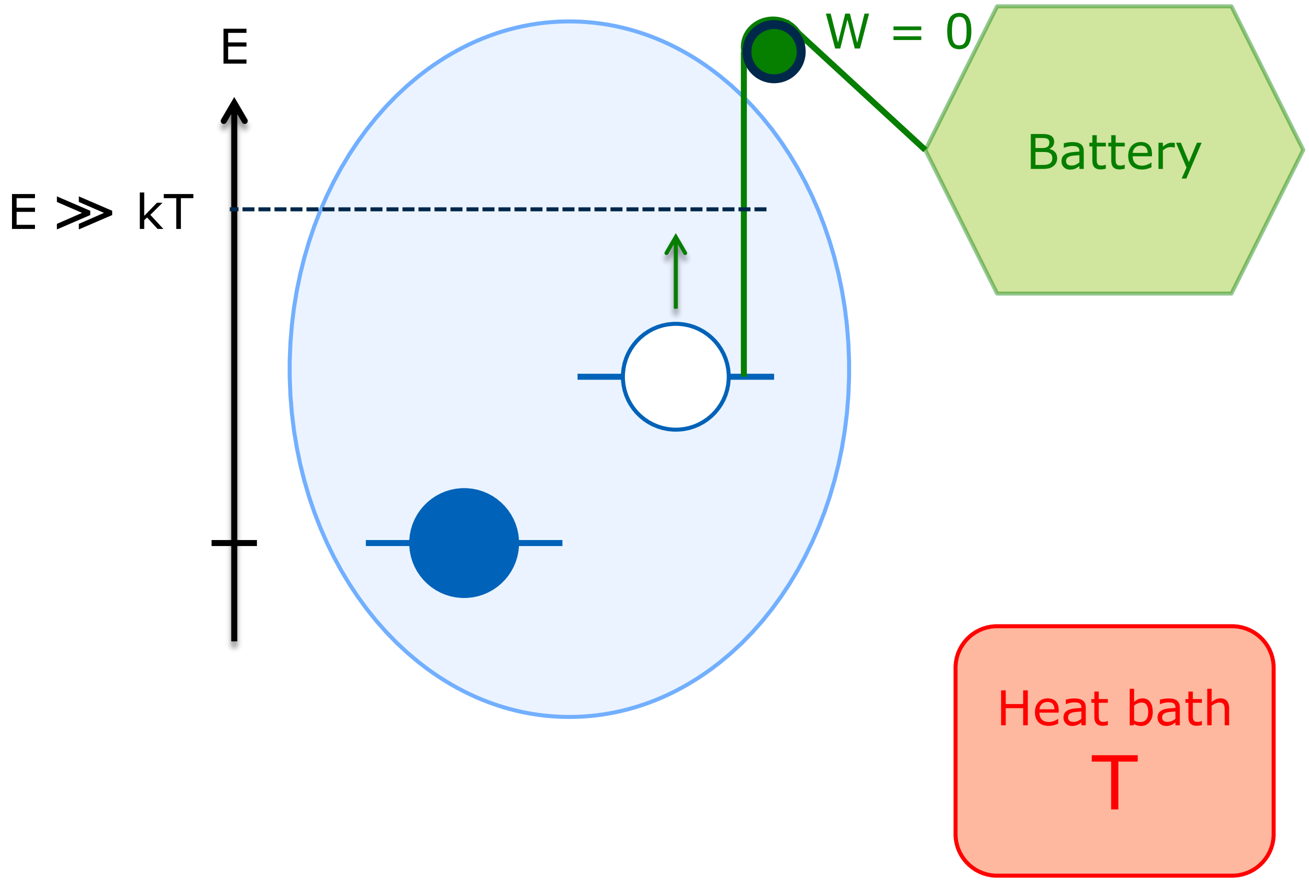
Szilard engine protocol



The qubit is in a known state and isolated from the bath

t < 0

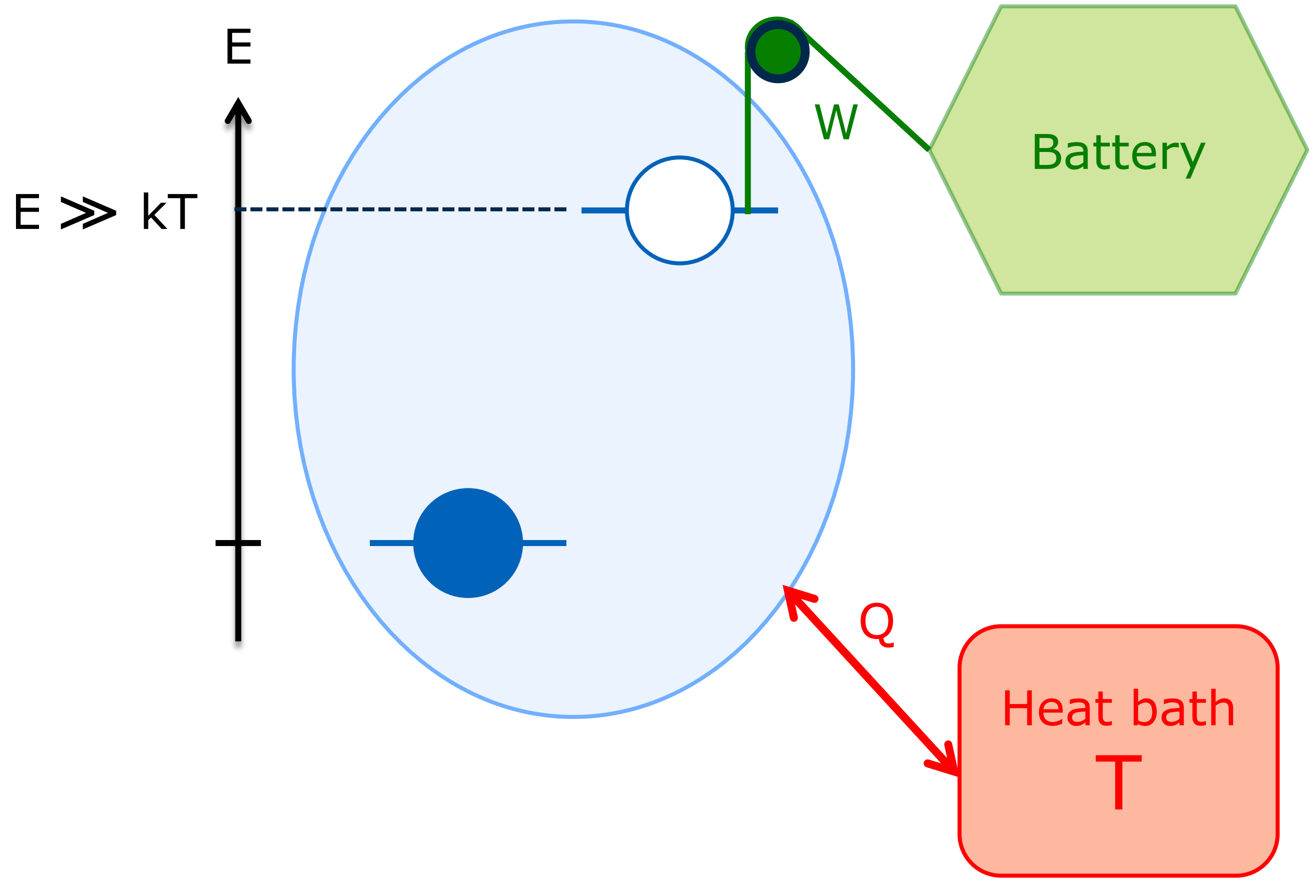
Szilard engine protocol



The empty state is raised with no work cost

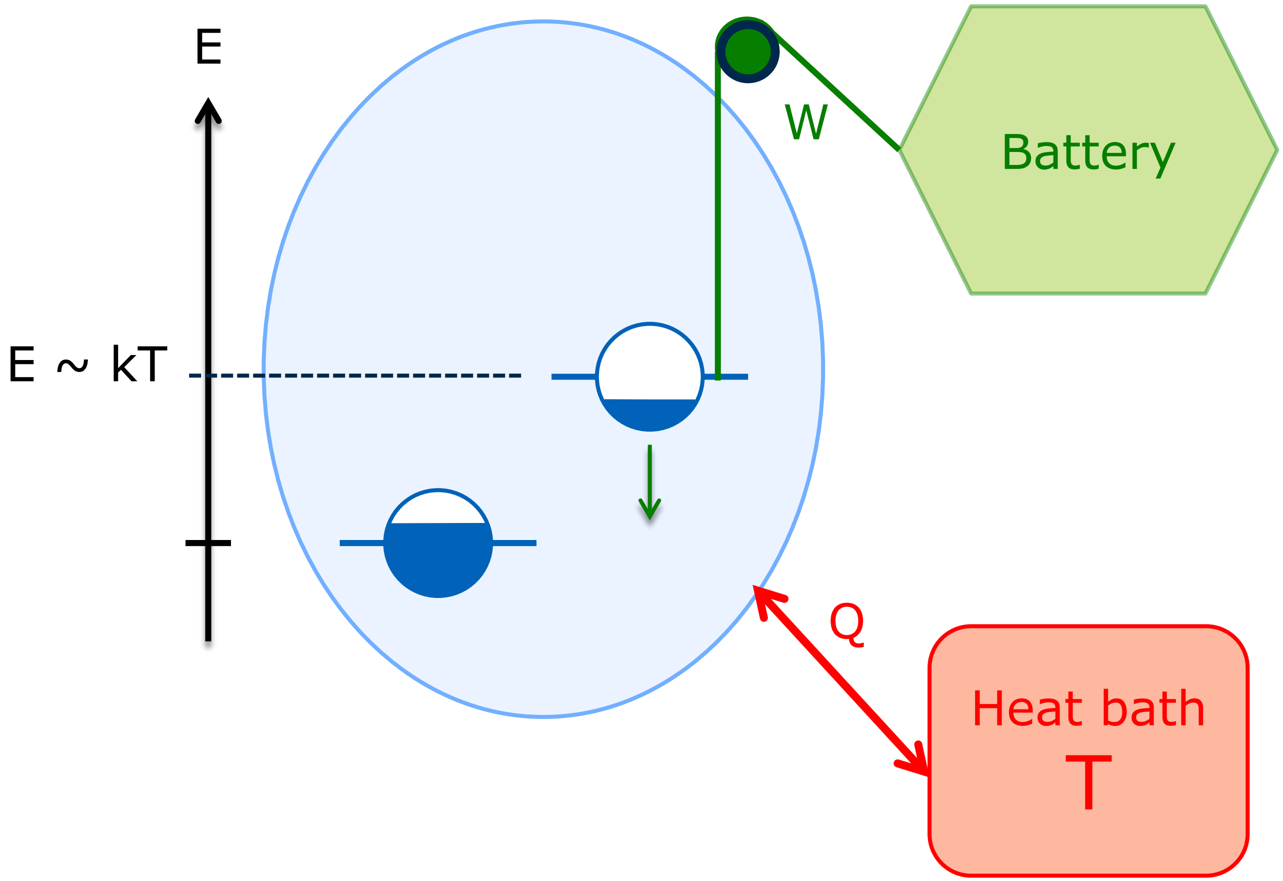
t < 0

Szilard engine protocol



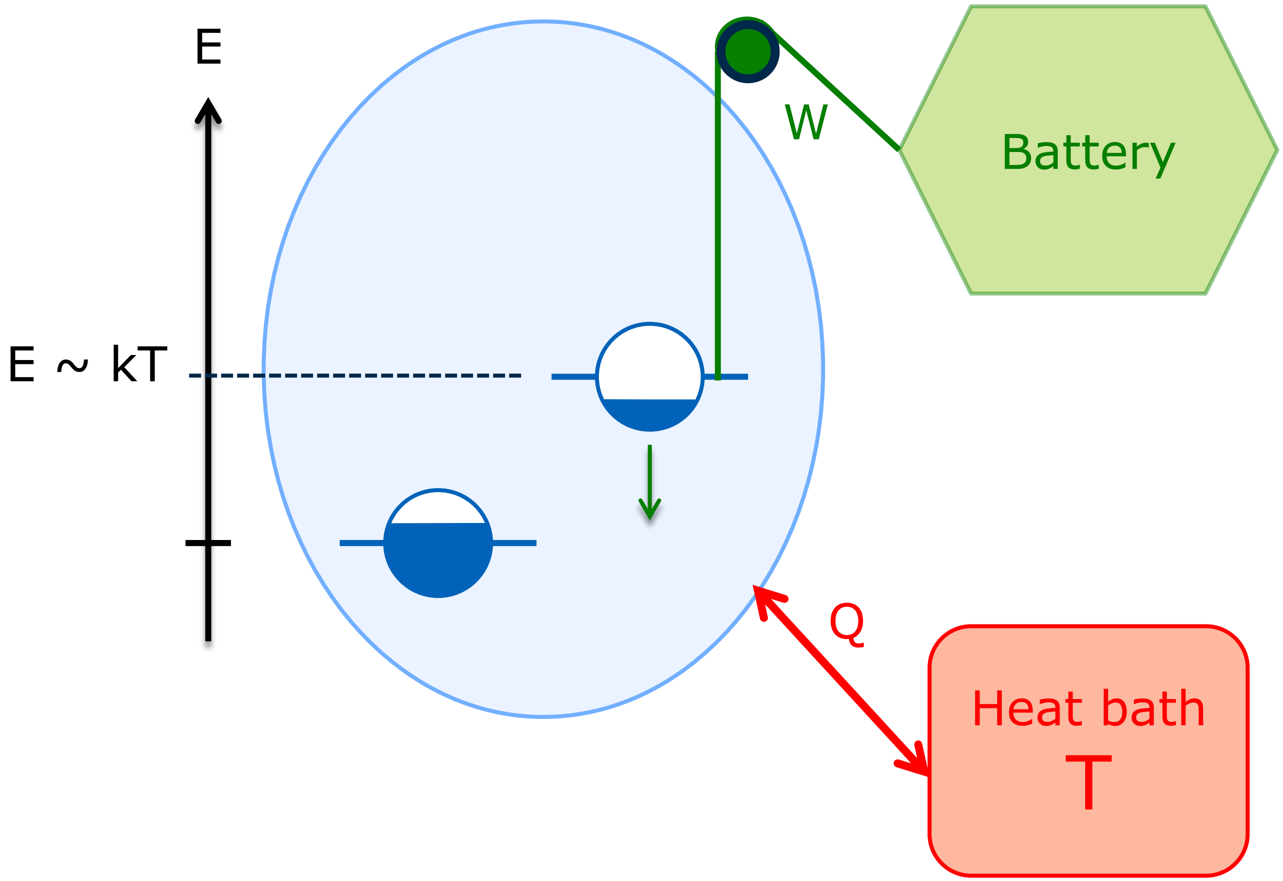
The qubit is put in equilibrium with the bath

t = 0



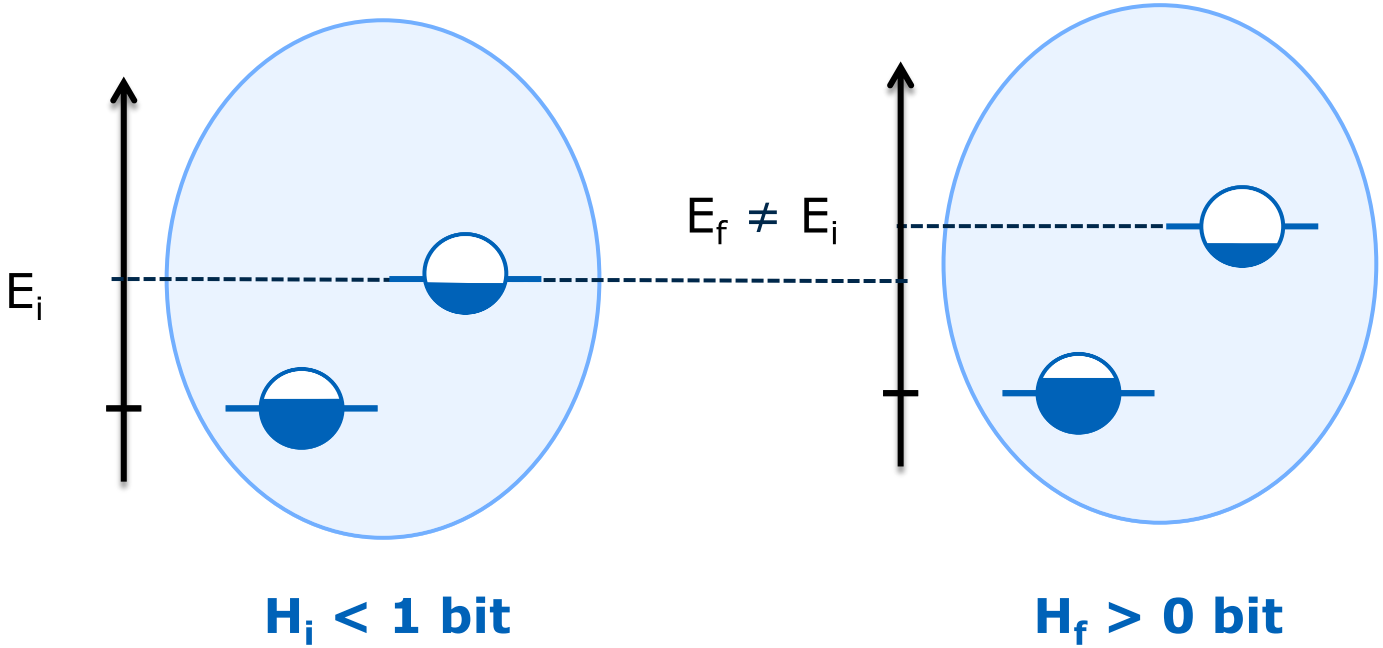
The empty state is lowered very slowly. For an energy low enough it gets populated.

$0 < t < t_f$



Then lowering the occupied state enables to extract work

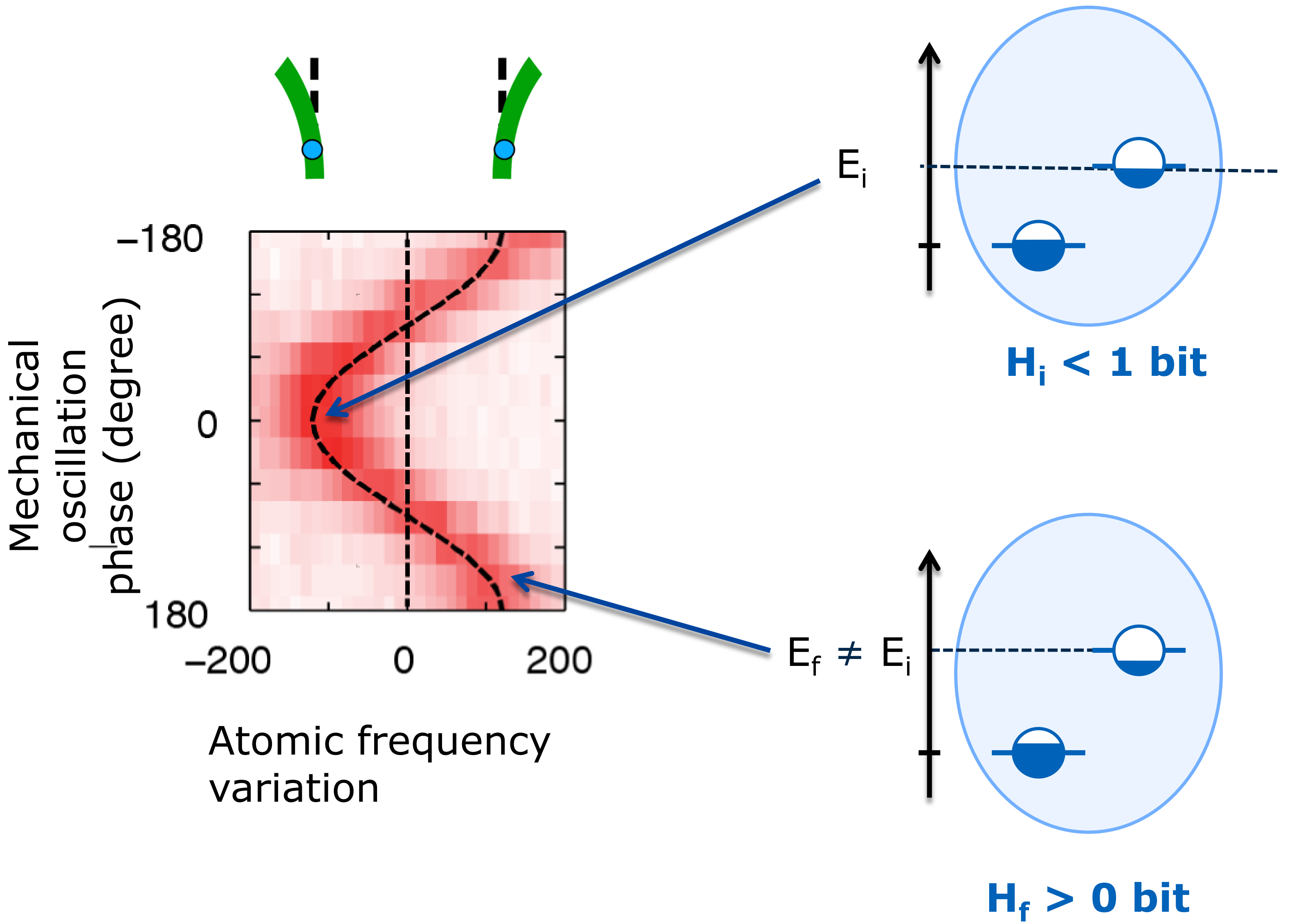
$\mathbf{0 < t < t_f}$



General formula for reversible work cost

$$-Q_L = W_L - \Delta U_L = kT(H_f - H_i)$$ (Clausius' Law)

internal energy of the bit: $U_L = P(E)\,E$



Mechanical oscillation phase (degree)
−180
0
180
−200
0
200
Atomic frequency variation
E_i
$E_f \neq E_i$
H_i < 1 bit
H_f > 0 bit

Modelling

Hamiltonian:

$$H = \underbrace{\frac{\hbar\omega_0}{2}\sigma_z}_{\textit{Qubit}} + \underbrace{\frac{\hbar g_m}{2}\left(b+b^\dagger\right)\left(\sigma_z+1\right)}_{\textit{Optomech. coupling}} + \underbrace{\hbar\Omega\left(b^\dagger b + 1/2\right)}_{\textit{Oscillator}}$$

If the oscillator and the qubit are connected to a thermal bath:

$$\dot{\rho} = \mathcal{L}_m\rho + \mathcal{L}_{int}\rho + \mathcal{L}_q\rho$$

Damping Γ_m of the oscillator (→ $\mathcal{L}_m\rho$)

Damping γ of the qubit (→ $\mathcal{L}_q\rho$)

Semi-classical regime

Hamiltonian:

$$H = \underbrace{\frac{\hbar\omega_0}{2}\sigma_z}_{\text{Qubit}} + \underbrace{\frac{\hbar g_m}{2}(b+b^\dagger)(\sigma_z+1)}_{\text{Optomech. coupling}} + \underbrace{\hbar\Omega(b^\dagger b + 1/2)}_{\text{Oscillator}}$$

If the oscillator and the qubit are connected to a thermal bath:

$$\dot{\rho} = \mathcal{L}_m \rho + \mathcal{L}_{int} \rho + \mathcal{L}_q \rho$$

Damping Γ_m of the oscillator (→ $\mathcal{L}_m$)

Damping γ of the qubit (→ $\mathcal{L}_q$)

$g_m << \gamma$ **semi classical regime**

The correlations between oscillator and qubit die quicker than they are created !

Semi-classical Bloch equations

Expansion to first order in $\varepsilon = g_m/\gamma$:

$$\begin{cases} \dot{P}_e(t) = -\gamma(2\bar{n}+1)P_e(t) + \gamma\bar{n} \\ \dot{s}(t) = -ig_m(\beta(t)+\beta^*(t))s(t) - \frac{\gamma}{2}(2\bar{n}+1)s(t) \\ \dot{\beta}(t) = -i\Omega\beta(t) - ig_m P_e(t) - \Gamma_m\beta(t) \\ \dot{N} = -ig_m P_e(t)(\beta^*(t)-\beta(t)) - \Gamma_m N + \Gamma_m n_m \end{cases}$$

qubit population $P_e = \dfrac{\langle\sigma_z\rangle + 1}{2}$

qubit dipole $s = \langle\sigma_-\rangle$

Mech. amplitude $\beta = \langle b\rangle = x + ip$

Mech. population $N = \langle b^\dagger b\rangle$

Wallquist et al., New J. Phys, 10, 095019 (2008)
Rabl, Phys. Rev. B 82

Semi-classical Bloch equations

Expansion to first order in $\varepsilon = g_m/\gamma$:

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Effective atomic frequency: $\omega_q(t) = \omega_0 + \frac{g_m}{x_{zpf}} x(t)$

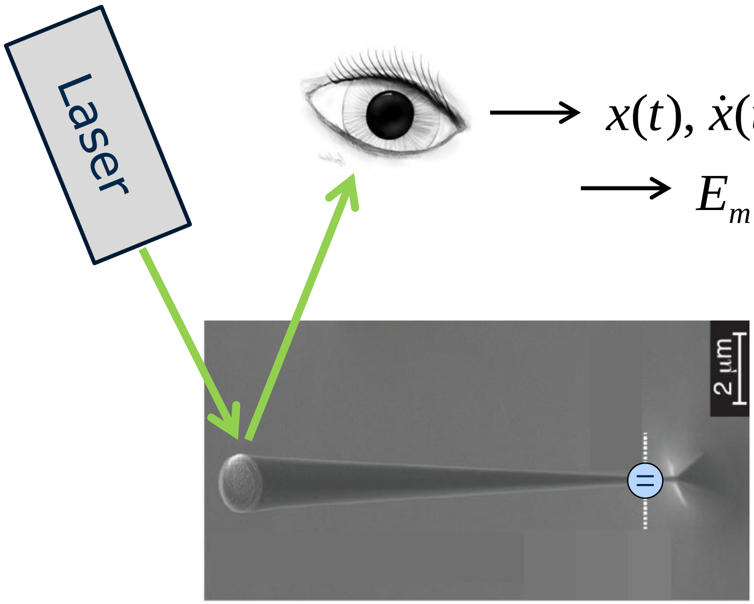
Mechanical energy variation: $\Delta E_m = \hbar\Omega\Delta N = -\hbar\int P_e(t)\, d\omega_q(t) = -W(t)$

The oscillator stores work in its own mechanical energy !

Semi-classical Bloch equations

Effective atomic frequency: $\omega_q(t) = \omega_0 + \frac{g_m}{x_{zpf}} x(t)$

Mechanical energy variation: $\Delta E_m = -\hbar \int P_e \, d\omega_q = -W(t)$



$\longrightarrow x(t), \dot{x}(t)$

$\longrightarrow E_m(t)$

→ Deflexion measurement enables to measure the average work directly in that setup !

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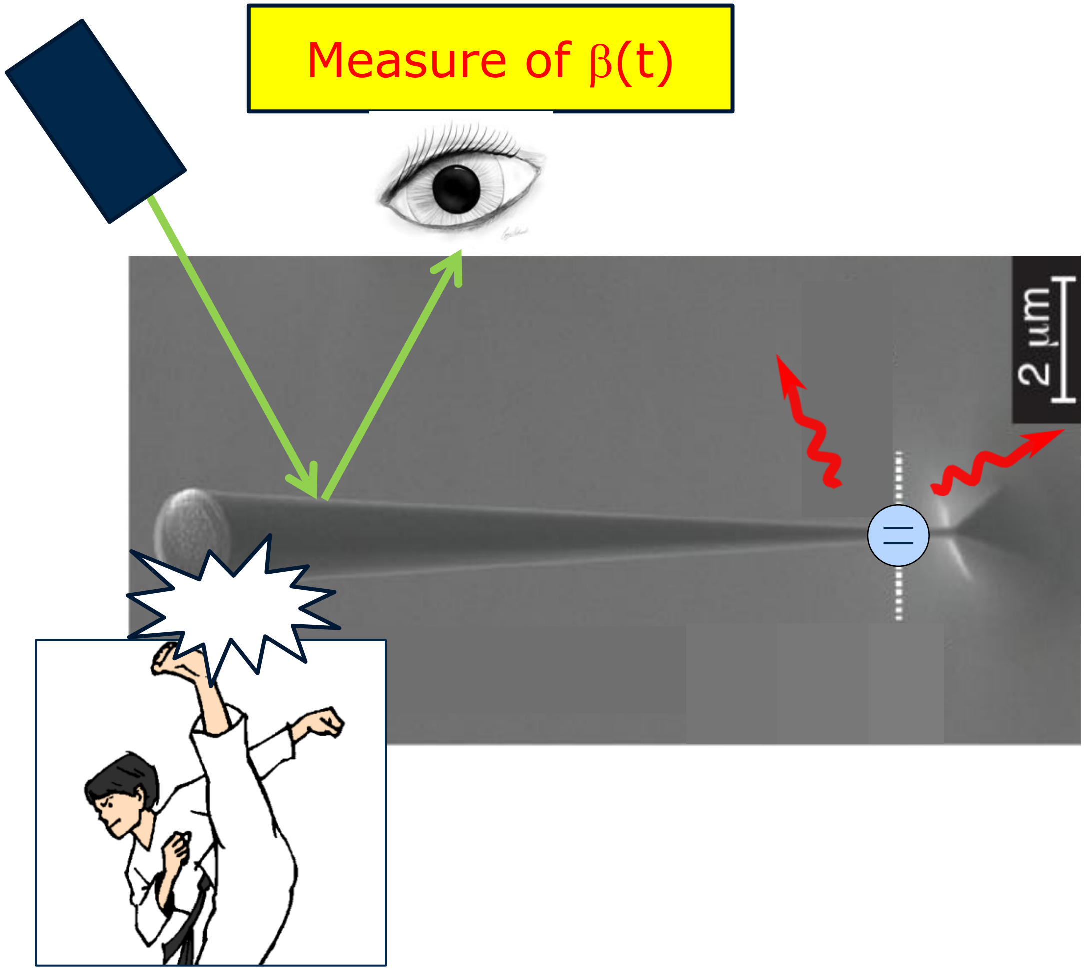
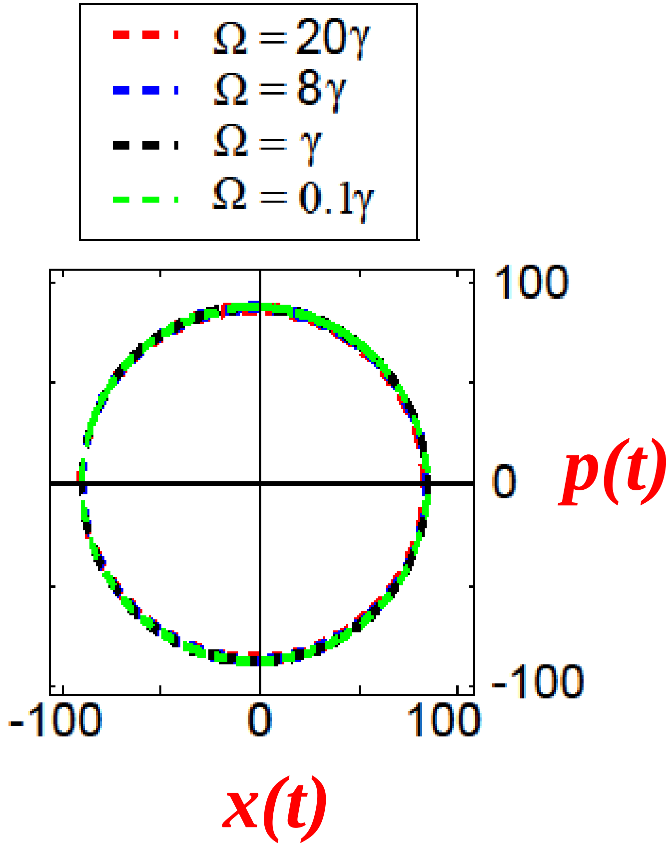
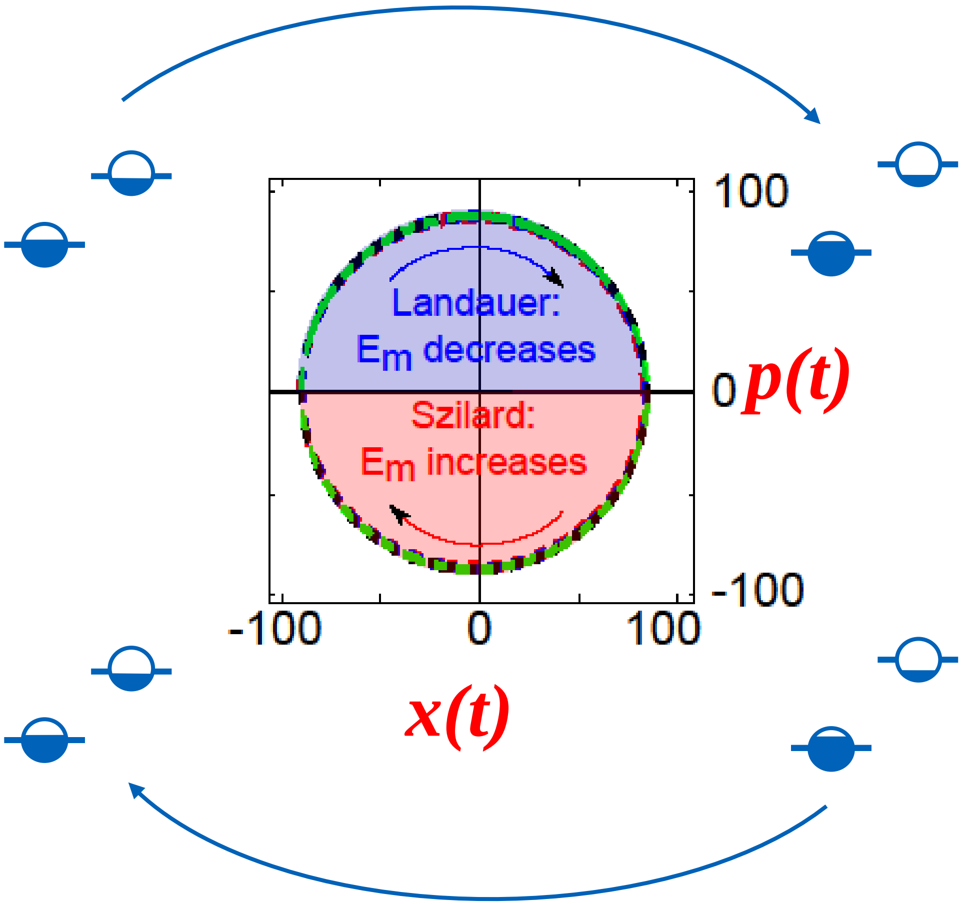
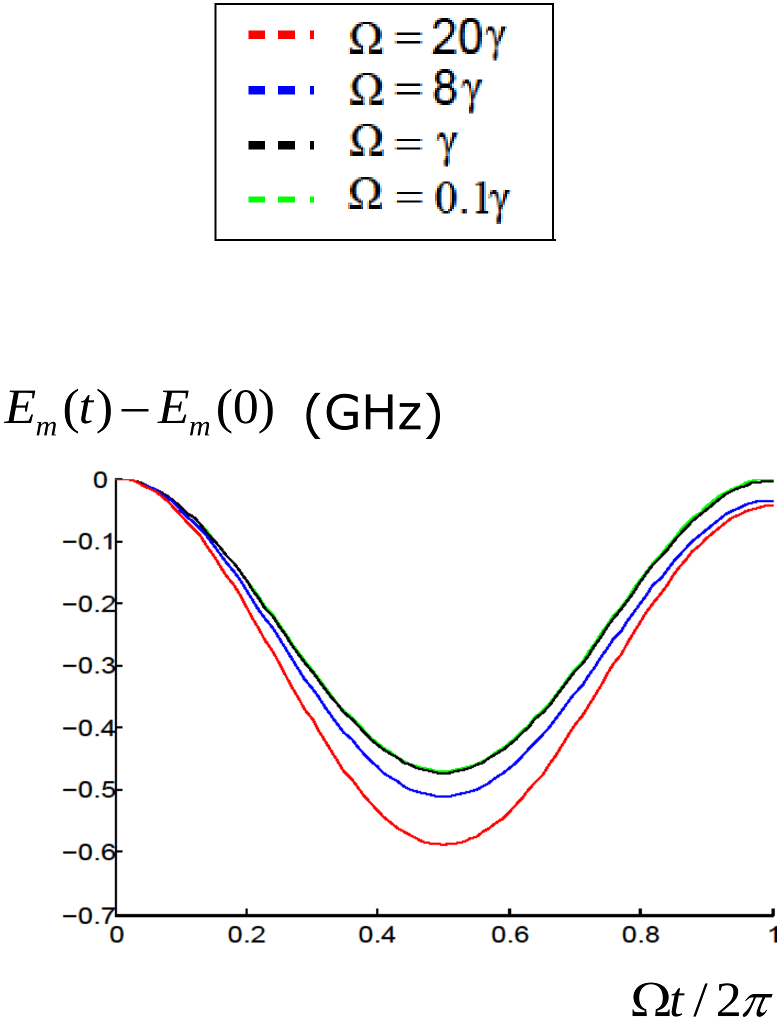


At t=0, we kick the oscillator and let it evolve …

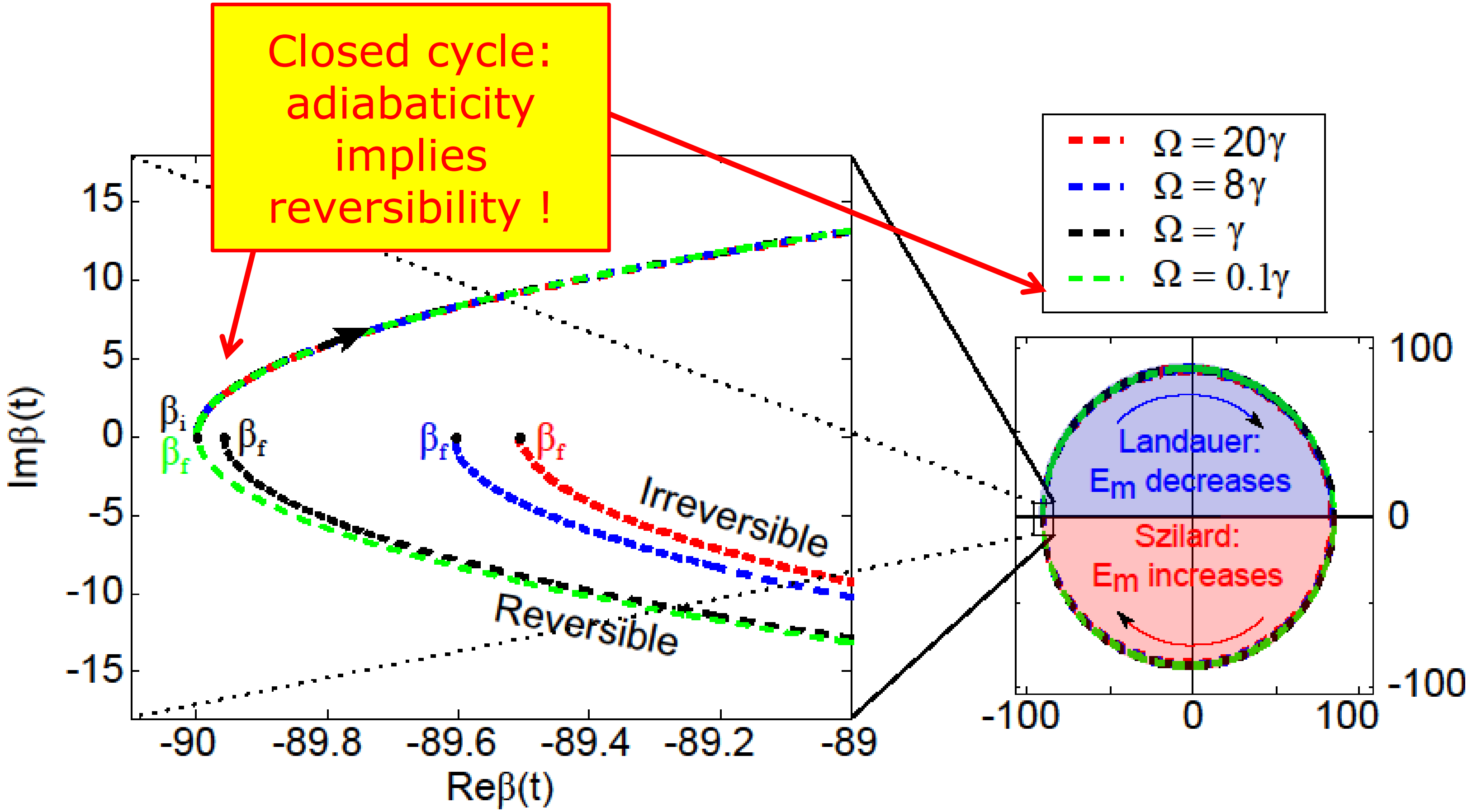
Implementation with the hybrid system

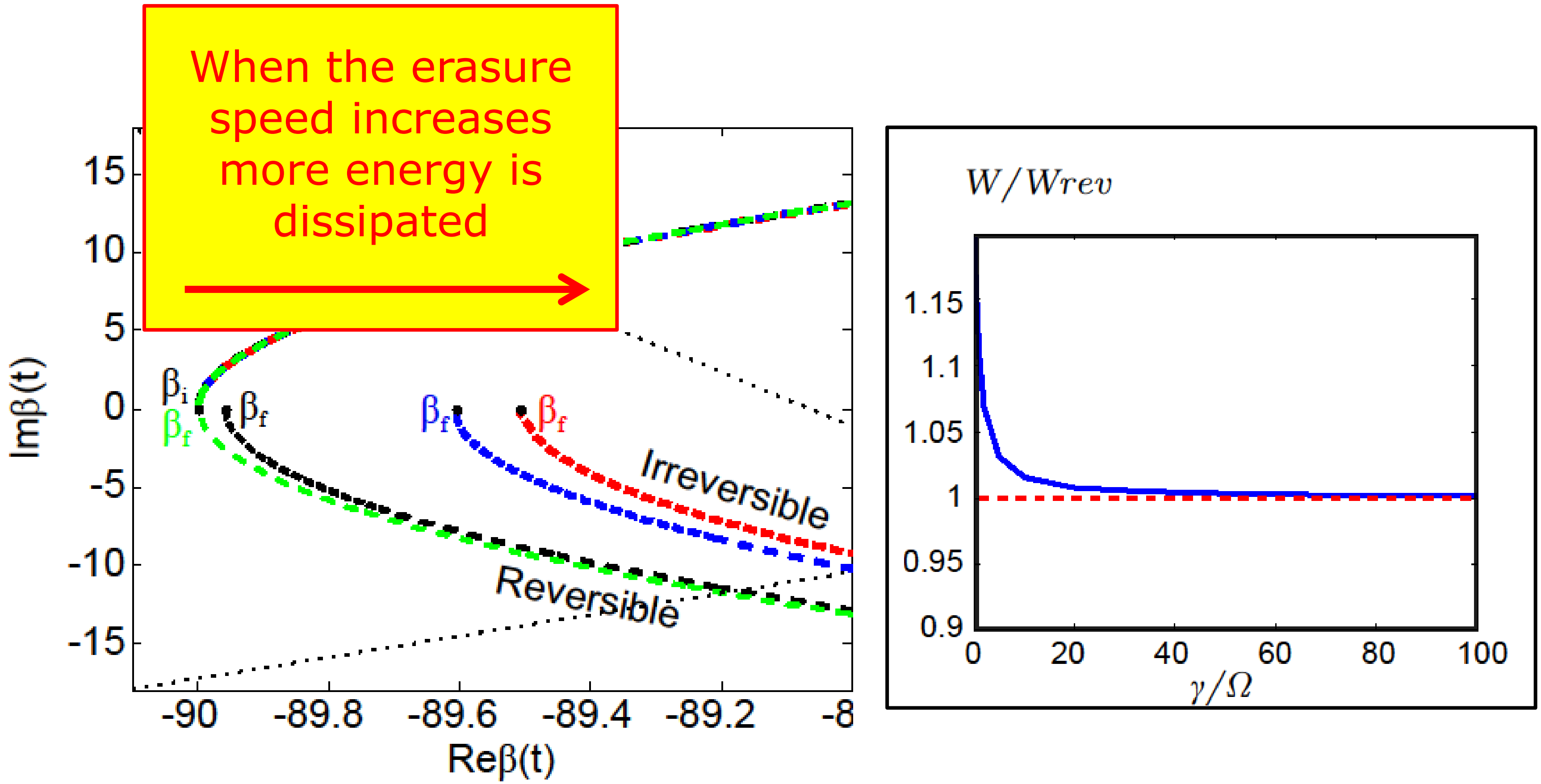




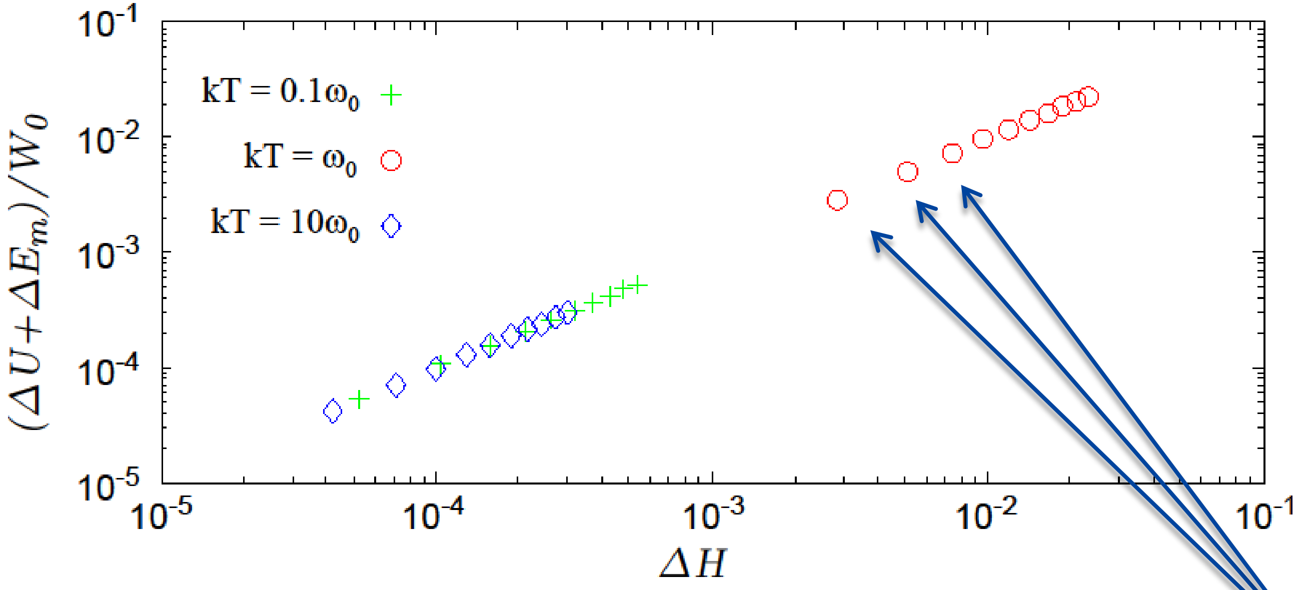


N.B.: at T = 4K, $kT = 80\,\text{GHz}$


Closed cycle: adiabaticity implies reversibility !
Ω = 20γ
Ω = 8γ
Ω = γ
Ω = 0.1γ
βi
βf
Irreversible
Reversible
Imβ(t)
Reβ(t)
15
10
5
0
-5
-10
-15
-90
-89.8
-89.6
-89.4
-89.2
-89
Landauer:
Em decreases
Szilard:
Em increases
100
0
-100


When the erasure speed increases more energy is dissipated
15
10
5
0
-5
-10
-15
-90
-89.8
-89.6
-89.4
-89.2
-8
Imβ(t)
Reβ(t)
β_i
β_f
β_f
β_f
β_f
Irreversible
Reversible
W/Wrev
1.15
1.1
1.05
1
0.95
0.9
0
20
40
60
80
100
γ/Ω

Heat exchange for different initial kicks and different temperatures



Shannon entropy variation

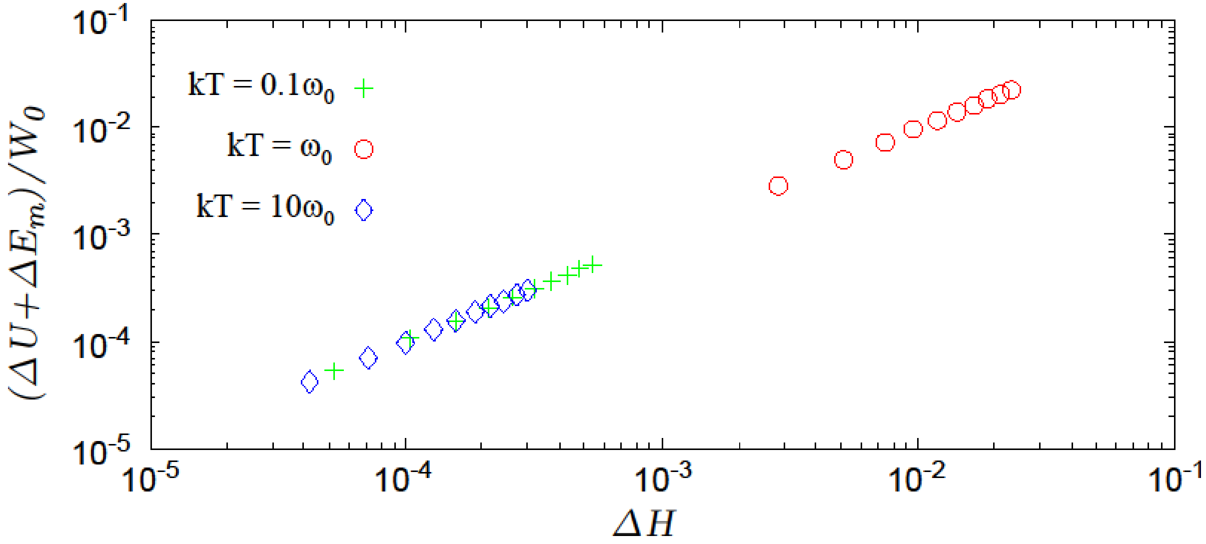
$\Omega = 0.1\gamma$

Clausius law checked

$$\Delta Q = -kT \log 2 \Delta H$$

Different values of the maximal elongation which determines ΔH

Heat exchange for different initial kick and different temperature



$\Omega = 0.1\gamma$

Clausius law checked

$$\Delta Q = kT \log 2 \Delta H$$

Conclusion

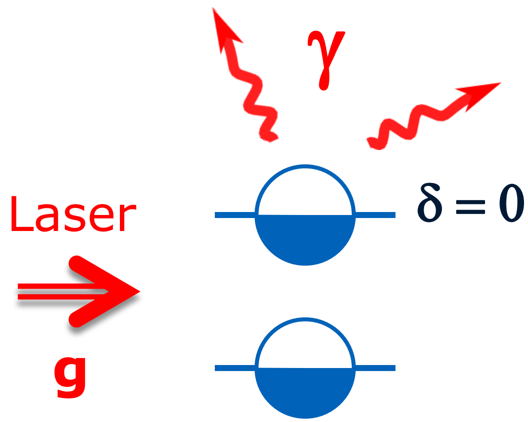
- Reversible cycles of information-to-energy conversions with realistic parameters!

- Restriction: oscillator elastic regime $|\omega_q(t) - \omega_0| << \omega_0$
  $\longrightarrow$ Only incomplete erasures $|\Delta H| < 1$ …

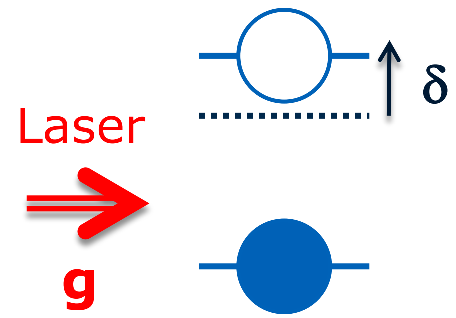
Outline

- I) Measuring work in a hybrid opto-mechanical system
- II) Information to energy conversions in the hybrid system
- III) Information to energy conversions in a driven qubit

An optical version of the erasure protocol



Resonantly driven qubit = Totally mixed state

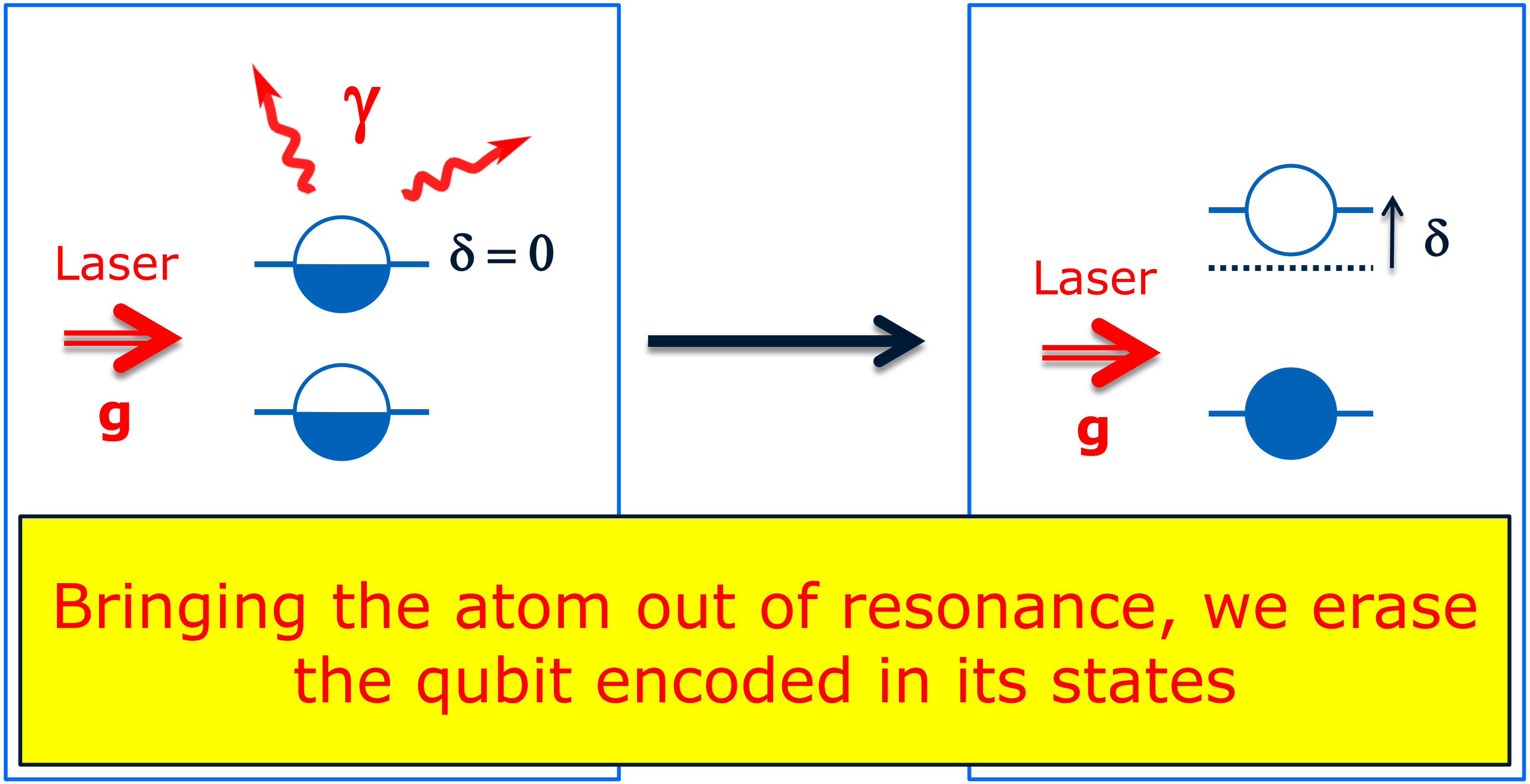


Non resonantly driven atom = ground state

γ spontaneous emission rate of the qubit
g classical Rabi frequency (intensity of the laser)
δ qubit-laser detuning (frequency difference)

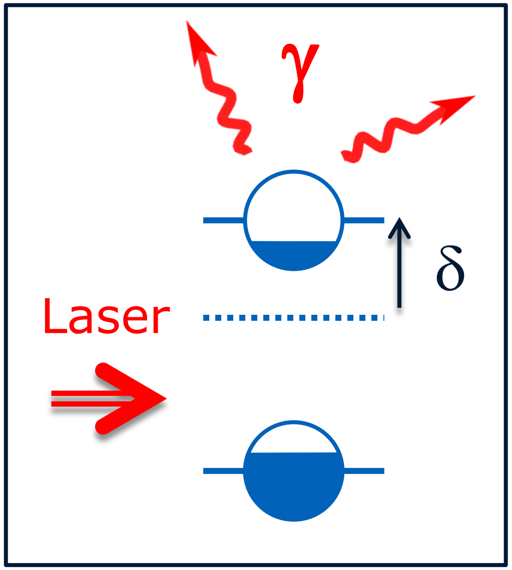
Saturated regime

$g \gg \gamma$



Bringing the atom out of resonance, we erase the qubit encoded in its states

γ spontaneous emission rate of the qubit
g classical Rabi frequency (intensity of the laser)
δ qubit-laser detuning (frequency difference)



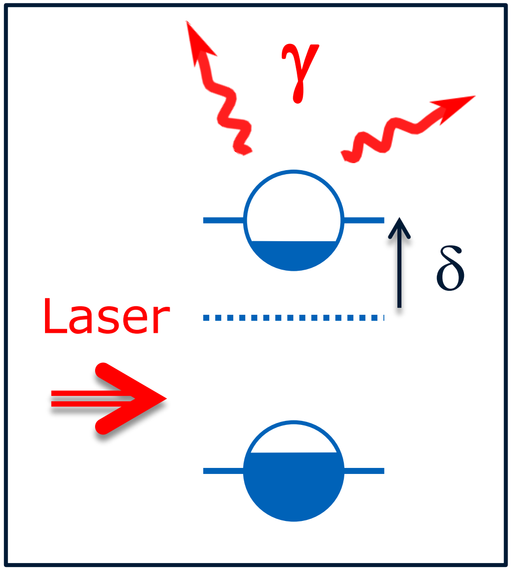
For a fixed δ, after a time 1/γ , the population of the excited state is in steady state:

$$P_e^{ss}(\delta) = \frac{1/2}{1+(\delta/g)^2}$$

$$\neq \quad P_e^{eq}(\delta) = \frac{e^{-\hbar(\omega_0+\delta)/kT}}{1+e^{-\hbar(\omega_0+\delta)/kT}}$$

$= 0$ at zero temperature

Slow (adiabatic) erasure



Adiabatic condition: $\Omega << \gamma$

Then the qubit remains in the steady-state

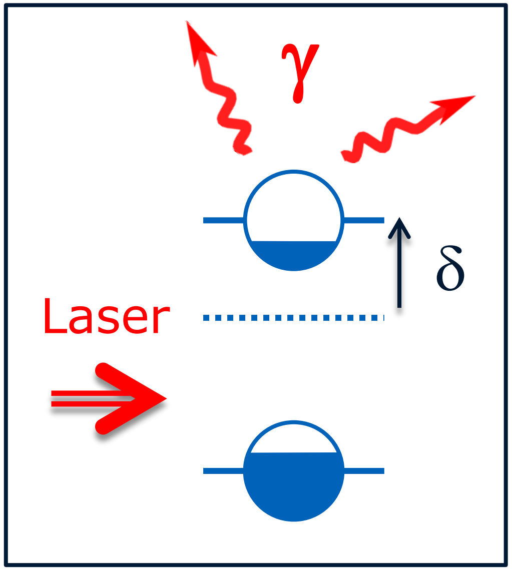
$$P_e^{ss}(\delta) = \frac{1/2}{1+(\delta/g)^2}$$

Minimum work to bring the qubit out of resonance adiabatically

$$W_L = \hbar \int_0^{\infty} P_e^{ss}(\delta) d\delta$$

Strong analogy with Landauer's minimal work W_0

Fast (non adiabatic) erasure



Fast mechanical oscillations: $\Omega \sim \gamma$

$$P_e(t) \neq P_e^{ss} = \frac{1/2}{1+(\delta/g)^2}$$

More work needed to erase the qubit fast

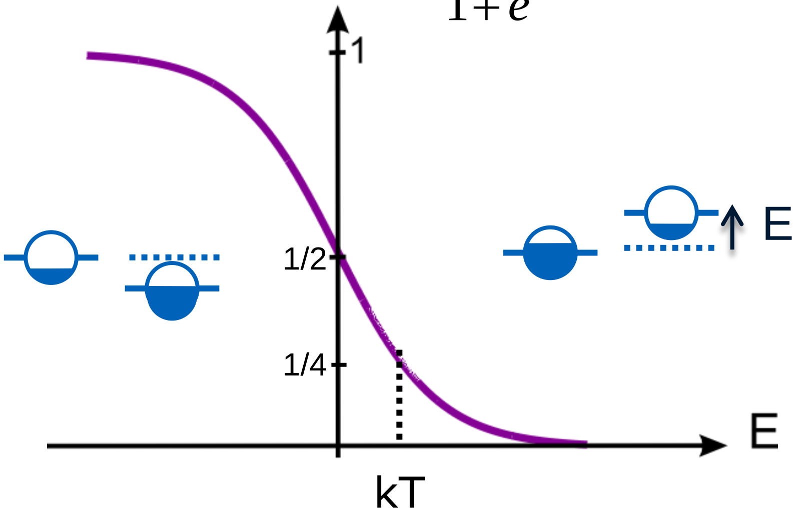
$$W = \hbar \int_0^\infty P_e(\delta) d\delta > W_L$$

Strong analogy with Landauer's minimal work W_0

See Steady-state thermodynamics formalism, e.g. : Esposito et al., Phys. Rev. E 76, 031132 (2007)
Oono et al., Progr. of Theor. Physics Supp. No. 130, 1998

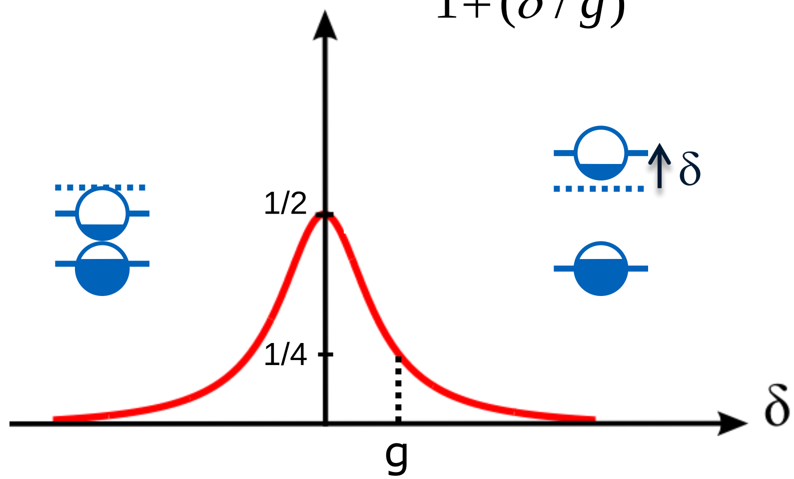
Thermal bath

$$P_e(E) = \frac{e^{-E/kT}}{1+e^{-E/kT}}$$



Laser drive

$$P_e(\delta) = \frac{1/2}{1+(\delta/g)^2}$$

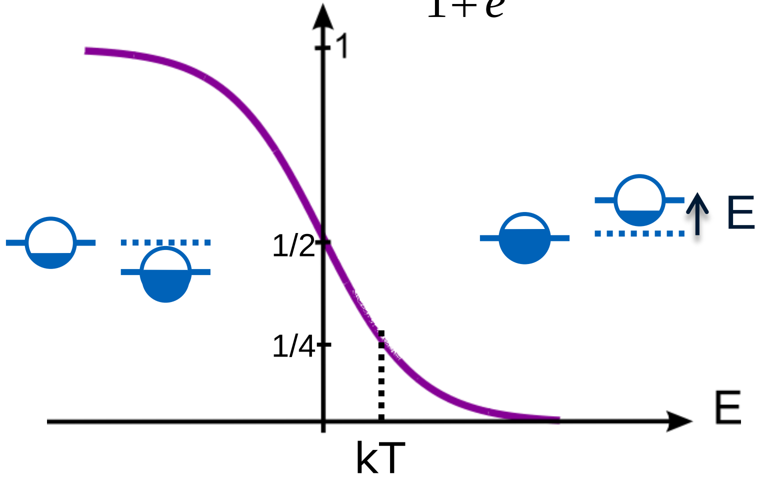


- The right side of the plot is very similar
- Behaviour is different for negative detuning

A new value of the elementary work

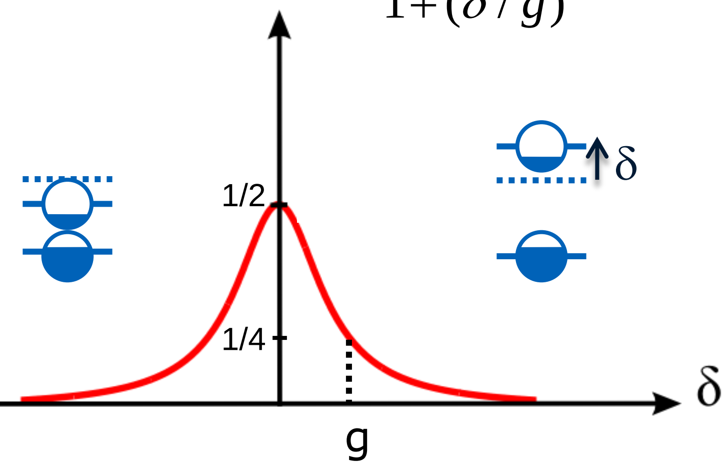
Thermal bath

$$P_e(E) = \frac{e^{-E/kT}}{1+e^{-E/kT}}$$



Optical bath

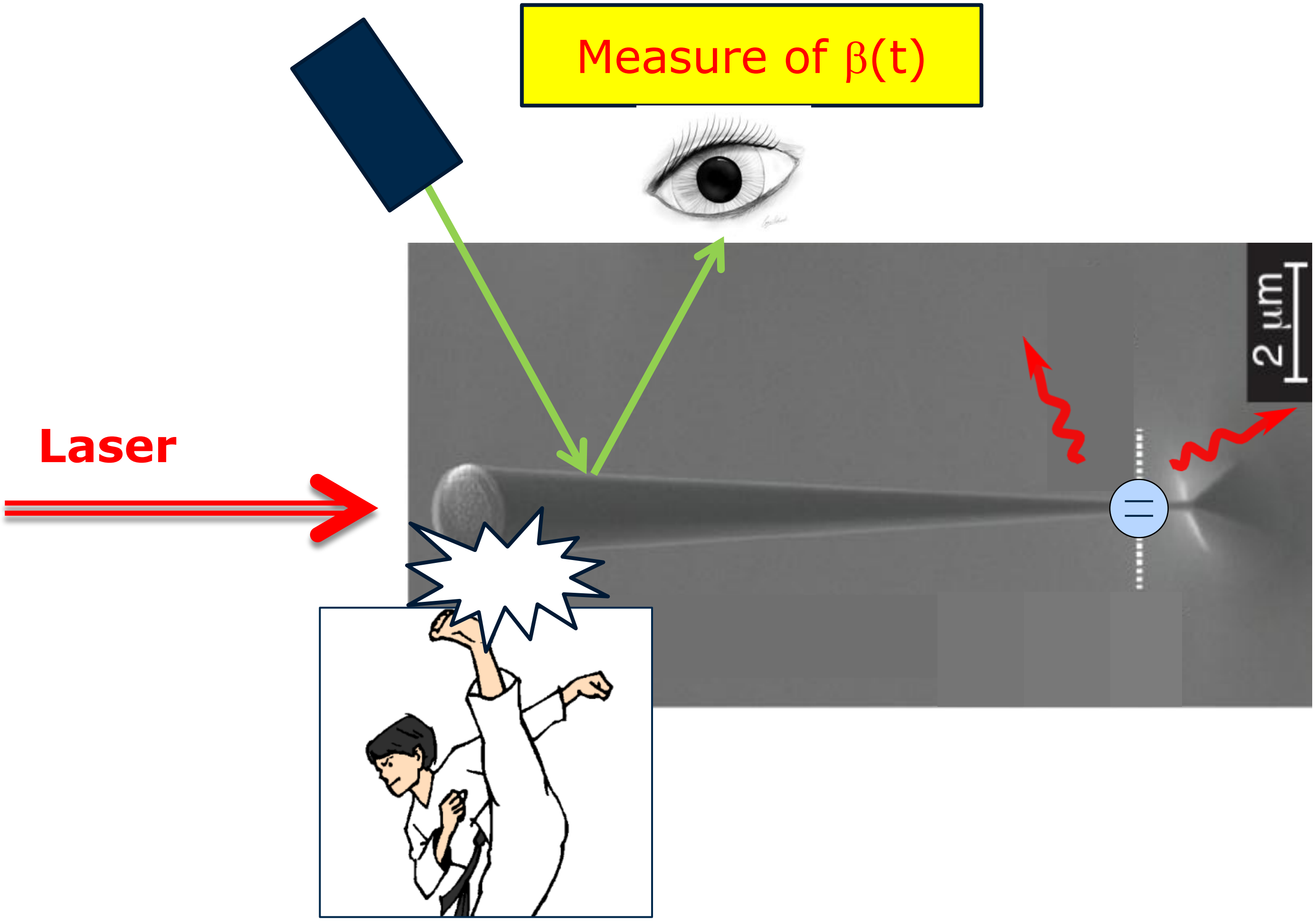
$$P_e(\delta) = \frac{1/2}{1+(\delta/g)^2}$$



$$W_0 = kT\ln 2 \quad \longleftrightarrow \quad W_L = \hbar\int_0^\infty P_e(\delta)d\delta = \hbar g\,\frac{\pi}{4\sqrt{2}}$$

Rabi frequency (laser intensity)

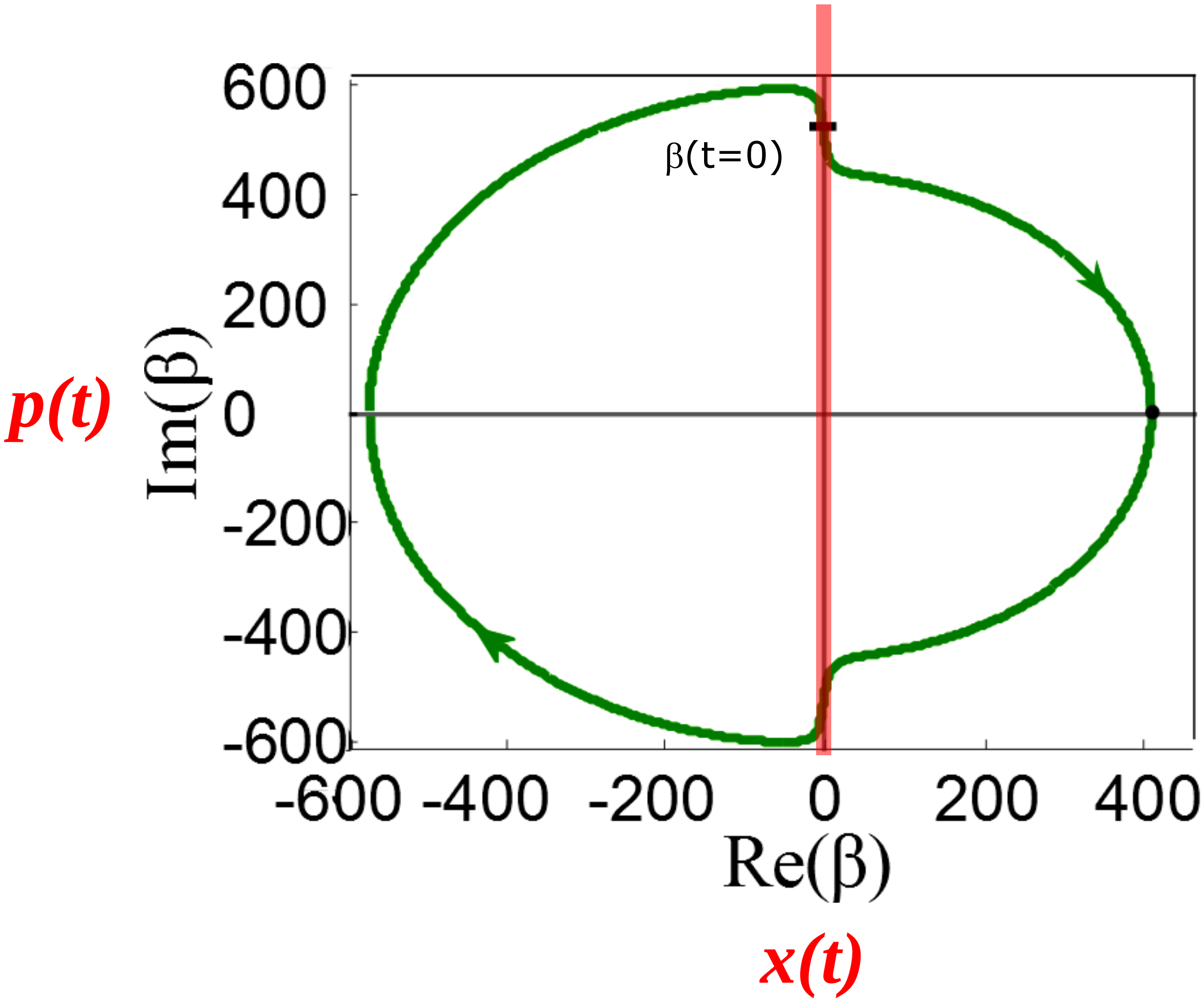
Implementation



At t=0, we kick the oscillator and let it evolve …

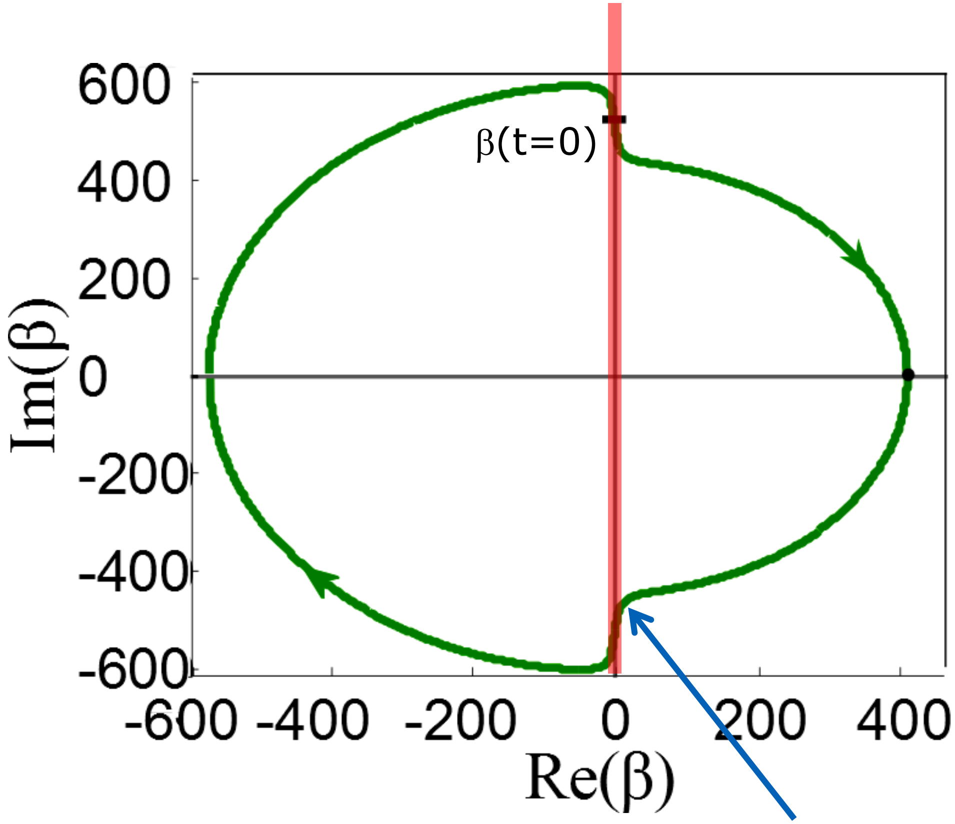
Observing work exchanges

Resonance with the laser



Observing work exchanges

Resonance with the laser



Typically W_L corresponds to:

Δx = 0.4 pm
Amplitude: 1.2 pm
Signal/Shot noise = 40
Signal/Thermal noise = 0.3

(g = 3 GHz, g_m = 30 MHz,
$\beta_0 = 10^2$, $\Omega/2\pi$ = 550 kHz,
T = 100 mK)

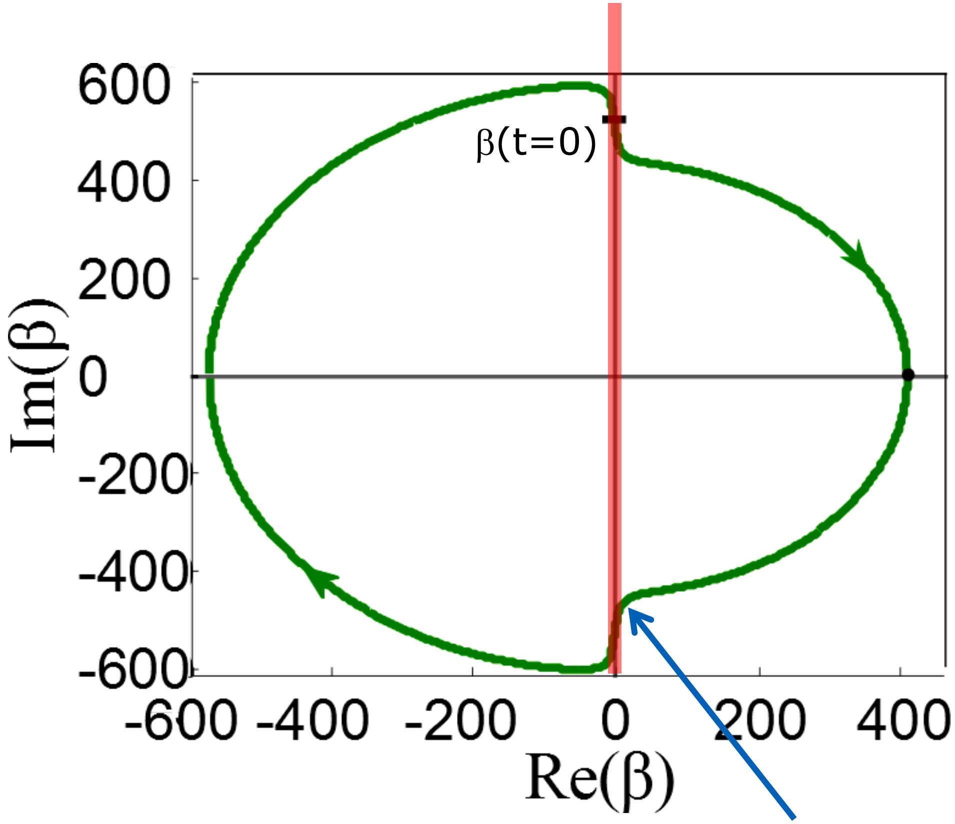
→ Measurable with current deflexion techniques

B. Sanii et al. PRL 104 (2010)

Variation of |β| when leaving or coming in resonance → exchange of work

Observing work exchanges

Resonance with the laser



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Signal/Thermal noise = 0.3

(g = 3 GHz, g_m = 30 MHz, $\beta_0 = 10^2$, $\Omega/2\pi$ = 550 kHz, T = 100 mK)

NB: $g_m > \Omega$ needed

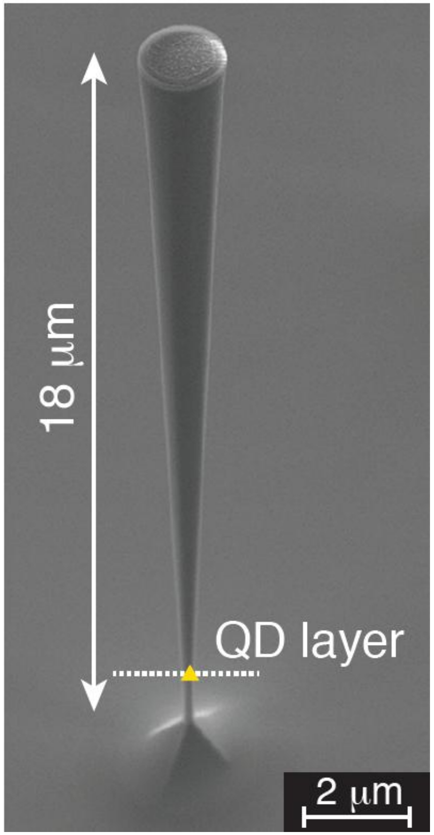
Variation of $|\beta|$ when leaving or coming in resonance → exchange of work

Conclusion

- A set up enabling reversible information-to-energy conversion in a qubit
  - In contact with a thermal Bath
  - Driven by a laser
- Direct observation of work exchanges in a quantum battery
- Application: optical Carnot engine reaching maximum efficiency

More details in: arXiv:1309.5276

I.Yeo et al., Nature Nanotechnology 9, 106–110 (2014)



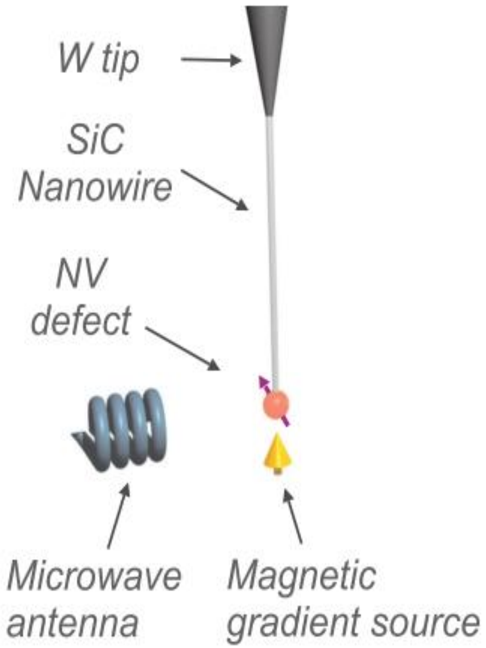
Now that the building blocks Landauer's erasure & Szilard engine are ensured, we can go to the fully quantum regime

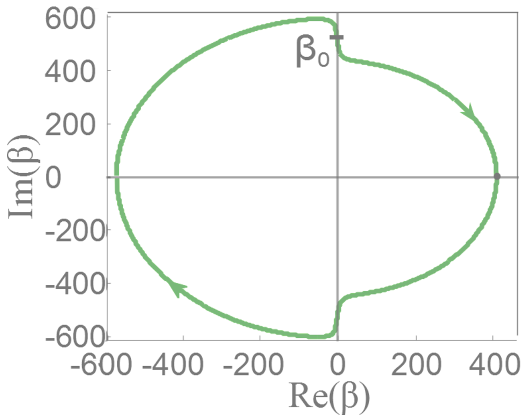
Ex: erasure cost of two entangled qubits

L. del Rio et al., Nature 474, 61--63 (2011)

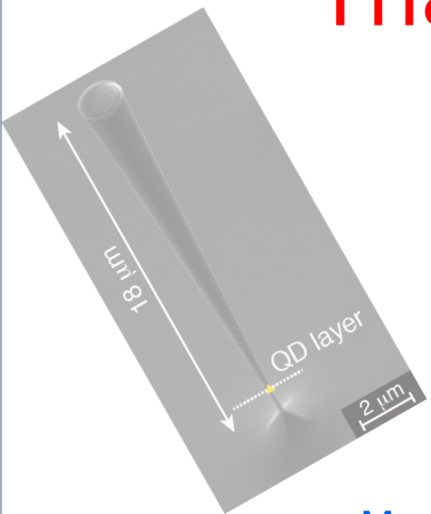
Oppenheim, J. et al., PRL 89, 180402 (2002)

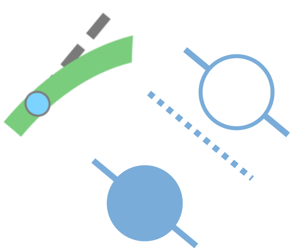
O. Arcizet et al., Nature Physics 7 (2011) 879





Thank you for your attention



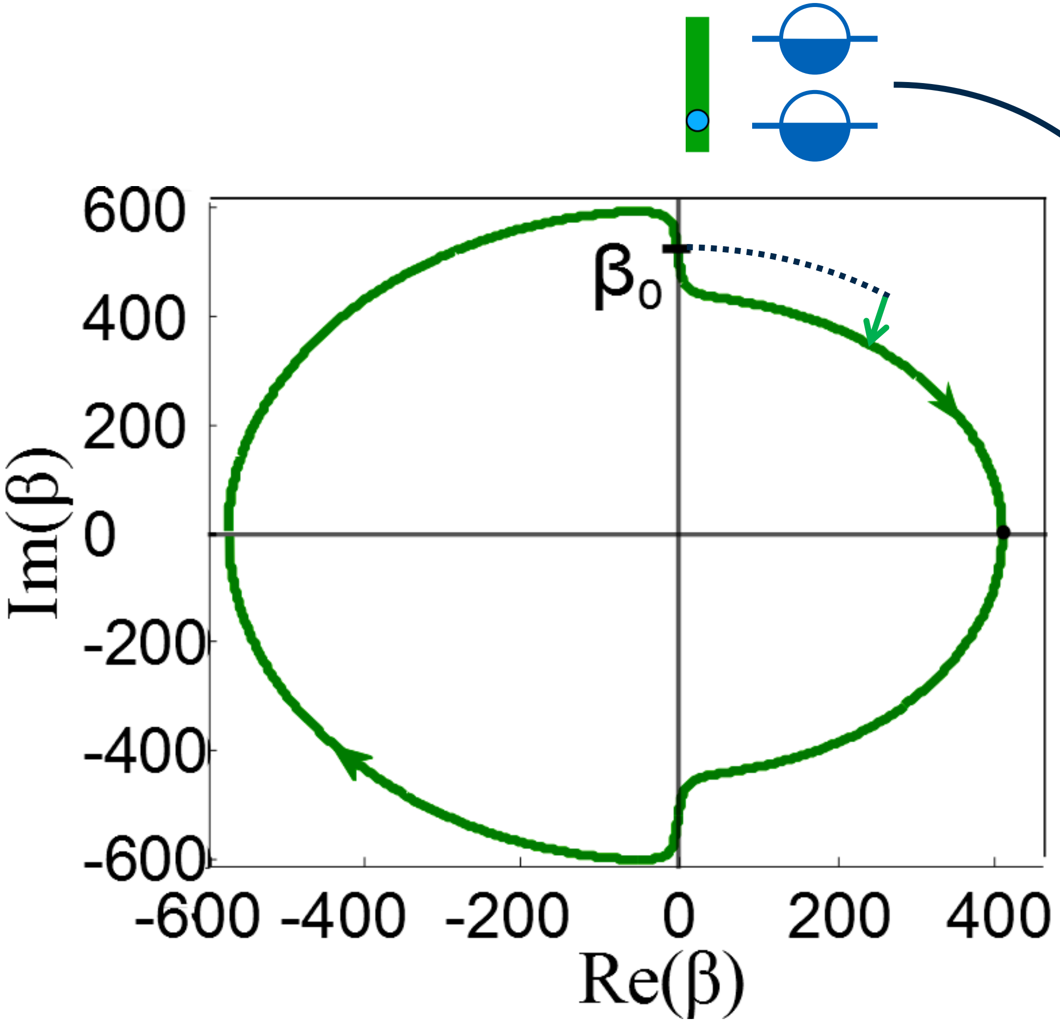


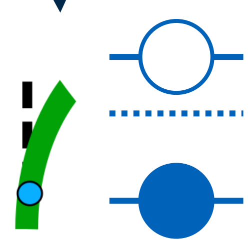
$$W_L = \hbar g \frac{\pi}{4}$$

More details in:
CE, Maxime Richard, Alexia Auffèves, arXiv:1309.5276

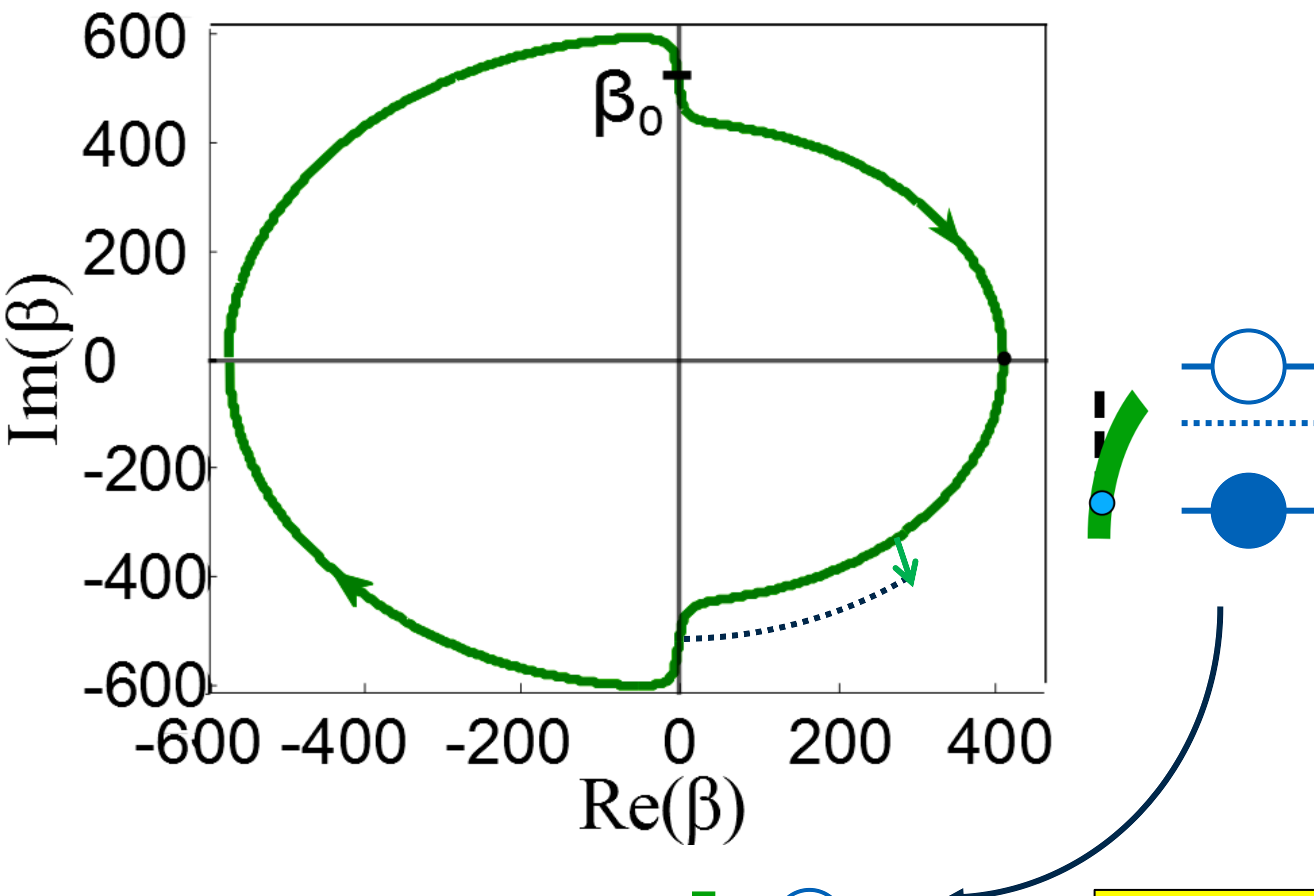
First quarter of oscillation
Landauer's erasure

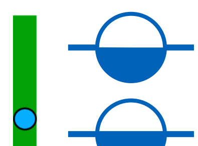
- W_L provided by the battery
- W_L dissipated in the bath
- Qubit erased





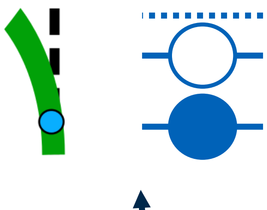
Second quarter of oscillation
Szilard's engine

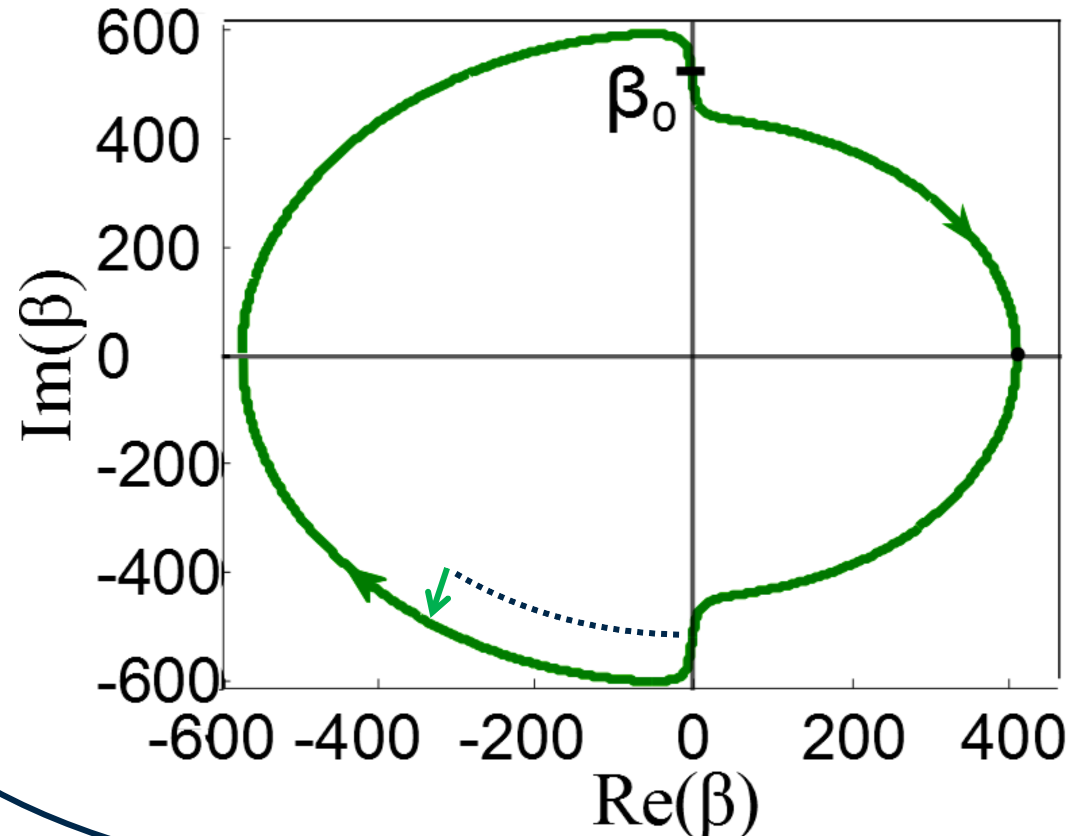




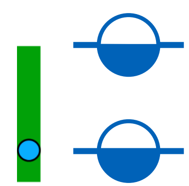
- W_L stored in the battery
- W_L extracted from the bath
- Qubit in a mixed state

Third quarter of oscillation
Inverse Landauer's erasure





- W_L stored in the battery
- W_L extracted from the bath
- Qubit erased

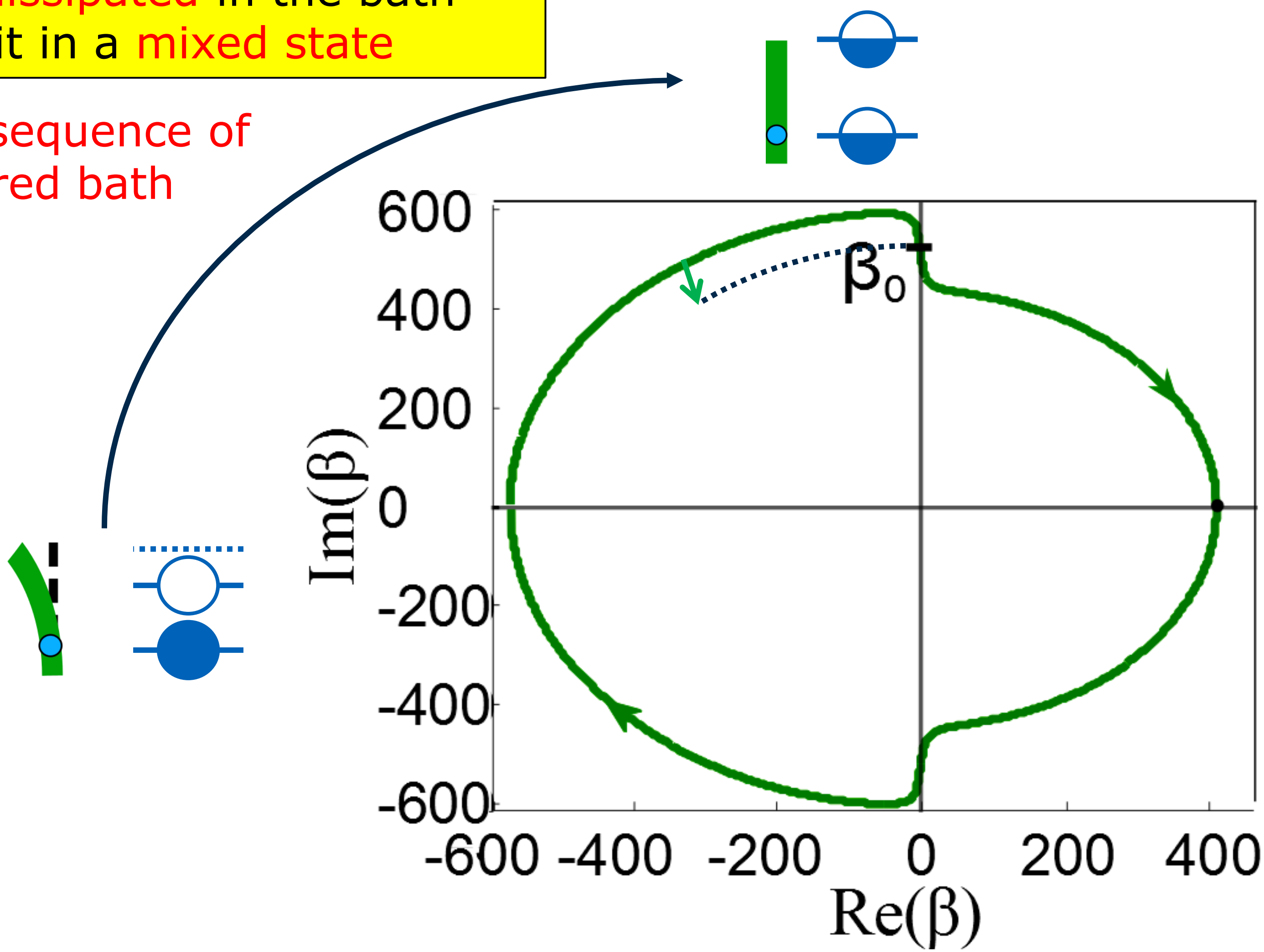


Consequence of colored bath

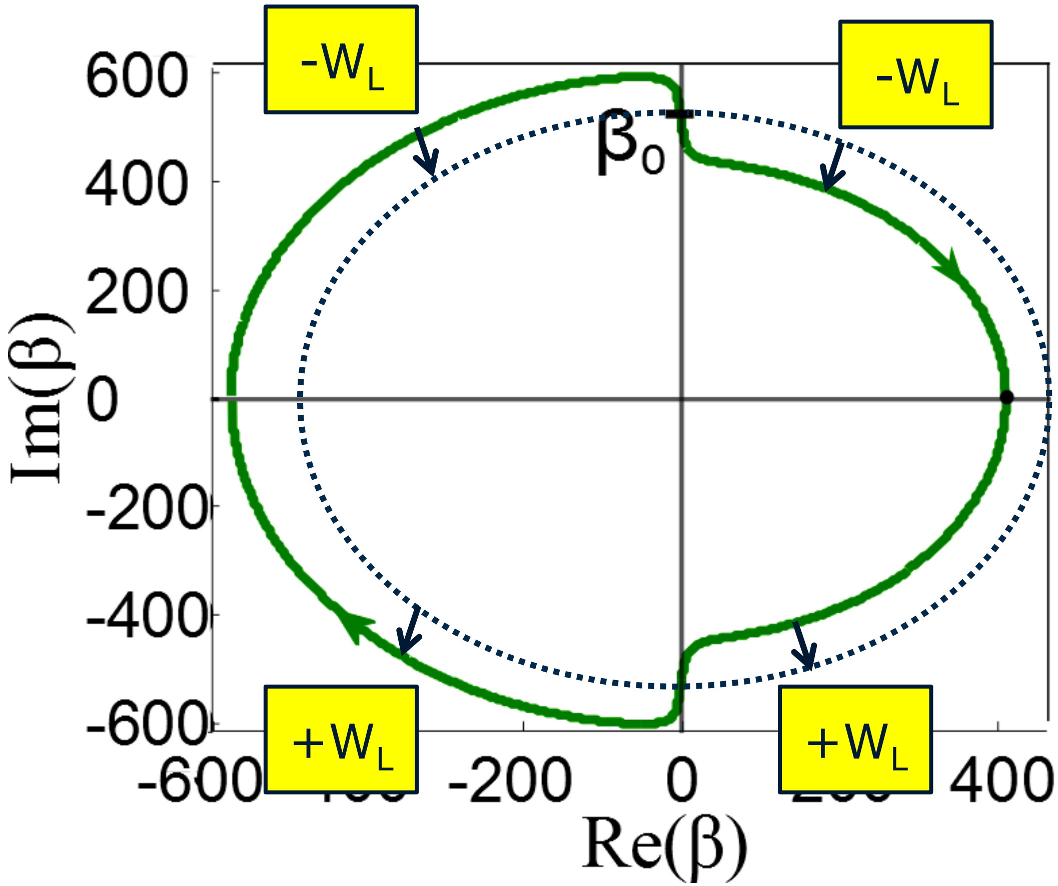
Inverse Szilard's engine

- W_L provided by the battery
- W_L dissipated in the bath
- Qubit in a mixed state

Consequence of colored bath



Optical Carnot engine



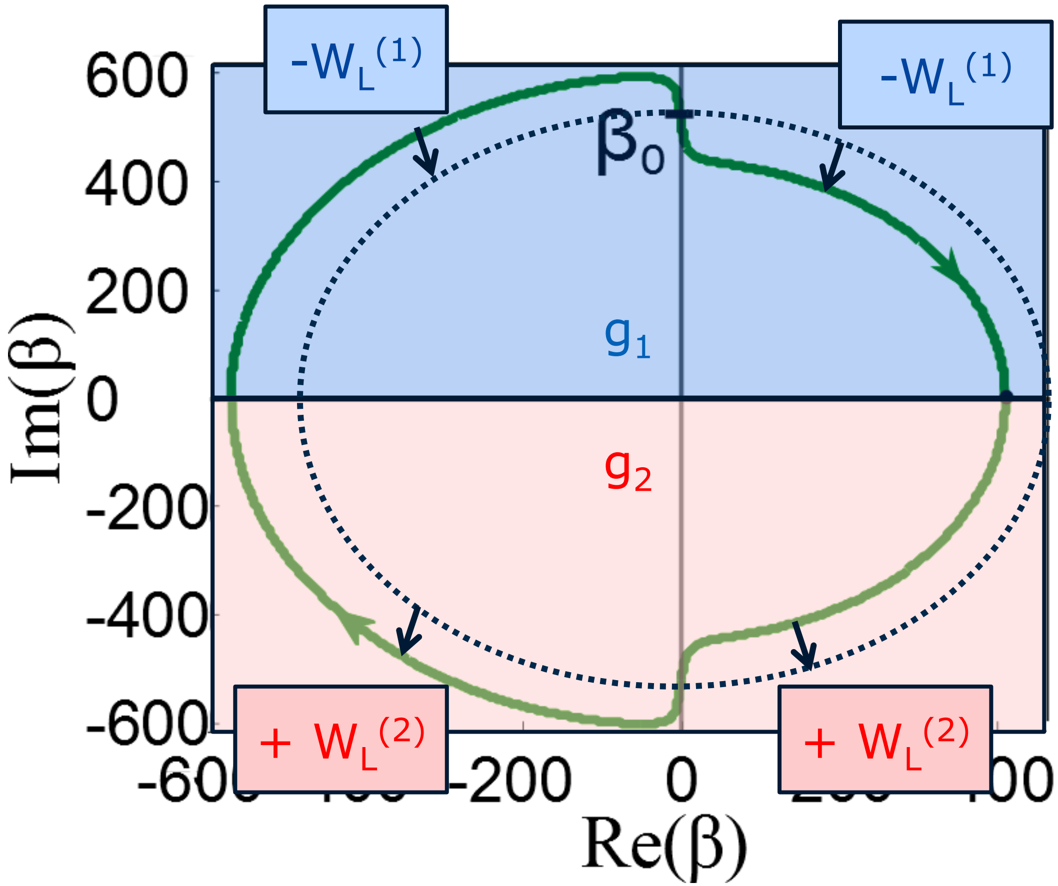
Elementary work

$$W_L = \hbar g \frac{\pi}{4}$$

Rabi frequency (temperature)

$$W_{stored} = -\hbar g \frac{\pi}{4} + \hbar g \frac{\pi}{4} + \hbar g \frac{\pi}{4} - \hbar g \frac{\pi}{4} = 0$$

Optical Carnot engine



Two different Rabi frequencies

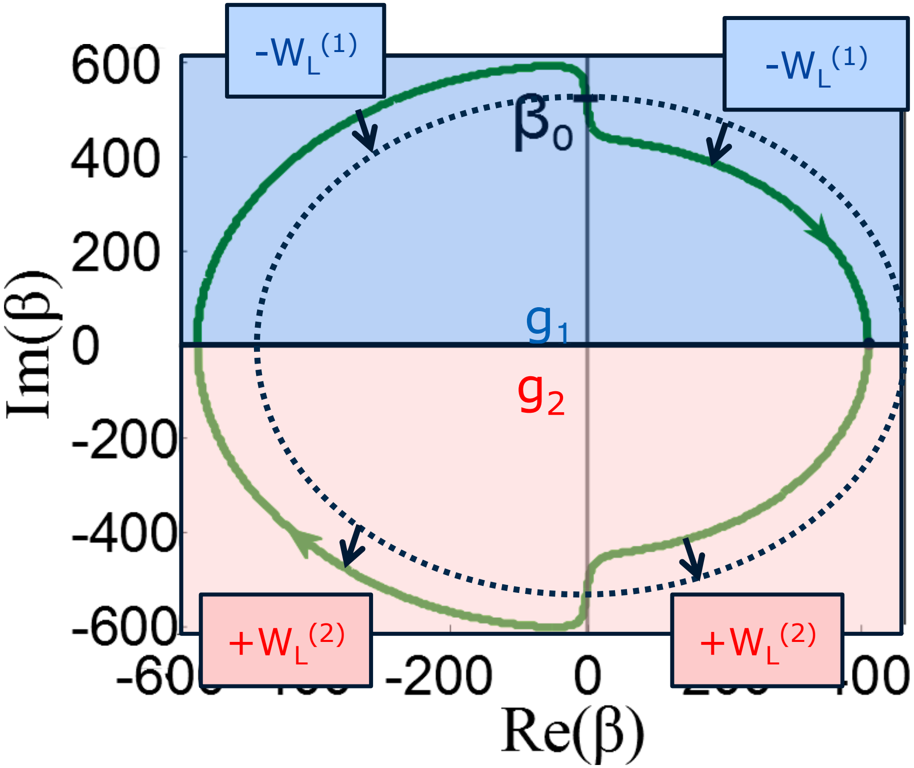
$g_2 > g_1$

=

thermalization of the qubit with a hot source and a cold source

$$W_{stored} = -\hbar g_1 \frac{\pi}{4} + \hbar g_2 \frac{\pi}{4} + \hbar g_2 \frac{\pi}{4} - \hbar g_1 \frac{\pi}{4} > 0$$

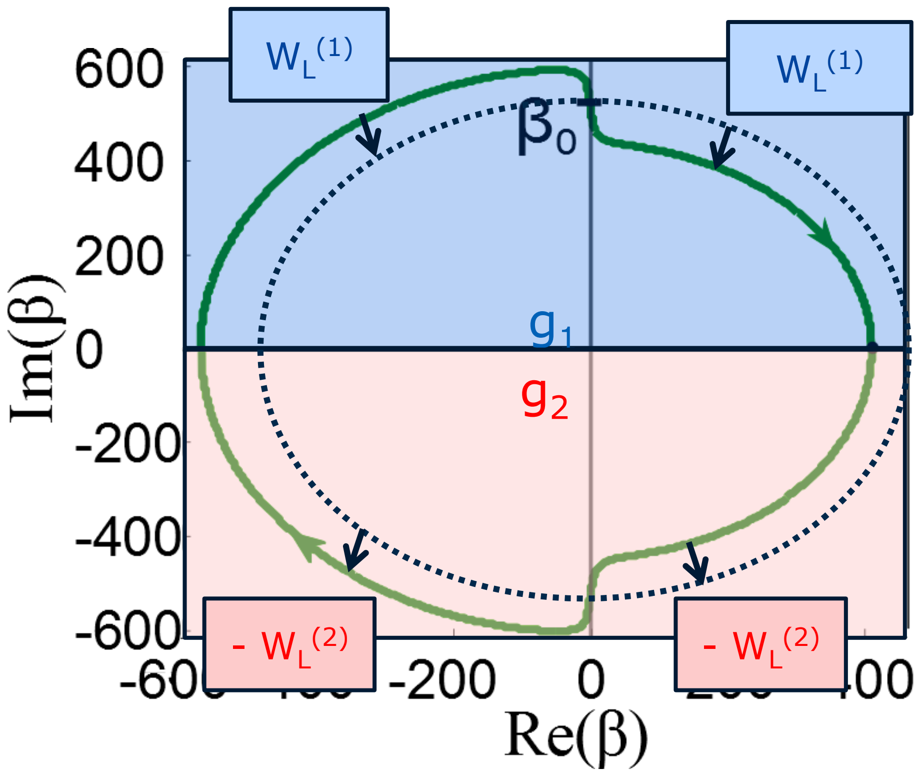
Carnot efficicieny in finite time



$$\eta = 1 - g_2 / g_1$$

$$\Leftrightarrow \quad \eta_C = 1 - T_2 / T_1$$

Carnot ideal efficiency reached with realistic parameters!



Carnot ideal efficiency reached

$$\eta = 1 - g_2 / g_1$$

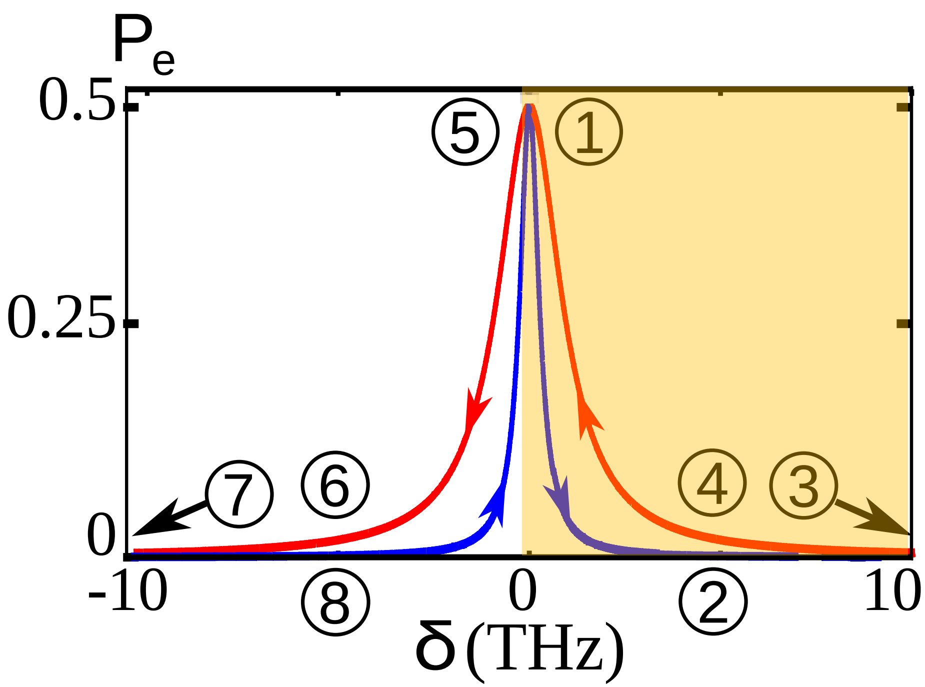
$$\Leftrightarrow \quad \eta_C = 1 - T_2 / T_1$$

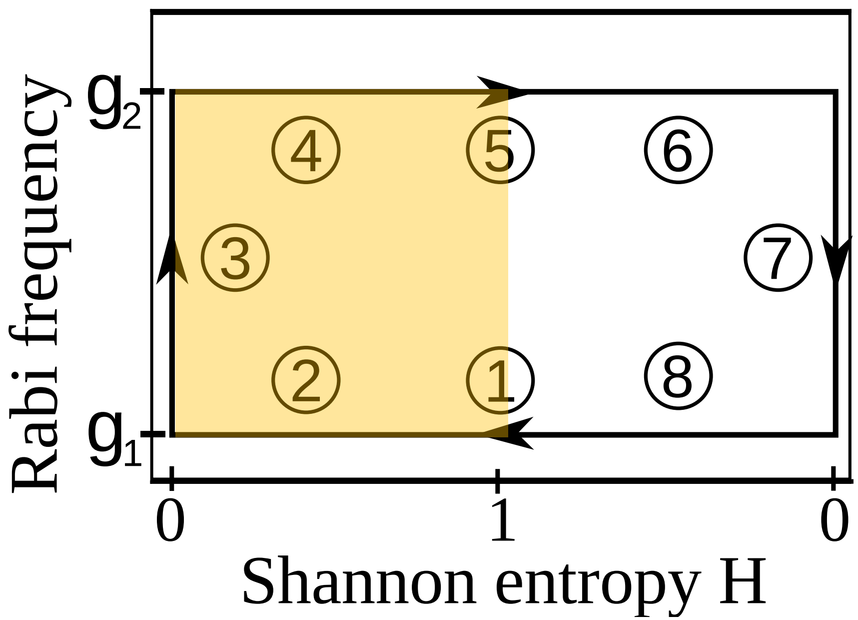
$\mathcal{P}$ = **10^{-17}** W

3 order of magnitudes over existing proposals of single qubit heat engines

O. Abah et al., PRL 109, 203006 (2012).

1st cycle: Landauer's erasure + Szilard engine



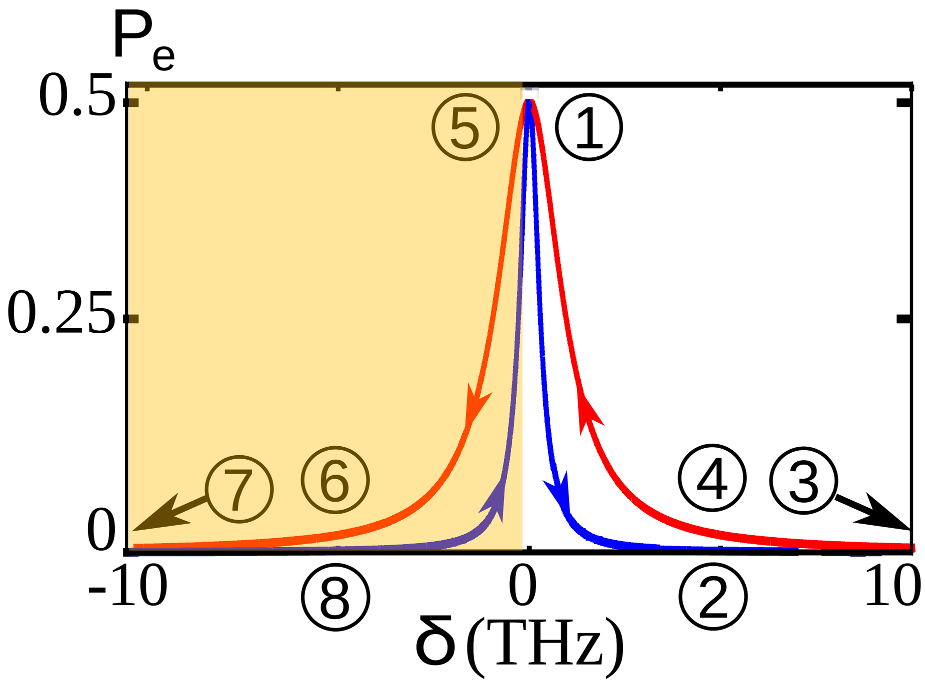


Equivalent to (P,V)

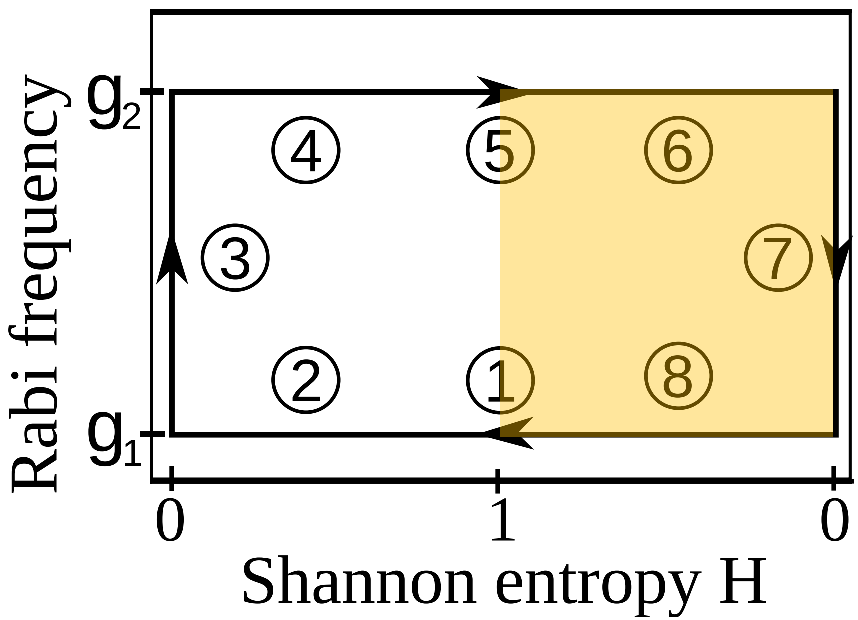
Equivalent to (S,T)

A double Carnot cycle

2nd cycle: *inverse* Landauer's erasure + *inverse* Szilard engine



Equivalent to (P,V)



Equivalent to (S,T)