Grenoble 29/09-01/10/2014

Workshop hbar kB

Jarzynski Equality and the Landauer's bound: an experimental approach

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Nature 483, 187-189 (2012)

2013 *EPL* **103** 60002 ; arXiv:1302.4417 ;

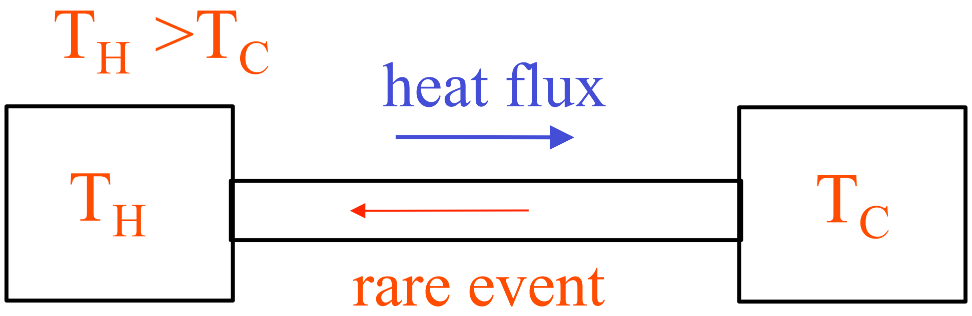
Detailed Jarzynski Equality applied on a Logically Irreversible Procedure

A few notes on stochastic thermodynamics from theory to experiment

S.Ciliberto, S. Joubaud, A. Petrosyan, JSTAT (2010) P12003 , arXiv:1009.3362

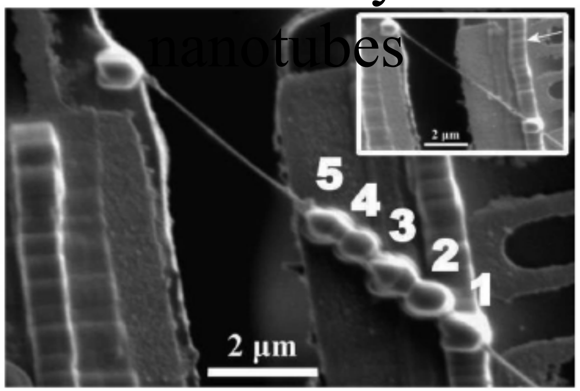
Fluctuations in out of equilibrium systems

Steady current through a system in contact between two reservoirs



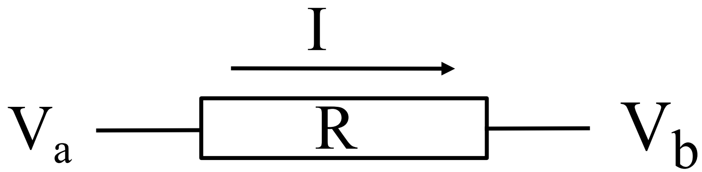
What is the probabilty that the heat flows from the cold to the hot reservoir ?

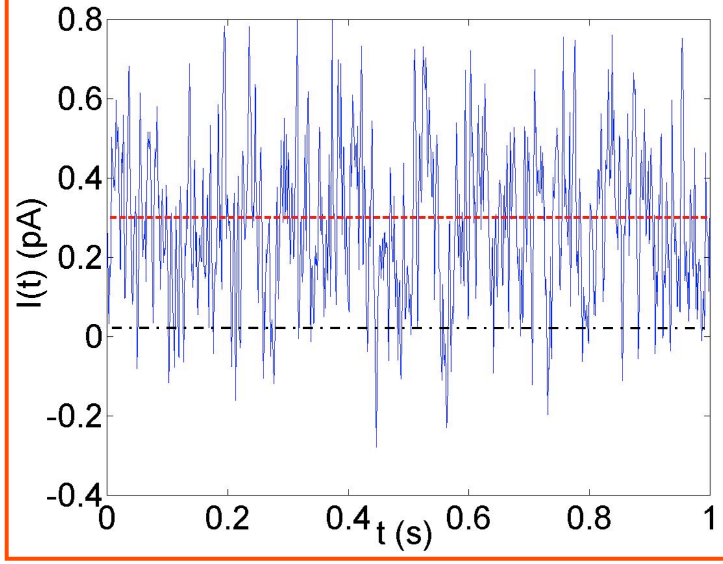
Thermal conductivity in nanotubes



C.W. Chang, et al.
PRL 101, 075903 (2008)

Electric current





R.Van Zon, et al
PRL 92, 130601 (2004)

N. Garnier, S. Ciliberto
PRE 71, 060101 (2005)

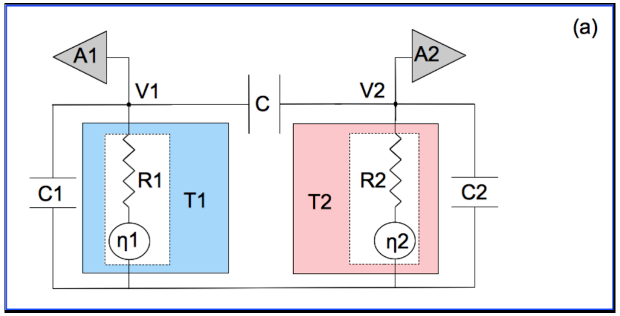
$$\bar{I} = \frac{(V_b - V_a)}{R}$$

Injected power

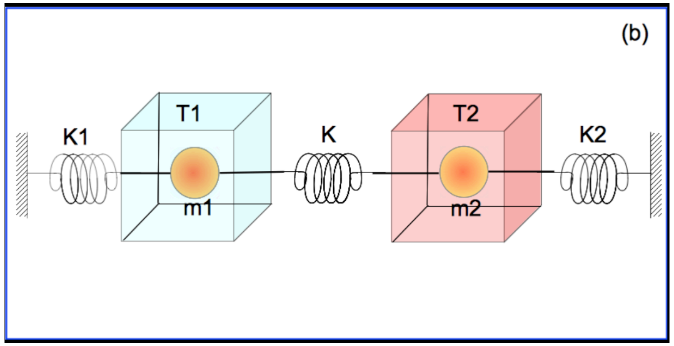
$$10^{-19} W$$

Applications of FT

Heat flux between two crcuits kept at different temperature

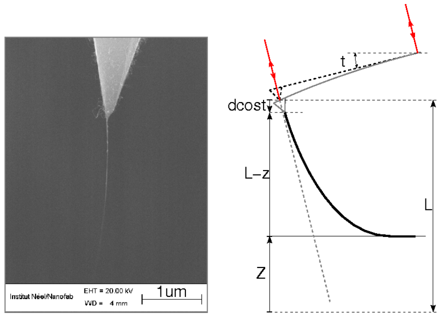


The Mechanical equivalent

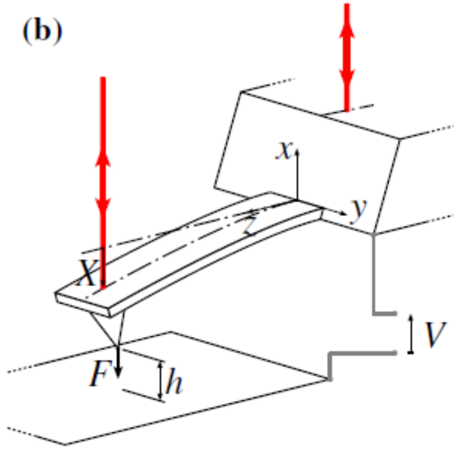


- Trapped Brownian particles
- Molecular motor
- Single molecule experiments

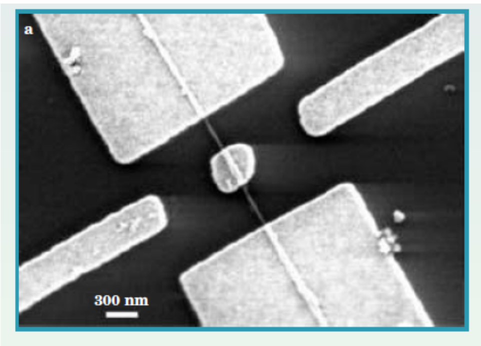
Fluctuations of injected and dissipated power in a harmonic oscillator.



Mechanical properties of nanotubes



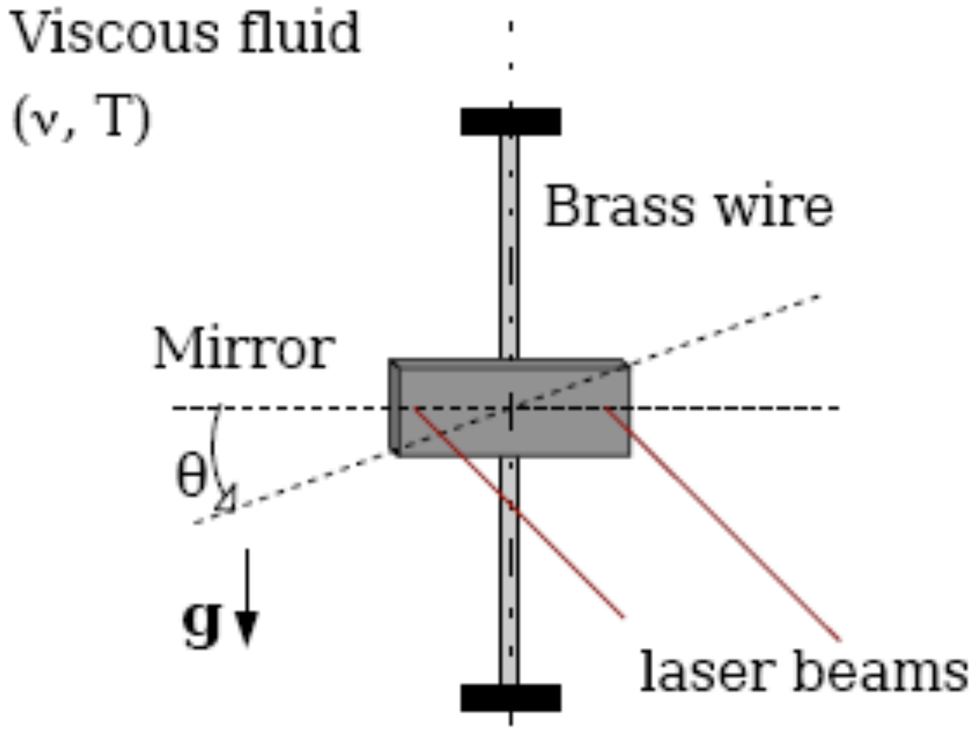
Dynamics of AFM tips



Micro Electro Mechanical Devices

Thermal rheometer

The torsion pendulum

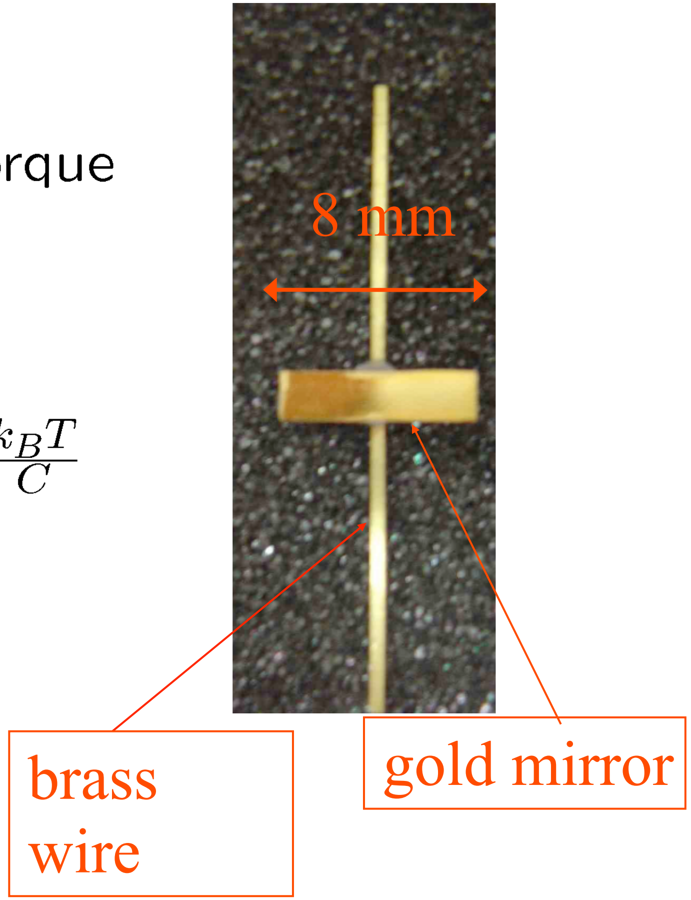


Elastic torque

$M_e = C\ \theta$

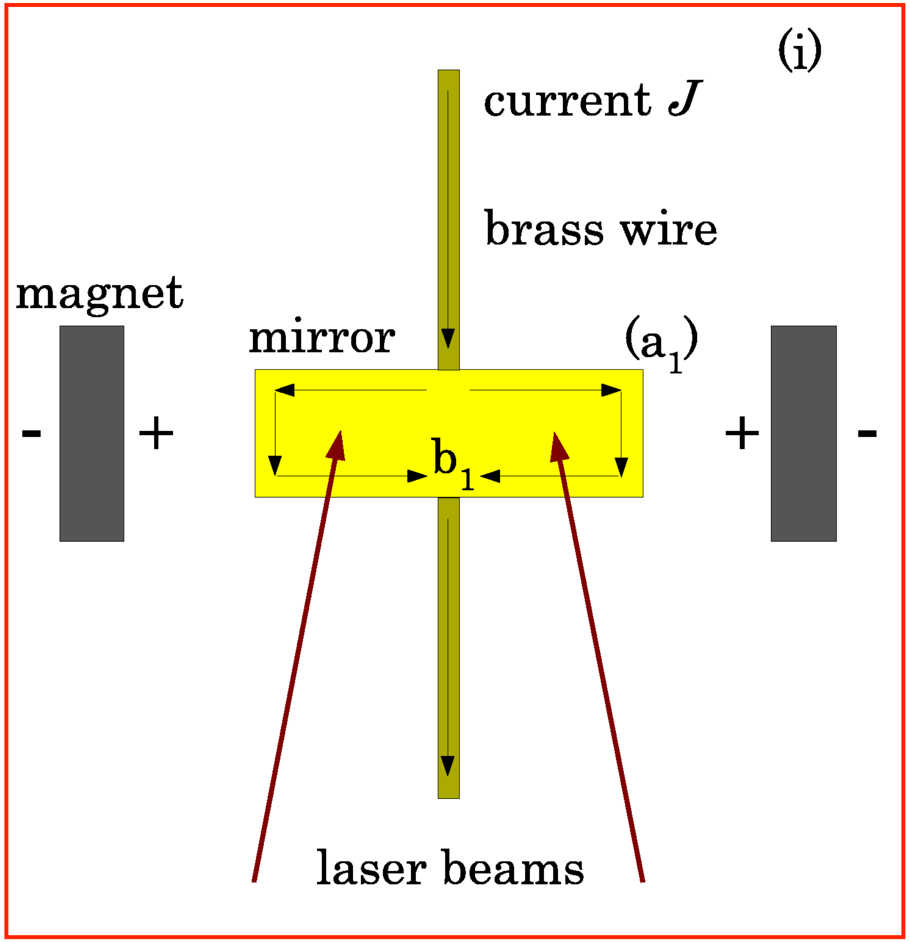
Variance

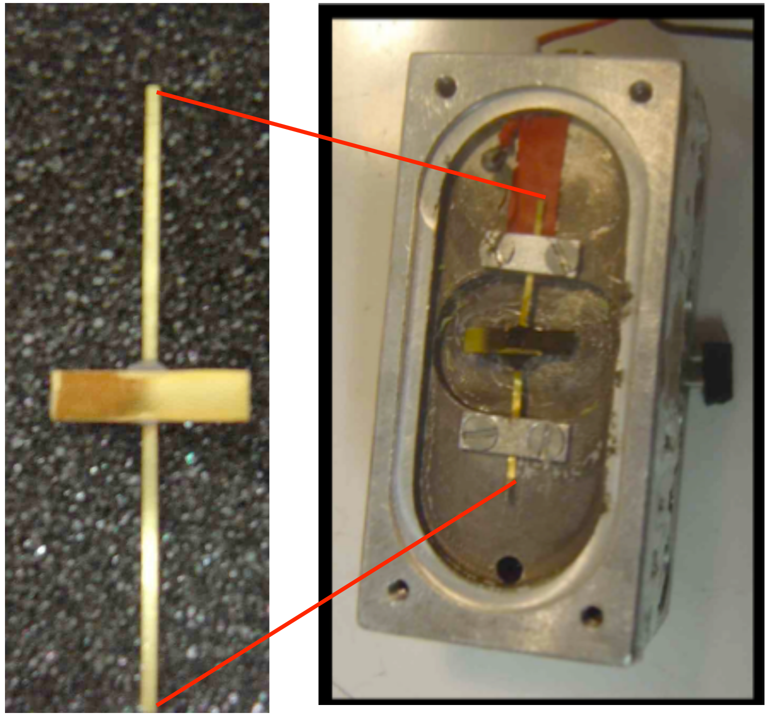
$< \theta^2 >= \frac{k_B T}{C}$



- stiffness $C = 4.7\ 10^{-4}$ Nm/rad
- typical displacement : $\sqrt{< \theta^2 >} = \sqrt{\frac{K_B\ T}{C}} \simeq$ 3nrad
- A differential interferometer is used to measure θ
- Measurement noise $\simeq$ 25 prad. Signal to noise ratio $\simeq$ 100.

External Forcing





The applied torque $M \propto J$

Typical applied torque < 50pN m

Equation of motion

ENS DE LYON

$$I_{\text{eff}}\,\ddot{\theta} + \int_{-\infty}^{t} G(t-t')\,\dot{\theta}(t')\;dt' + C\theta = M + \eta,$$

In Fourier space

$$[-I_{\text{eff}}\,\omega^2 + \hat{C}]\,\hat{\theta} = \hat{M},$$

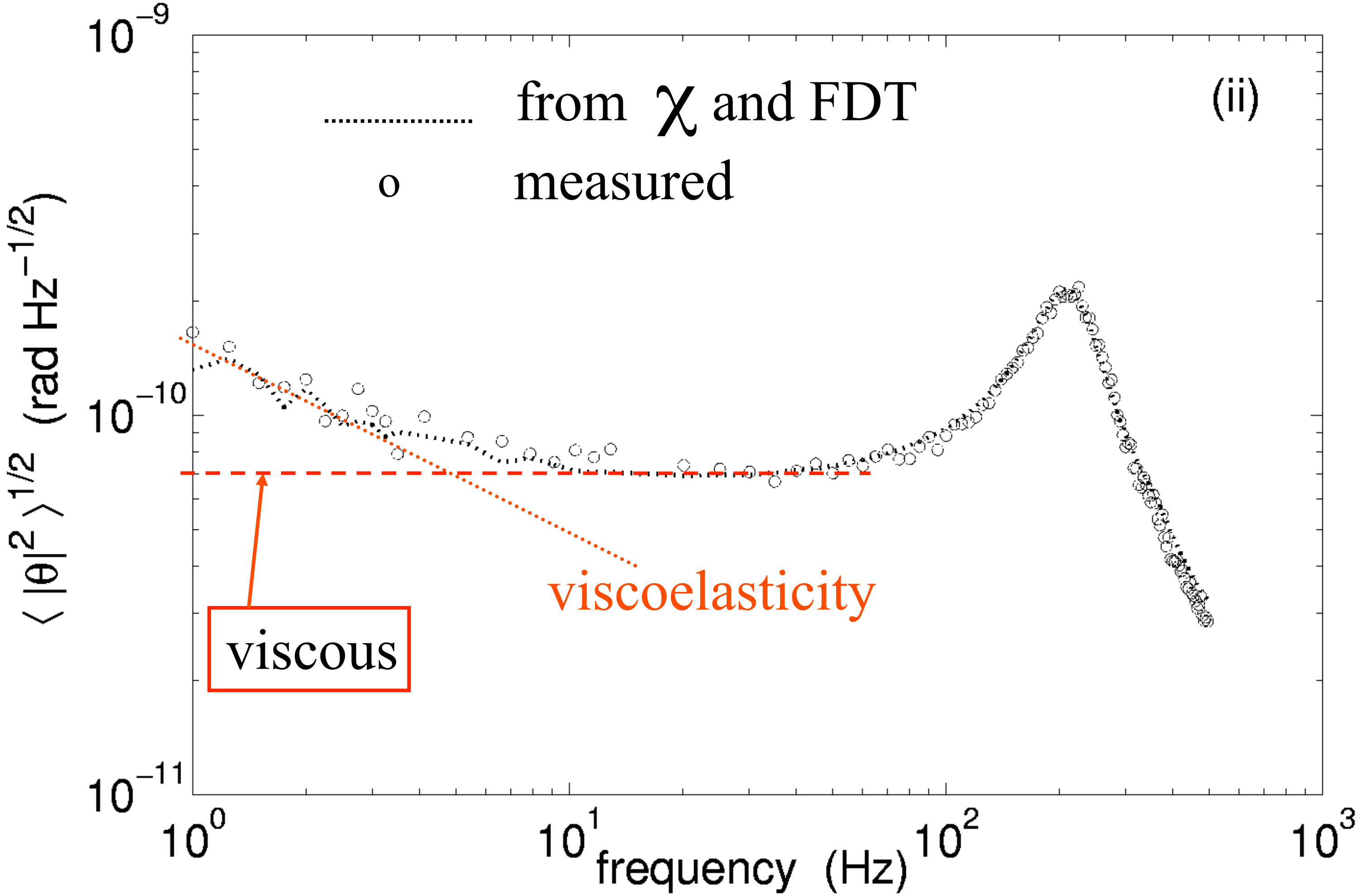
where $\hat{C} = C + i[C_1'' + \omega\nu]$ is the complex frequency-dependent elastic stiffness

The response function is $\hat{\chi} = \frac{\hat{\theta}}{\hat{M}}$

The thermal fluctuation power spectral density is given by FDT

$$\langle|\hat{\theta}|^2\rangle = \frac{4k_BT}{\omega}\,\mathrm{Im}\,\hat{\chi} = \frac{4k_BT}{\omega}\frac{C_1'' + \omega\,\nu''}{[-I_{\text{eff}}\,\omega^2 + C]^2 + [C_1'' + \omega\,\nu]^2}.$$

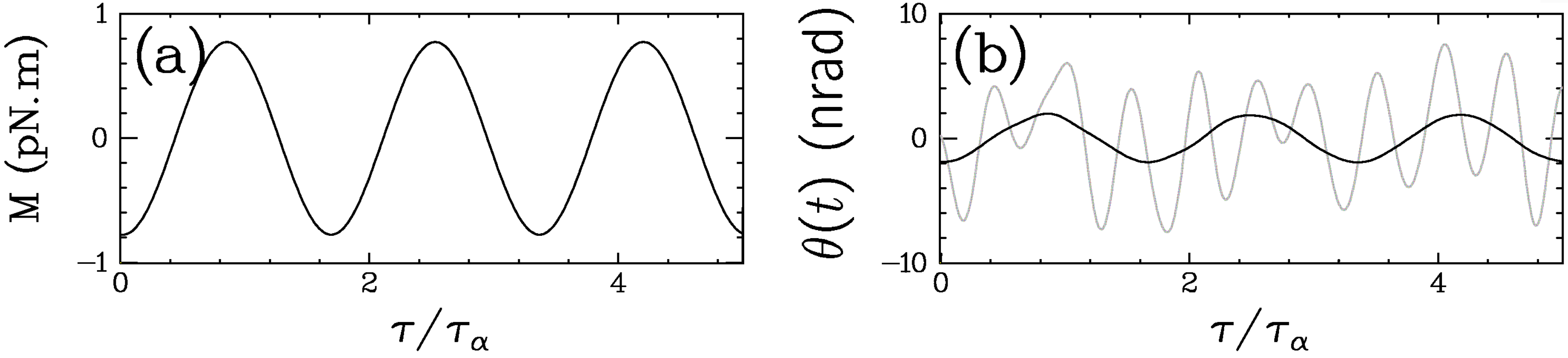
Fluctuation Spectrum



$$f_o = \sqrt{C/I_{\text{eff}}}/(2\pi) = 217\text{Hz}$$

relaxation time $\tau_\alpha = 2I_{\text{eff}}/\nu = 9.5\text{ms}.$

Work during periodic forcing



$$M(t) = M_o \sin \omega_d t$$

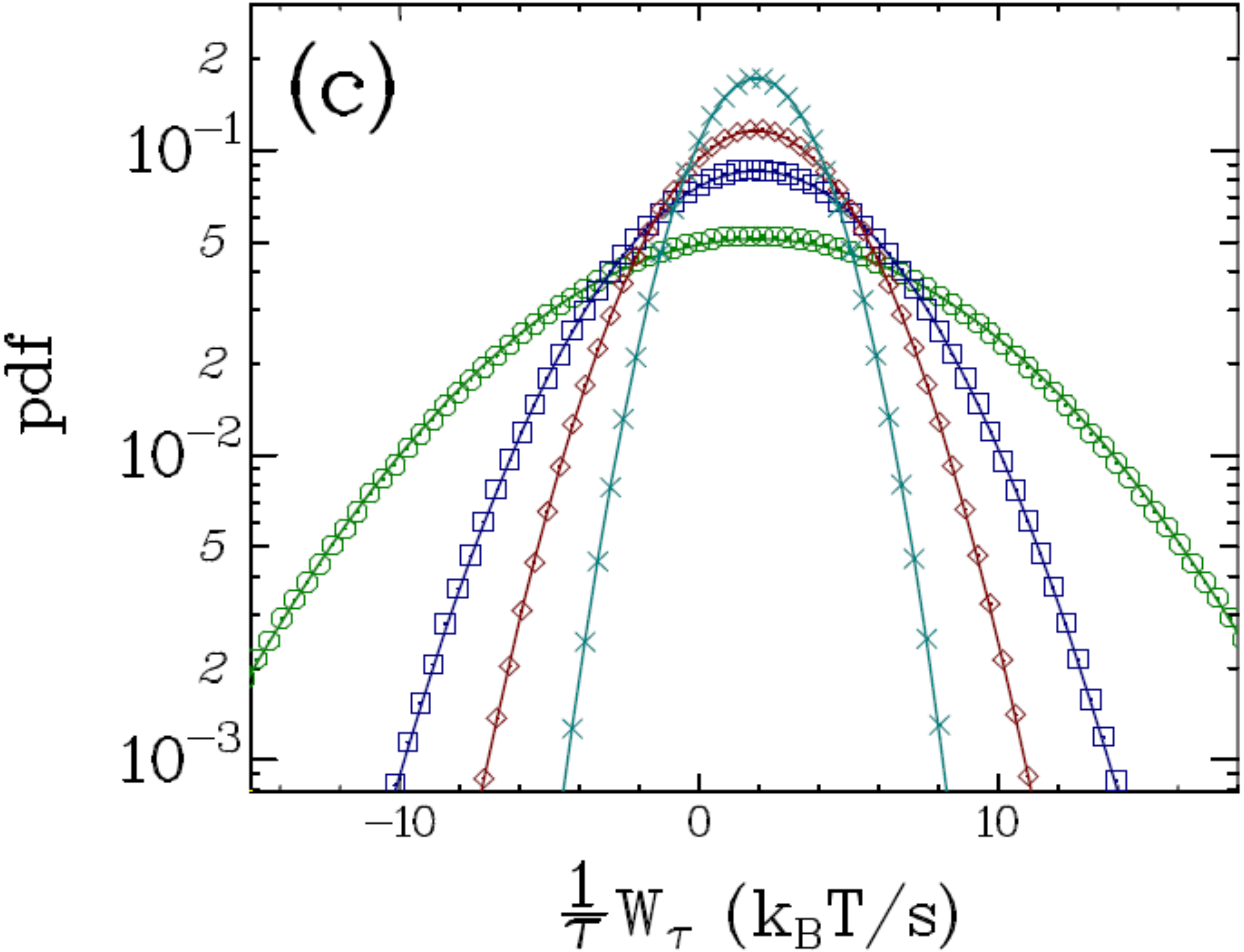
$$W_n = W_{\tau=\tau_n} = \int_{t_i}^{t_i+\tau_n} M(t) \frac{d\theta}{dt} dt \,,$$

with $\tau_n = n 2\pi / \omega_d$

W_τ is a fluctuating quantity

PDF of the work

$n = 7$ (○), $n = 15$ (□), $n = 25$ (◇) and $n = 50$ (×).



Energy Balance (I)

ENS DE LYON

Sekimoto K, Progress of Theoretical Phys. supplement (130), 17 (1998).

$$I_{\mathrm{eff}} \frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + \nu \frac{\mathrm{d}\theta}{\mathrm{d}t} + C\,\theta = M + \sqrt{2k_B T \nu}\,\eta,$$

- We multiply this equation by $\dot{\theta}$ and we get : $\frac{dU(t)}{dt} = P_{inj}(t) - P_{dis}(t)$
- The injected power : $P_{inj}(t) = M(t)\frac{\mathrm{d}\theta(t)}{\mathrm{d}t}$
- The dissipated power : $P_{diss}(t) = \nu \left[\frac{\mathrm{d}\theta(t)}{\mathrm{d}t}\right]^2 - \sqrt{2k_B T \nu}\ \ \eta(t)\ \frac{\mathrm{d}\theta(t)}{\mathrm{d}t}.$
- The internal energy : $U(t) = \left\{\frac{1}{2} I_{\mathrm{eff}} \left[\frac{\mathrm{d}\theta(t)}{\mathrm{d}t}\right]^2 + C\ \theta(t)^2\right\}.$

Energy Balance (II)

Sekimoto K, Progress of Theoretical Phys. supplement (130), 17 (1998).

$$\frac{dU(t)}{dt} = P_{inj}(t) - P_{dis}(t)$$

- We integrate over a time τ starting at a time t_i. We get:

$$\Delta U_\tau = U(t_i + \tau) - U(t_i) = W_\tau - Q_\tau$$

- W_τ is the work done on the system over a time τ :

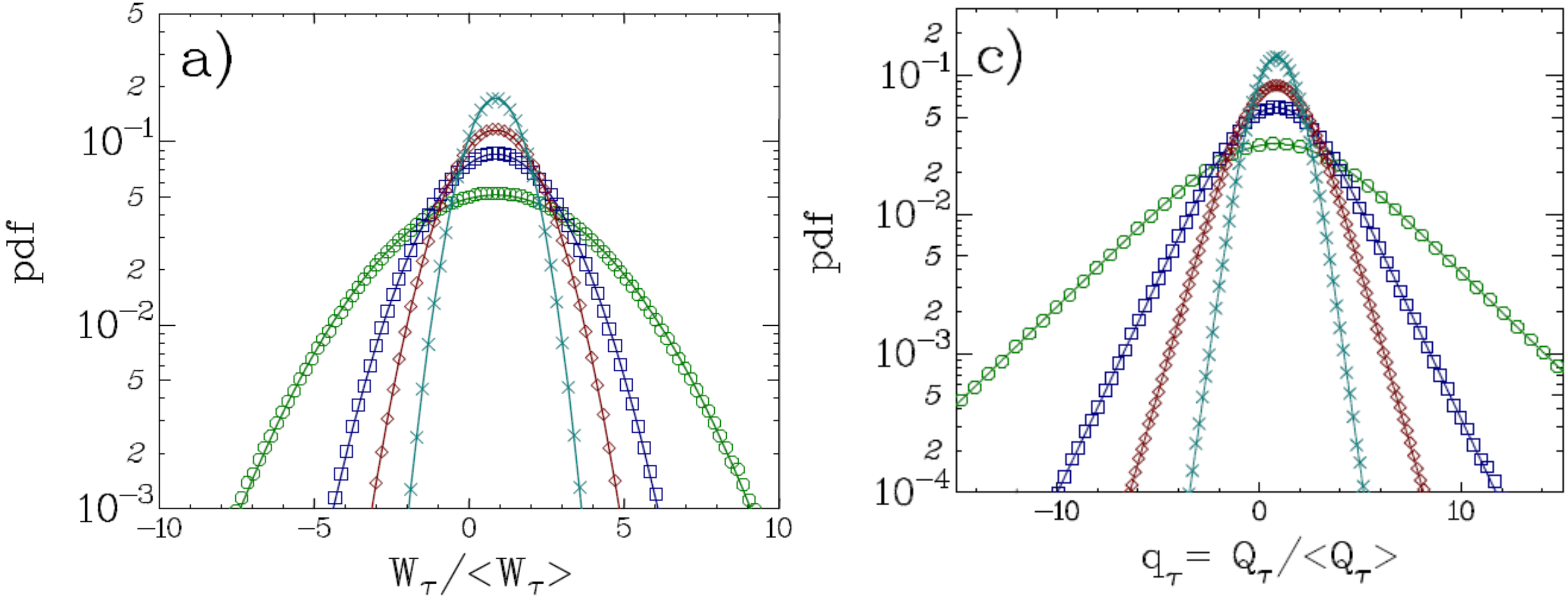
$$W_\tau = \int_{t_i}^{t_i+\tau} M(t') \frac{\mathrm{d}\theta}{\mathrm{d}t}(t') \mathrm{d}t'$$

- $Q_\tau = W_\tau - \Delta U_\tau$ is the heat dissipated by the system.

We study the fluctuations of W_τ, Q_τ and
the Fluctuation Theorem for these two quantities

PDF of the work and of the heat

$n = 7$ ($\circ$), $n = 15$ ($\square$), $n = 25$ ($\diamond$) and $n = 50$ ($\times$).



$$< W_\tau > = < Q_\tau > \simeq 0.04\ n\ (k_B T)$$

ENS DE LYON

Stationary State Fluctuation Theorem (SSFT)

(stochastic systems)

$$\log \frac{P(X_\tau)}{P(-X_\tau)} = \frac{X_\tau}{k_B T}\,\Sigma(\tau)$$

where $\Sigma(\tau) \rightarrow 1$ for $\tau \rightarrow \infty$

X_τ stands either for Q_τ or for W_τ

The Fluctuation Theorem fixes the symmetry of P(X) around zero

Transient Fluctuation Theorem (TFT)

At $\tau = 0$ the system is in equilibrium

$$\Sigma(\tau) = 1 \qquad \forall \tau$$

The Fluctuation Theorem (FT)

- 1993 First numercial evidence of fluctuations relations
  D. Evans, E.D.G. Cohen and G. P. Morris.

- 1994 Proof of the transient fluctuation theorem (TFT)
  D. Evans and D.J.Searles

- 1995 Proof of the Stationary State Fluctuation Theorem (SSFT) for
  dynamical systems. G. Gallavotti and E.D.G. Cohen.

- 1997 Later proofs of FT for systems with stochastic dynamics were given by
  J. Kurchan, J. Lebowitz and E. Spohn, J. Farago.

- 2003 R. van Zon and E.G.D. Cohen extended the results
  to the heat fluctuations in stochastic systems

- New kinds of relations for suitably defined entropies have been proposed for
  stochastic system.

Short comment on FT for Gaussian P(Xτ)

FT imposes that:

$$\boxed{\log \frac{P(X_\tau)}{P(-X_\tau)} = \frac{X_\tau}{k_B T}\,\Sigma(\tau)}$$

if $$P(X_\tau) = A\ \exp\left[-\frac{(X_\tau - <X_\tau>)^2}{2\delta_\tau^2}\right]$$

then from FT $$\delta_\tau^2 = 2\ k_B\ T\ \ <X_\tau>$$

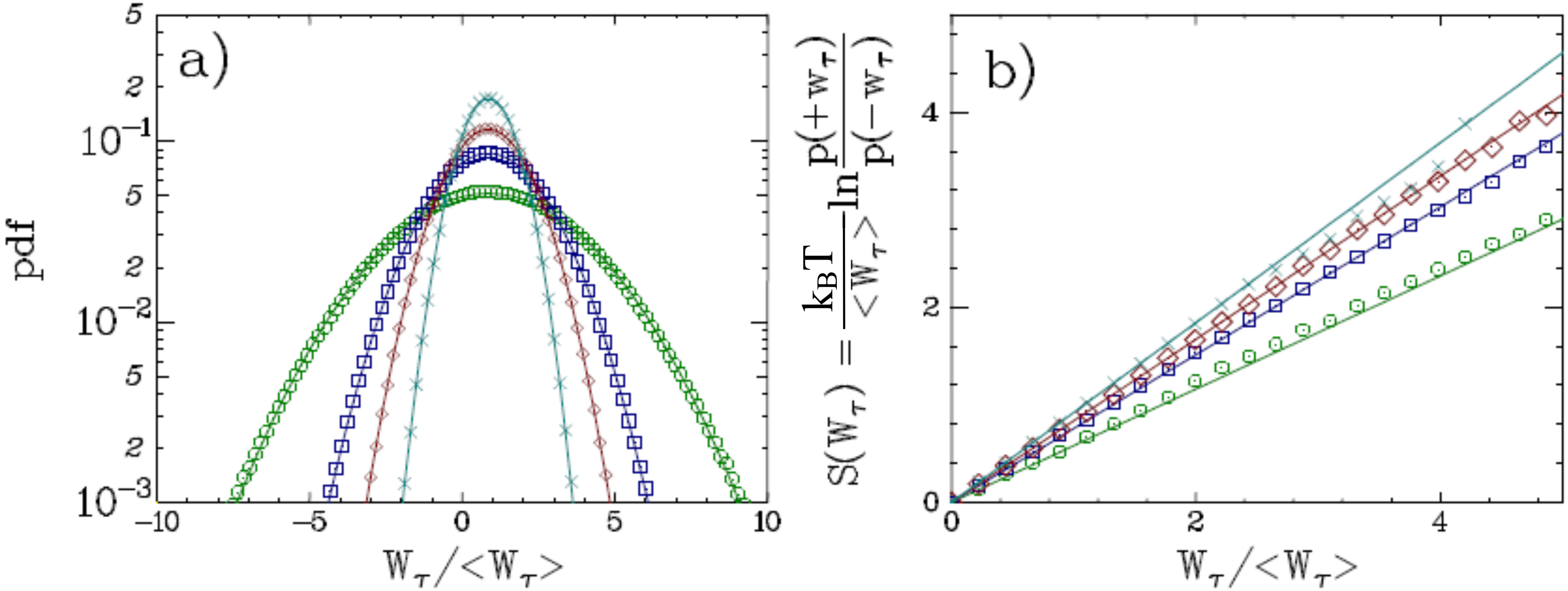
and $$\frac{\delta_\tau}{<X_\tau>} = \sqrt{\frac{2\ k_B\ T}{<X_\tau>}}$$

SSFT periodic forcing: W

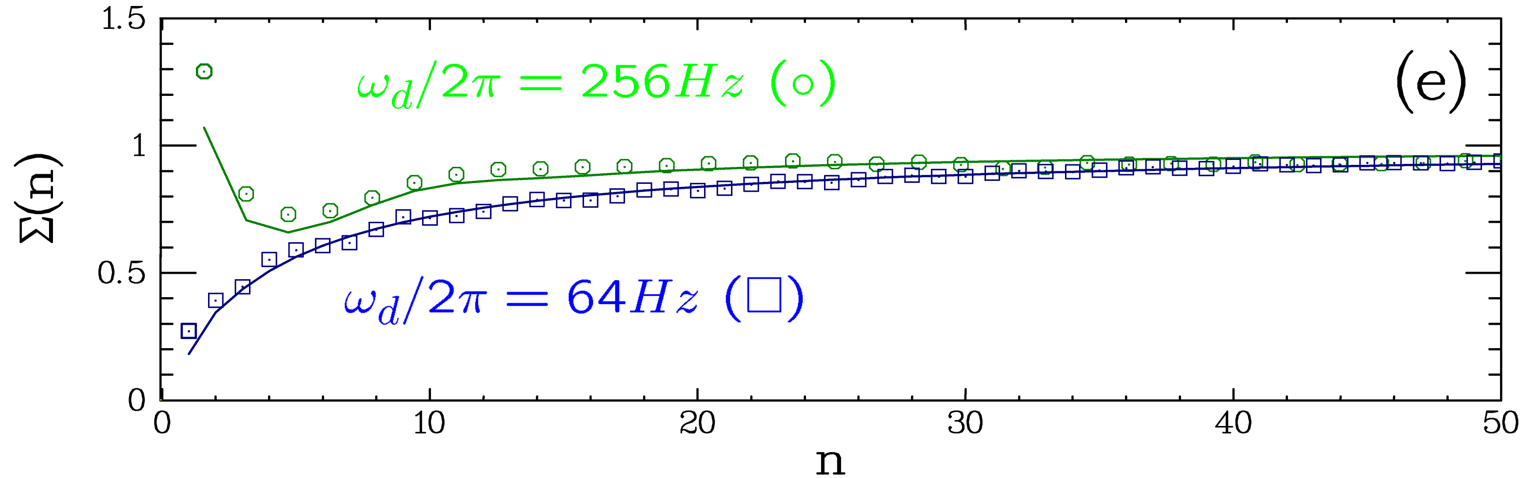
$$\frac{k_B\ T}{< W_\tau >} \log \frac{P(W_\tau)}{P(-W_\tau)} = \frac{W_\tau}{< W_\tau >} \Sigma(\tau)$$

$$\omega_d/2\pi = 64\text{Hz} < \omega_o/2\pi$$

$n = 7$ ($\circ$), $n = 15$ ($\square$), $n = 25$ ($\diamond$) and $n = 50$ ($\times$).



SSFT periodic forcing: Σ for W



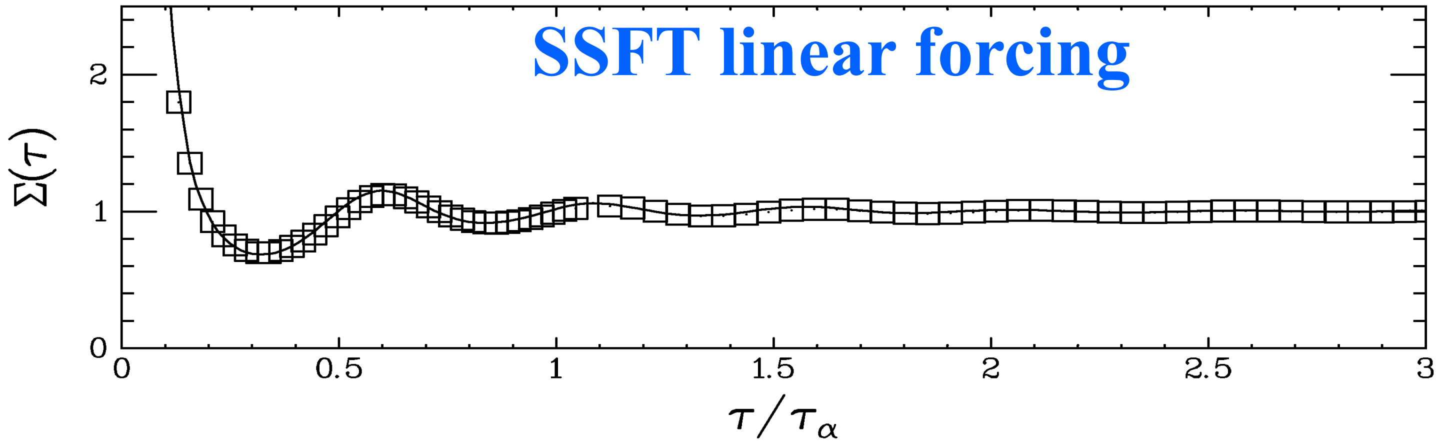
$$\frac{k_B\ T}{< W_\tau >} \log \frac{P(W_\tau)}{P(-W_\tau)} = \frac{W_\tau}{< W_\tau >} \Sigma(\tau)$$

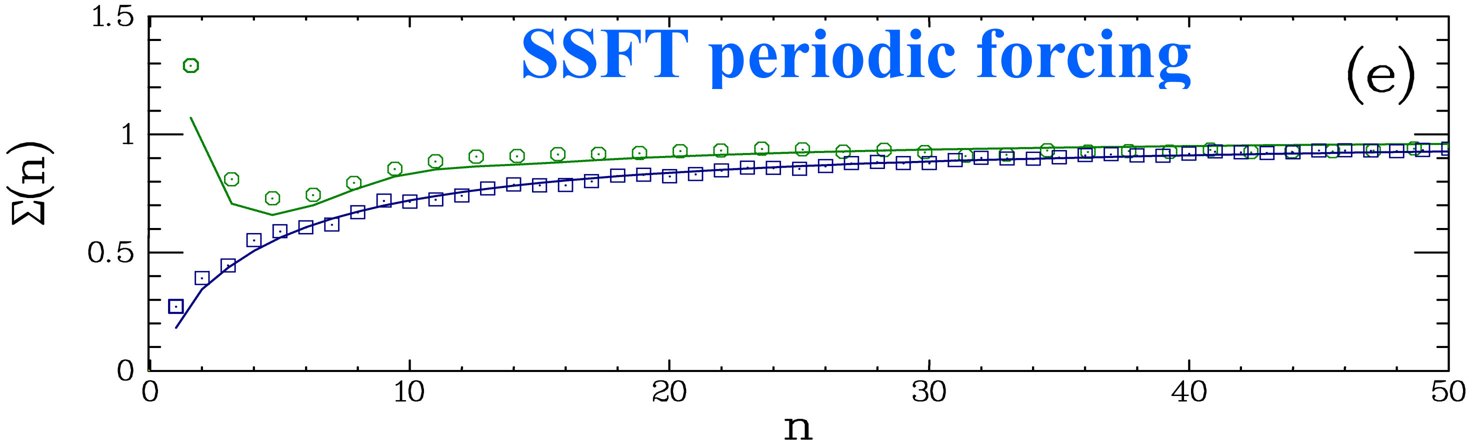
Analytically computed from the Langevin equation using two experimental observations

- The statistical properties of the bath are not modified by the driving
- The fluctuations of the work are Gaussian

S. Joubaud, N. B. Garnier, S. Ciliberto, J. Stat. Mech., P09018 (2007)

Non-Universality of $\Sigma(\tau)$ for W

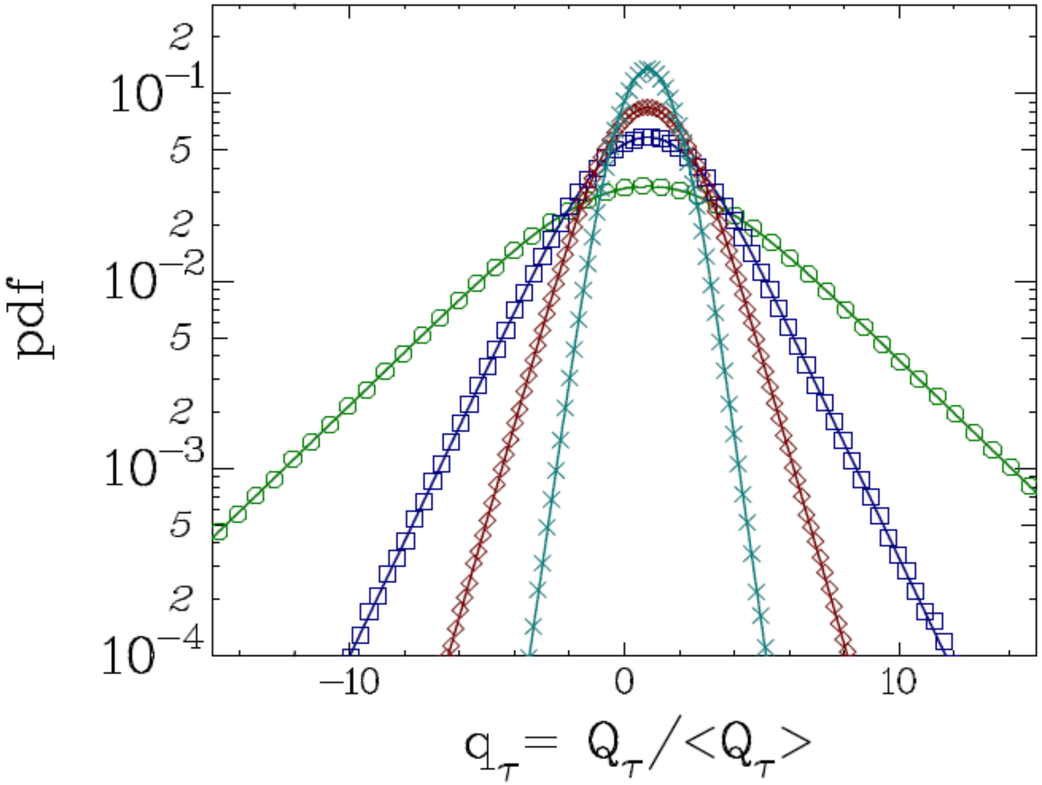


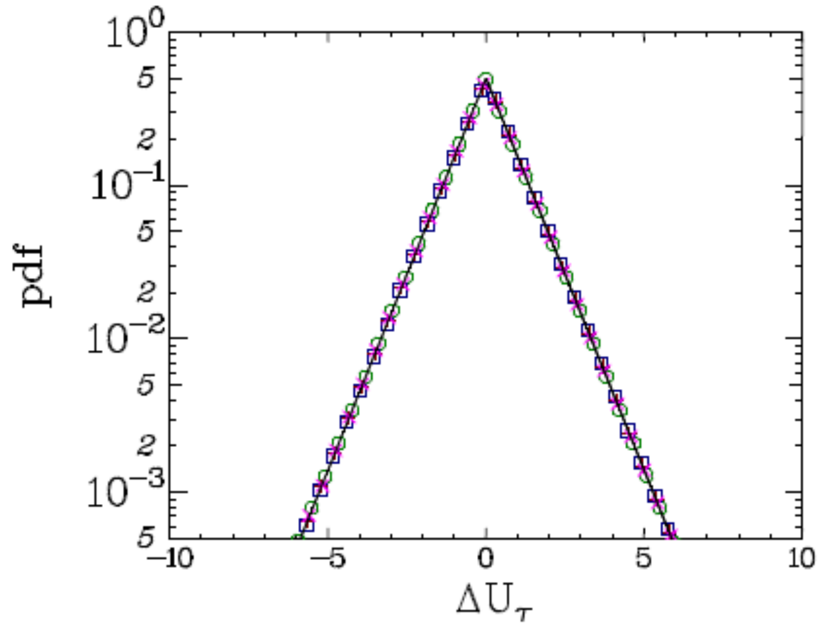


$$\log \frac{P(+W_\tau)}{P(-W_\tau)} = \frac{W_\tau}{k_B\ T}\,\Sigma(\tau)$$

SSFT periodic forcing: Q

$n = 7$ ($\circ$), $n = 15$ ($\square$), $n = 25$ ($\diamond$) and $n = 50$ ($\times$).



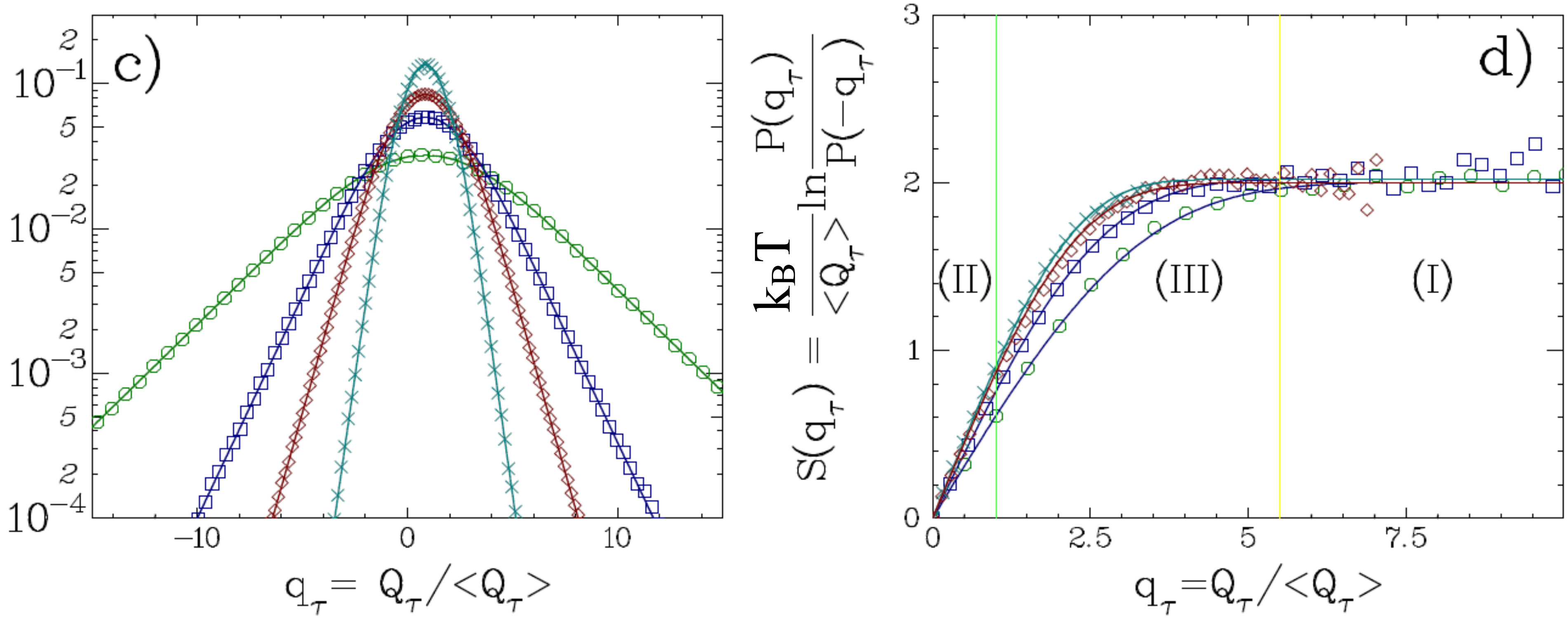


$$P(q) = \frac{\exp\left(\frac{\sigma^2}{2}\right)}{4}\left(\exp(q-\bar{q})\left[erfc\left(\frac{q-\bar{q}+\sigma_W^2}{\sqrt{2\sigma_W^2}}\right)\right] + \exp(-(q-\bar{q}))\left[erfc\left(\frac{-q+\bar{q}+\sigma_W^2}{\sqrt{2\sigma_W^2}}\right)\right]\right)$$

S. Joubaud, N. B. Garnier, S. Ciliberto, J. Stat. Mech., P09018 (2007)

SSFT periodic forcing: Σ for Q

$n = 7$ (○), $n = 15$ (□), $n = 25$ (◇) and $n = 50$ (×).



3 regions :

(I) Large fluctuations are exponential: $S(q_\tau) = 2$ for $q_\tau > 3$

(II) for $q_\tau < 2$, $S(q_\tau) = \Sigma(n)\ q_\tau$ with $\Sigma(n) \to 1$ for $n \to \infty$

(III) Smooth connection .

Trajectory dependent entropy

- U. Seifert, Phys. Rev. Lett., 95, 040602, (2005), for Langevin dynamics
- A. Puglisi, L. Rondoni, A. Vulpiani, J. Stat. Mech.: Theory and Experiment, P08010,(2006) for Markov process

Heat dissipated by the system towards the heat bath:

$$Q_\tau = W_\tau - \Delta U_\tau \, .$$

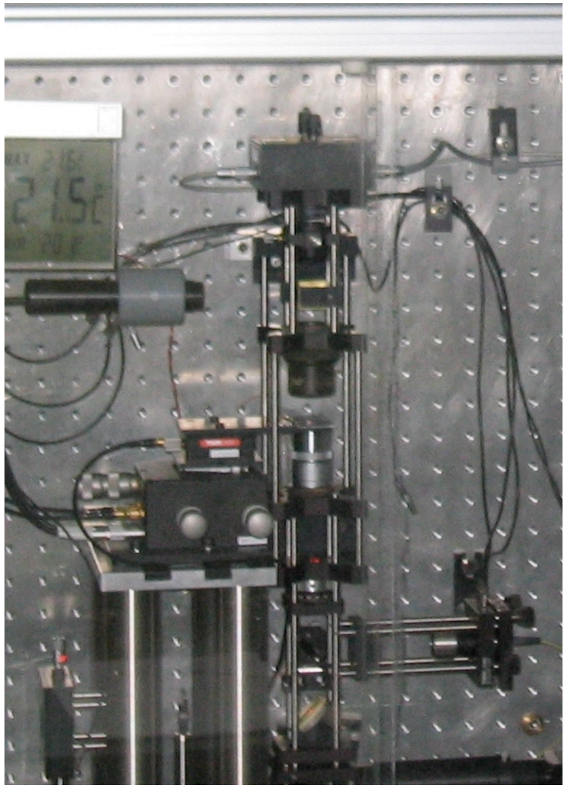
we define the entropy variation in the system during a time τ as :

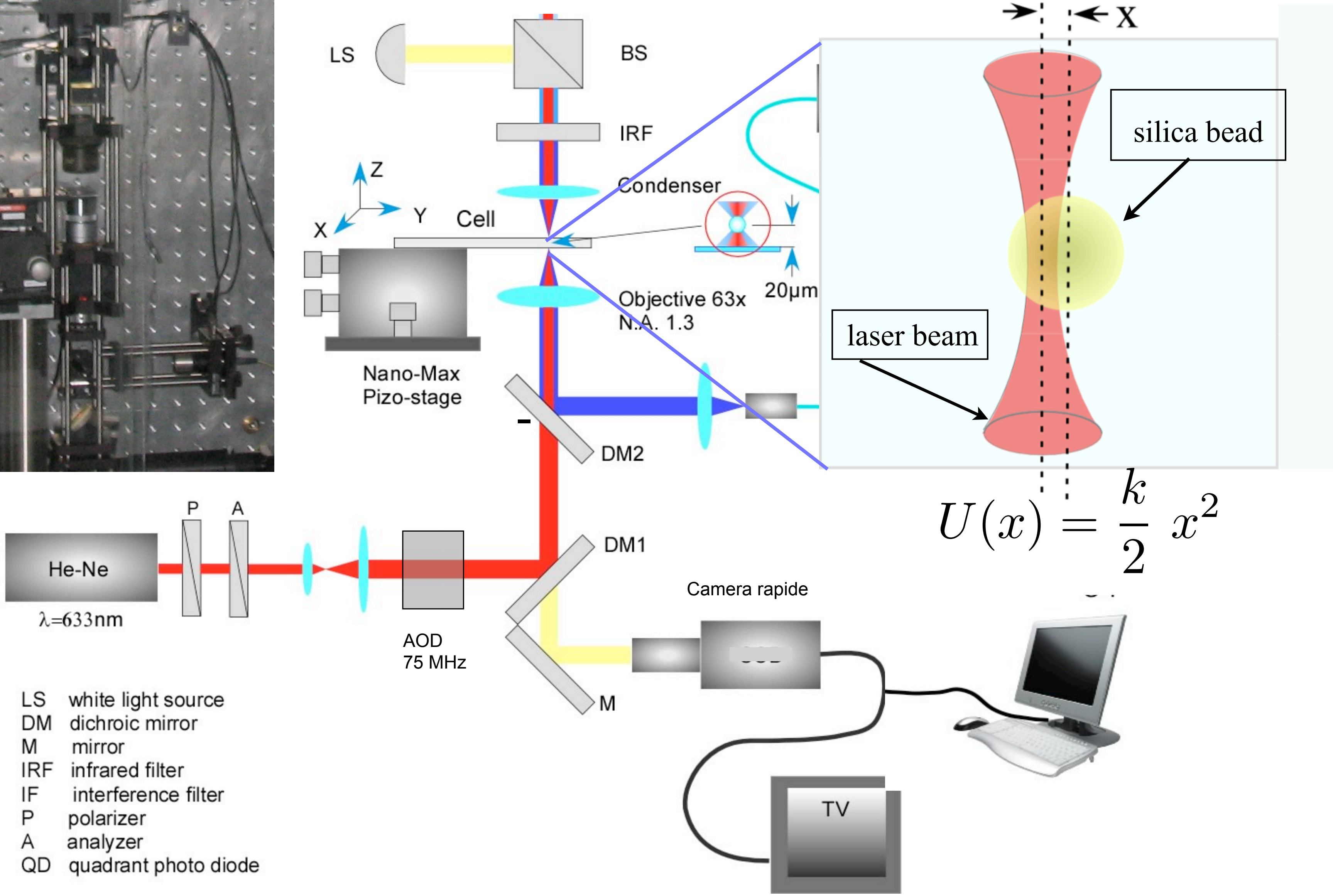
$$\Delta s_{\mathrm{m},\tau} = \frac{1}{T} Q_\tau$$

For thermostated systems, entropy change in medium behaves like the dissipated heat. The non-equilibrium Gibbs entropy is :

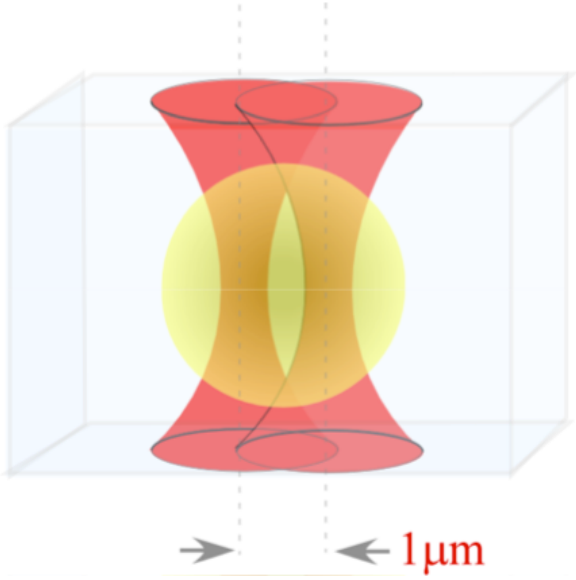
$$\langle s(t) \rangle = -k_B \int \; d\vec{x} \; p(\vec{x}(t), t, \lambda_t) \ln p(\vec{x}(t), t, \lambda_t)$$

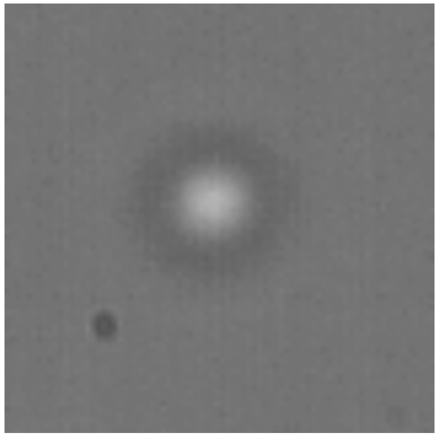
Experimental set-up Optical trap

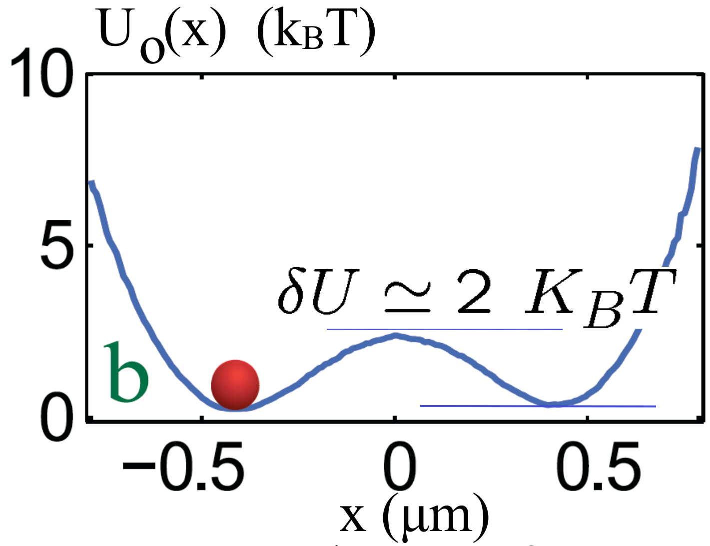




Brownian particle trapped by two laser beams







$$U_o(x) = a\ x^4 - b\ x^2\ +\ d\ x$$

The Kramers time

$$\tau_K = \tau_o\ \exp[\frac{\delta U}{k_B T}]$$

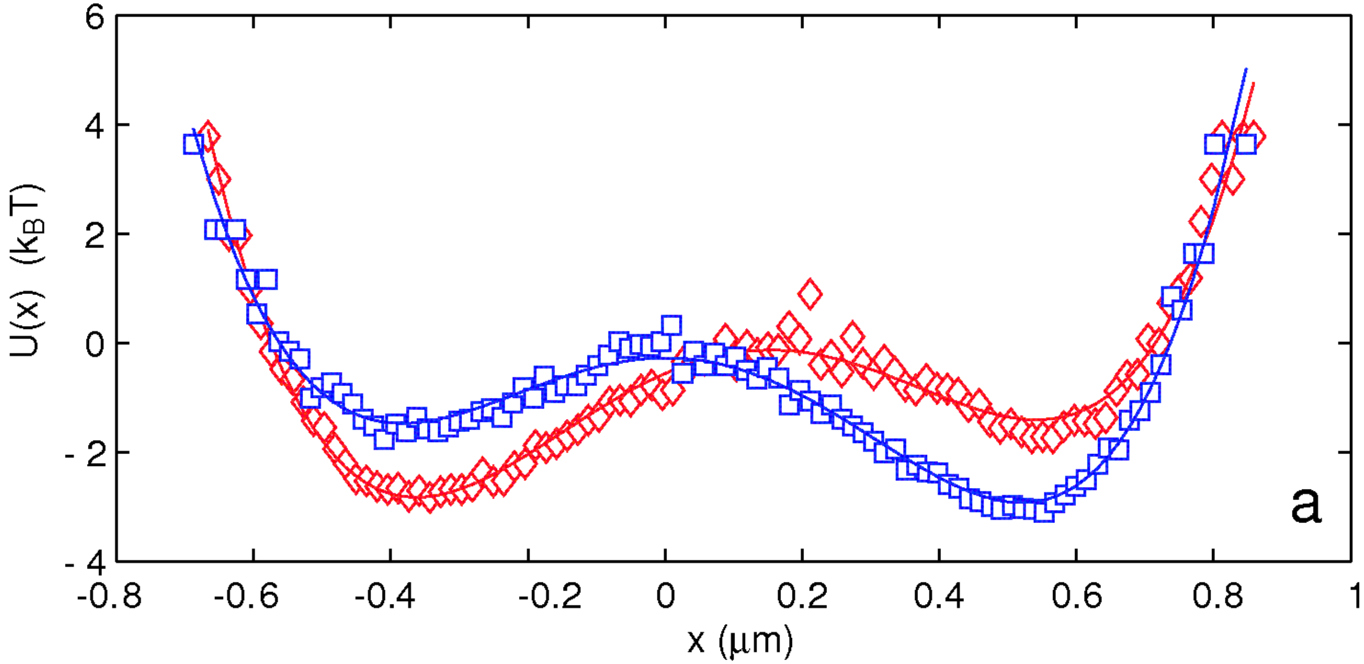
with $\tau_o = 1\ s$

Potential measured using detailed balance

$$\frac{\omega_{i \to j}}{\omega_{j \to i}} = e^{-\frac{\Delta U_{ij}}{k_B T}}$$

with $\Delta U_{i,j} = U(x_i) - U(x_j)$

Stochastic Resonance and FT



Kramer rate

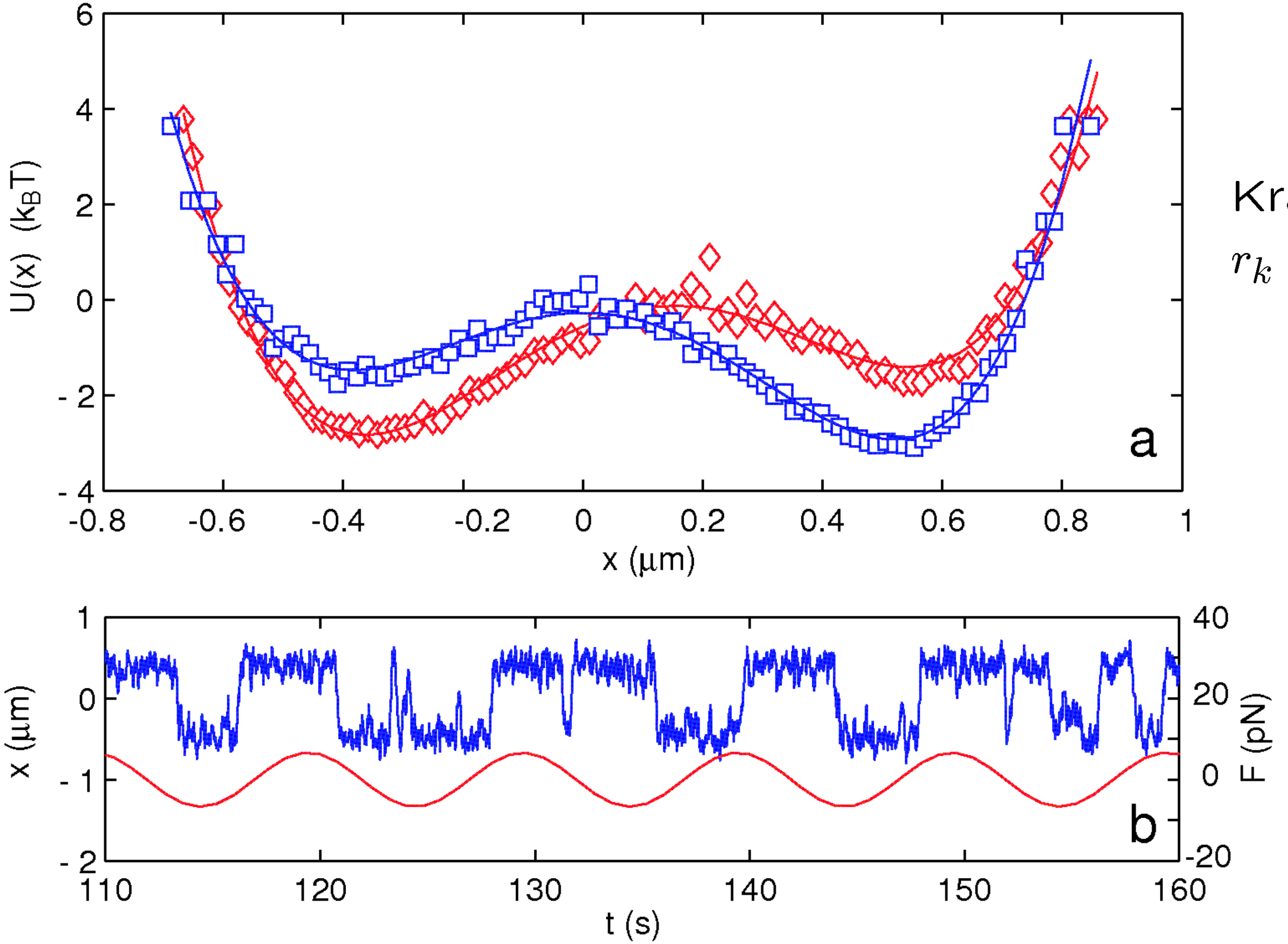
$$r_k = \tau_o^{-1} \exp(-\frac{\delta U}{k_B T})$$

$$U_0(x) = ax^4 - bx^2 - dx$$

$$U(x,t) = U_0(x) + U_p(x,t) = U_0 + c\ x\ \sin(2\pi f t),$$

$$\nu\ \dot{x} = -\frac{\partial U_o(x)}{\partial x} - c\ \sin(2\pi f t) + \eta$$

The non linear potential



Kramer rate

$r_k = \tau_o^{-1} \exp(-\frac{\delta U}{k_B T})$

f=0.1Hz

At $f \simeq r_k$ the hops of the particle synchronise with the external forcing

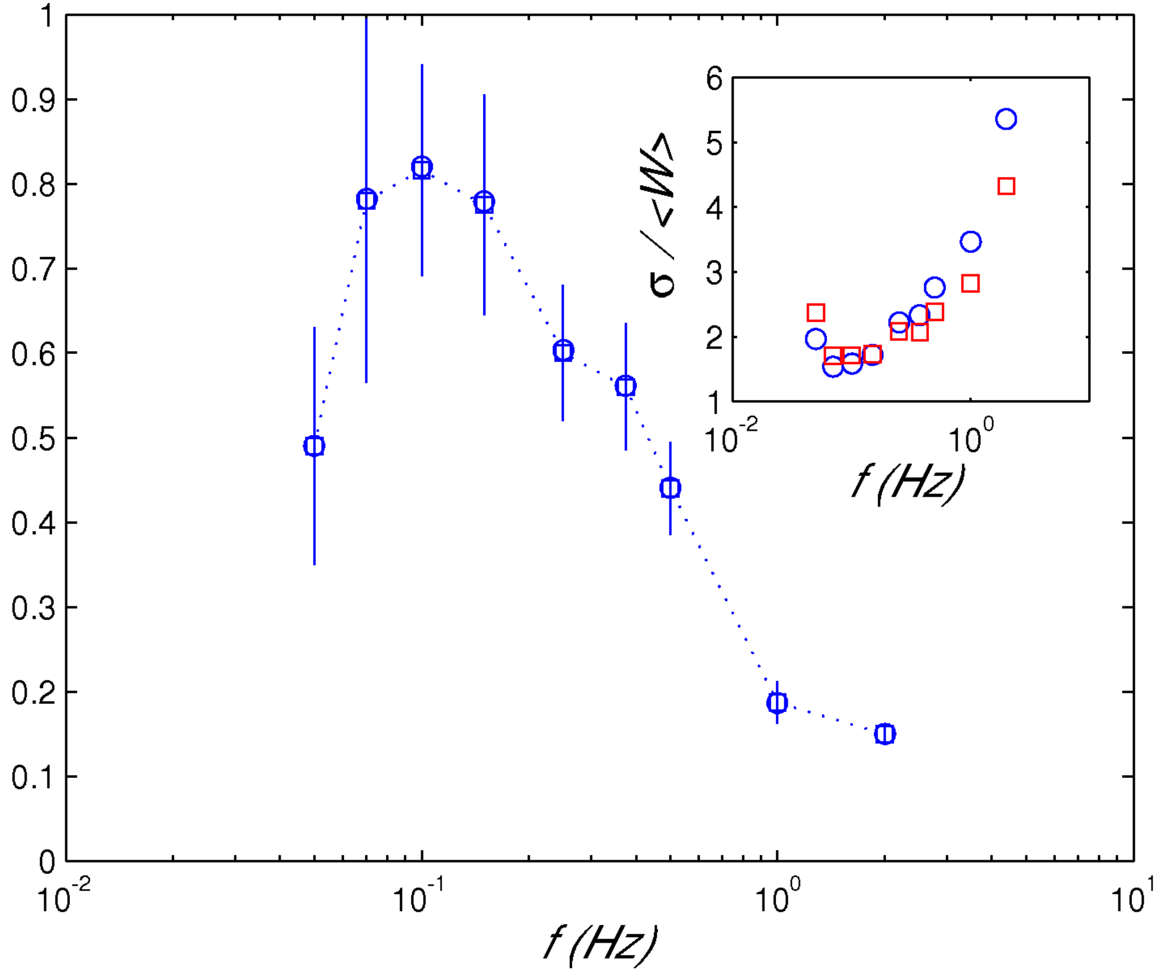
Stochastic Resonance

At $f \simeq r_k$ the hops of the particle synchronise with the external forcing

$$W_\tau = c \int_{t_i}^{t_i+\tau_n} \dot{x}\ \sin(2\pi f t) dt \qquad \text{with } \tau_n = n/f$$

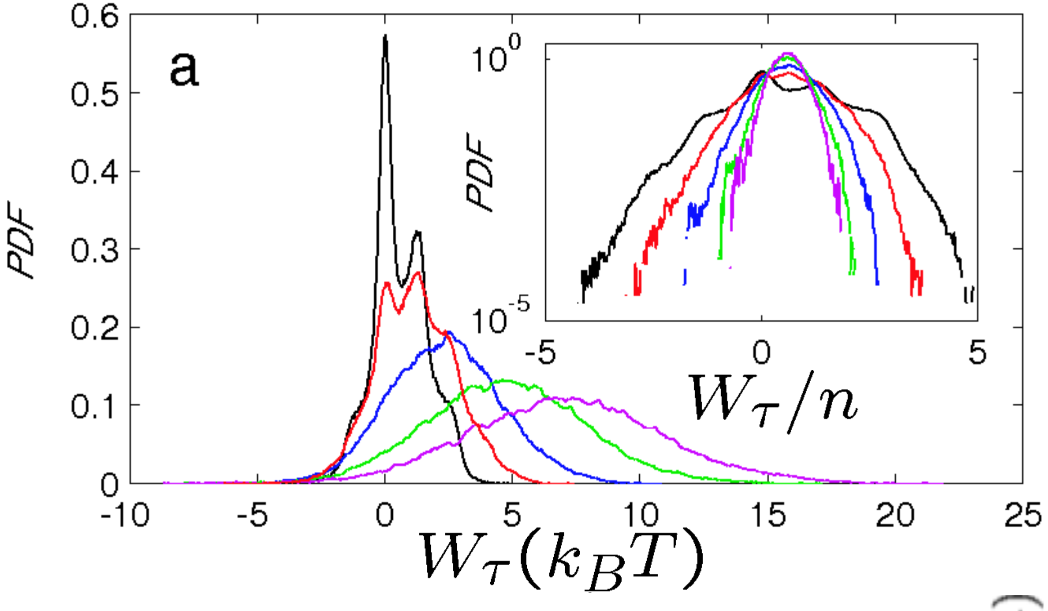
$< W_\tau >$

$n = 1$



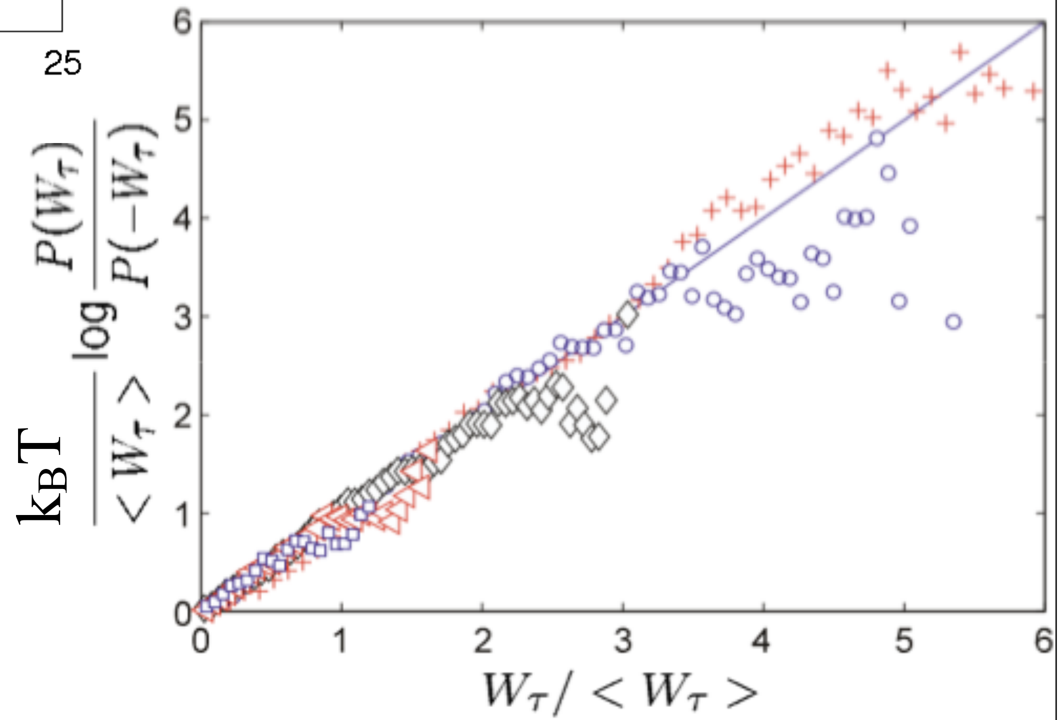
Fluctuation Theorem for W

=0.25Hz and τ= n / f



$n = 1$, 4, 8 and 12

$n = 1$ (+), 2 (○), 4 (◇), 8 (△), 12 (□)



What is FT useful for ?

- Several interesting consequences of FT such as the Jarzynski and Crooks equalities are useful to compute the free energy difference bewteen two equilibrium states using any kind of transformation

- Hatano-Sasa relation and the fluctuation dissipation theorem for non equilibrium steady states(NESS). These are useful to compute the response function of NESS

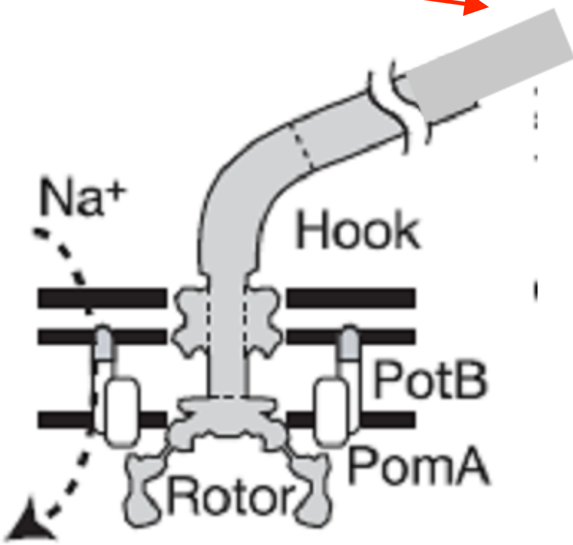
- FT allows the measure of tiny amount of heat exchange bewteen the system and its heat bath. (example: application to aging and biological systems)

- Measure of the offset of a variable

- Measure of the mean injected power.

Molecular motor and FT

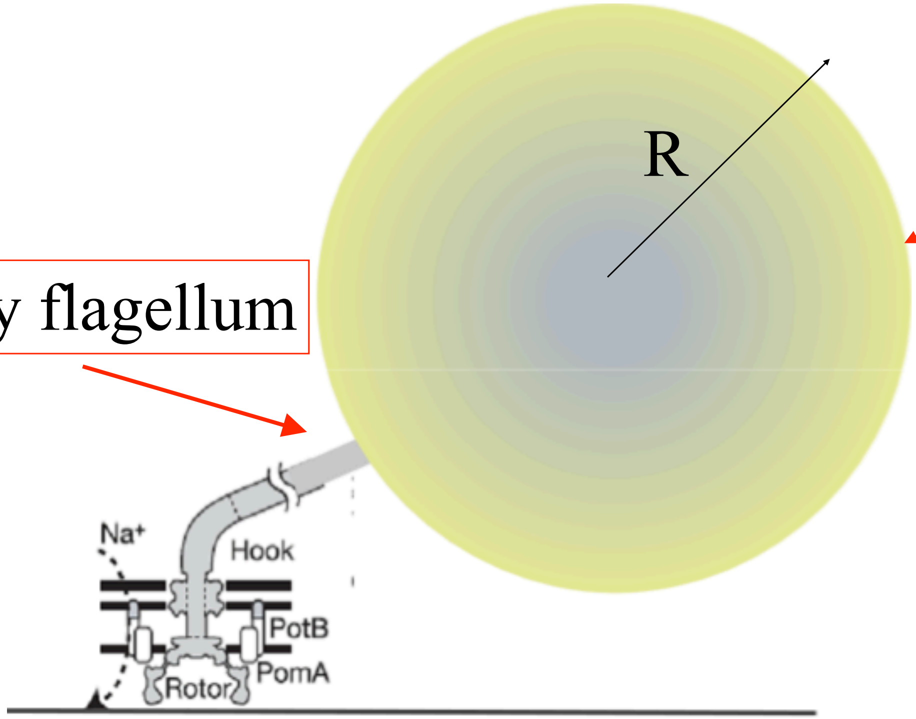
Sticky flagellum



Molecular motor and FT

polystyrene bead

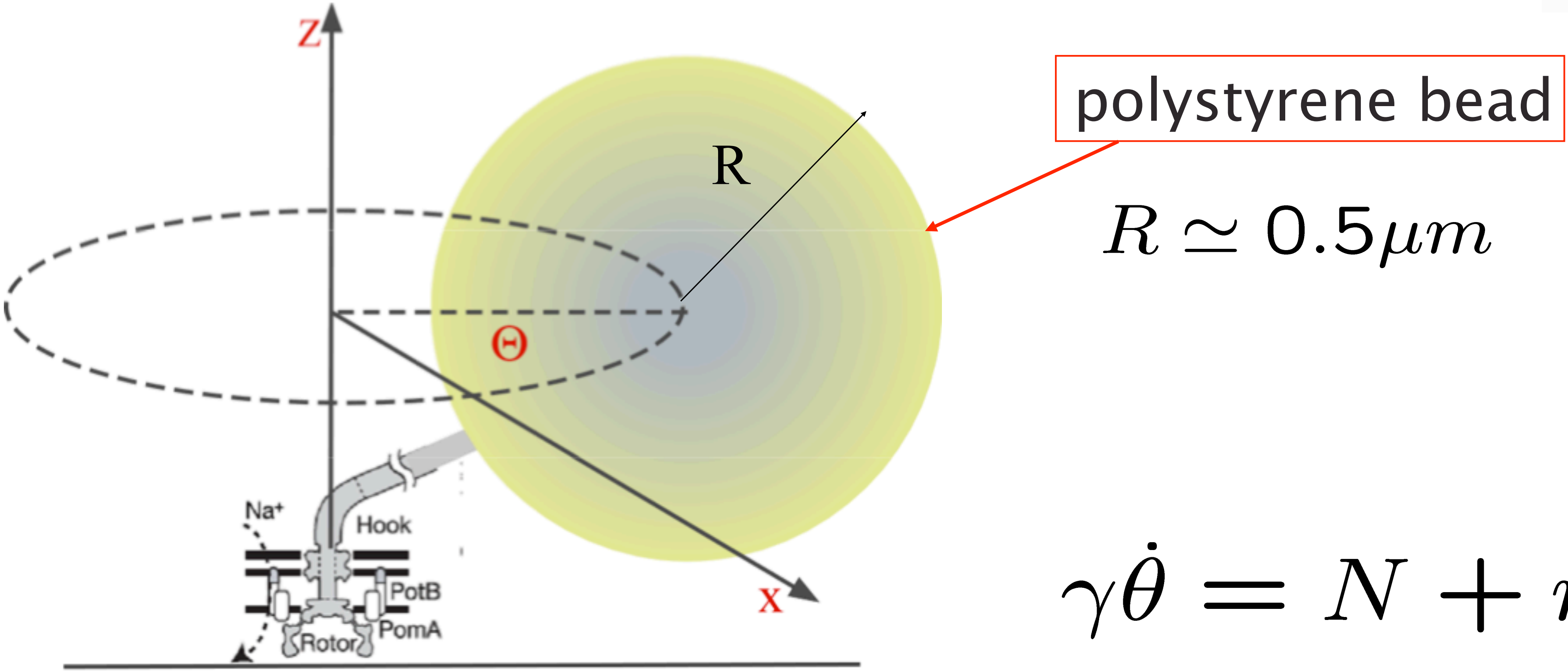
Sticky flagellum



$$R \simeq 0.5\mu m$$

(drawing not in scale)

Molecular motor and FT



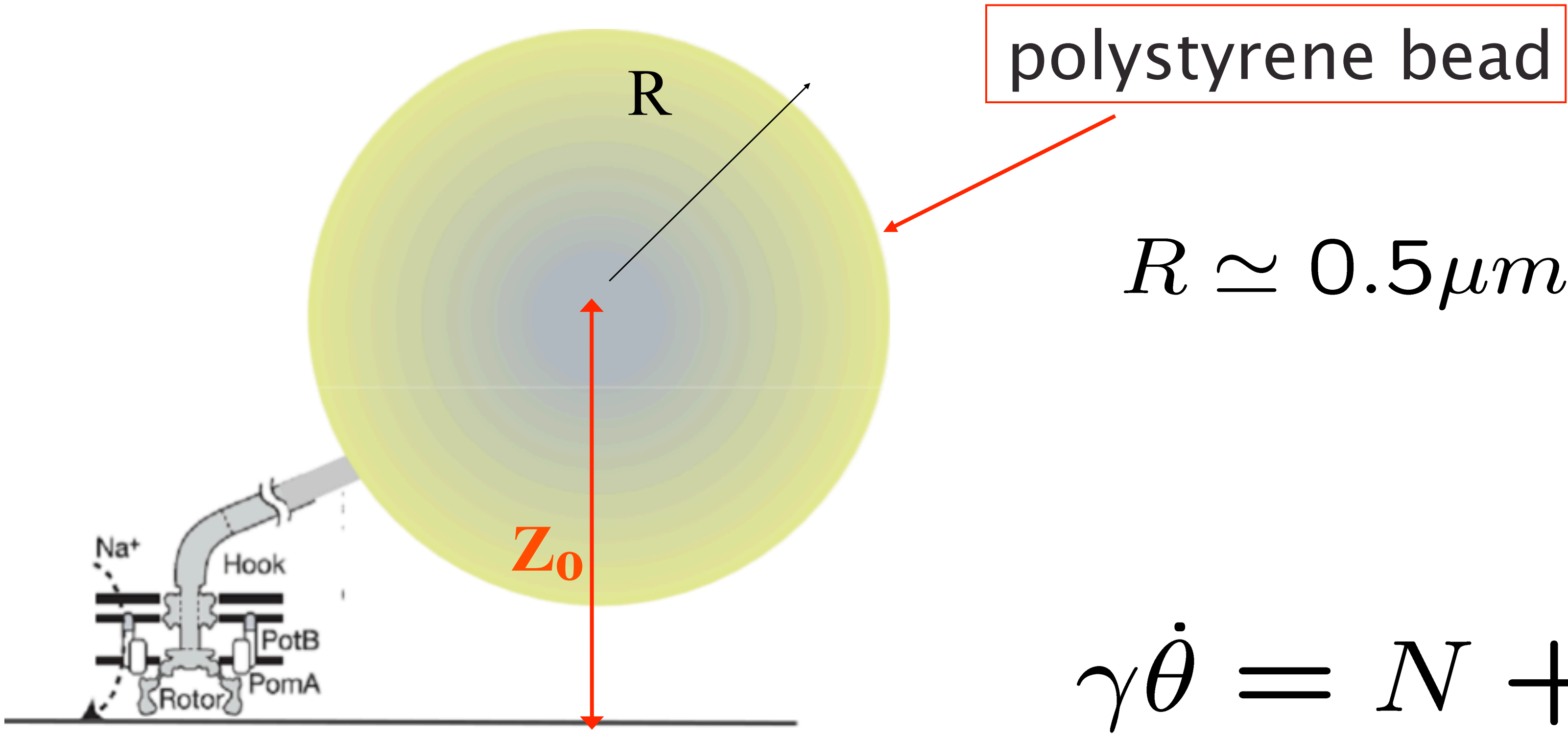
$$R \simeq 0.5\mu m$$

$$\gamma\dot{\theta} = N + \eta$$

Standard method to determine the torque **N**

$$N = \frac{<\dot{\theta}>}{\gamma}$$

Molecular motor and FT



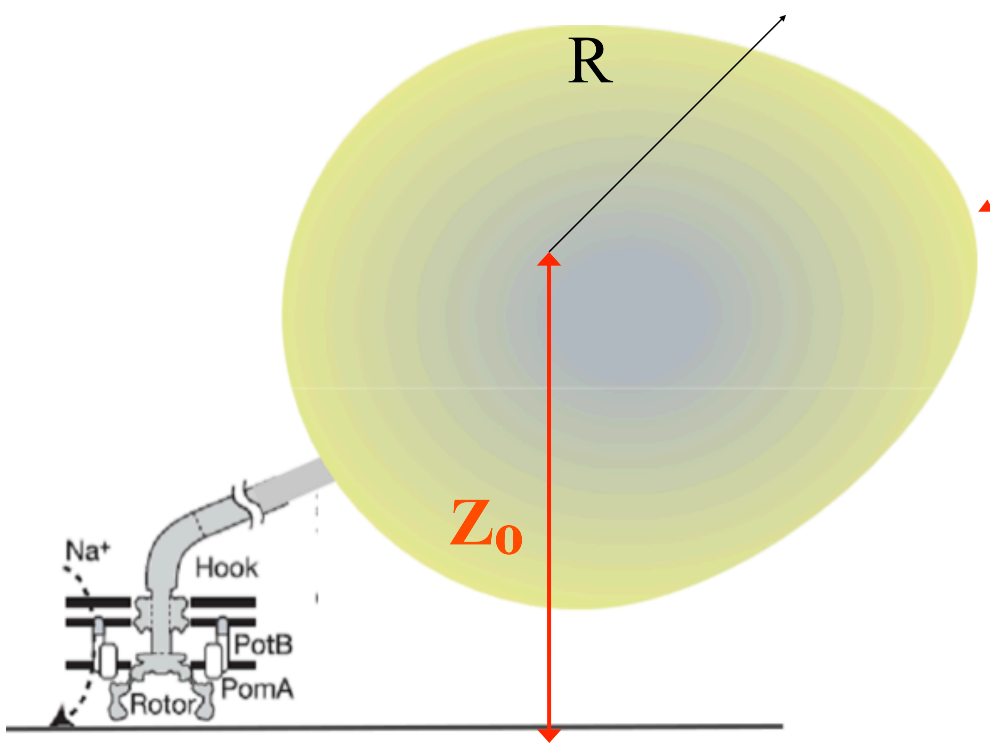
$$R \simeq 0.5 \mu m$$

$$\gamma \dot{\theta} = N + \eta$$

Standard method to determine the torque **N**

$$N = \frac{<\dot{\theta}>}{\gamma} \qquad \text{but} \qquad \gamma(R, Z_0)$$

Molecular motor and FT



$$R \simeq 0.5\mu m$$

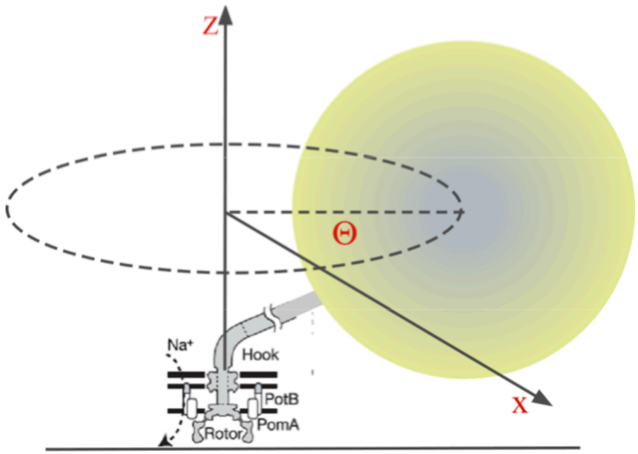
$$\gamma\dot{\theta} = N + \eta$$

Standard method to determine the torque **N**

$$N = \frac{<\dot{\theta}>}{\gamma}$$ but $$\gamma(R, Z_0)$$

and of the shape

Molecular motor and FT



New method based on FT to determine the torque **N**

$$\gamma \dot{\theta} = N + \eta$$

$$W_\tau = N \int_t^{t+\tau} \dot{\theta} \; dt = N \; \Delta\theta_\tau \quad \text{where } \Delta\theta_\tau = (\theta(t+\tau) - \theta(t))$$

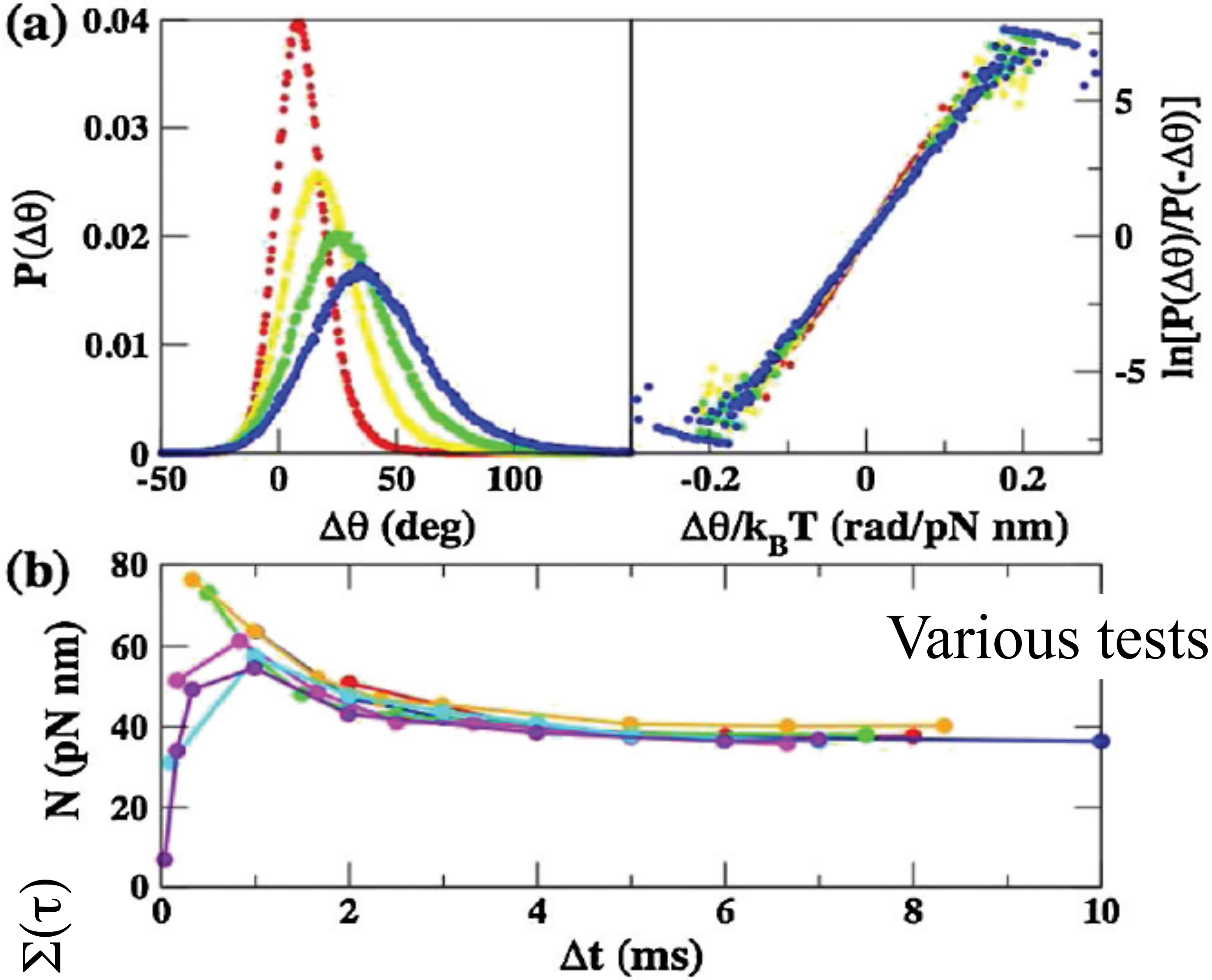
SSFT for W_τ :
$$\log\left(\frac{P(\Delta\theta_\tau)}{P(-\Delta\theta_\tau)}\right) = \Sigma(\tau) \; N \; \frac{\Delta\theta_\tau}{k_B T}$$

with $\Sigma(\tau) \to 1$ for $\tau \to \infty$

γ is not needed

Molecular motor and FT

$$\log\left(\frac{P(\Delta\theta_\tau)}{P(-\Delta\theta_\tau)}\right) = \Sigma(\tau)\ N\ \frac{\Delta\theta_\tau}{k_B T}$$



Kumiko Hayashi et al.,
PRL 104, 218103 (2010)

Various tests with different beads (size and shape)

Jarzynski equality

Consider a system whose energy is: $H(\Gamma, \lambda)$
Here $\lambda(t)$ is an externally controlled parameter.
We consider a transformation from an initial equilibrium state, $\lambda = A$ to another equlibrium state $\lambda = B$. Thus we have

$$H(\Gamma_r, B) - H(\Gamma_0, A) = W^J$$

where

$$W^J \; = \int_0^{\tau} dt \, \frac{d\lambda}{dt} \, \frac{\partial H}{\partial \lambda}$$

If ΔF is the free energy difference between the two equilibrium states A and B then the **Jarzynski Equality** (JE) states that:

$$\boxed{< \; \exp(-\beta \; W^J) \; > = \exp(-\beta \Delta F)}$$

If W^J has a Gaussian PDF then the JE takes a simple form:

$$\Delta F = < W^J > - \frac{\sigma_W^2}{2 \; K_B \; T}$$

Crooks identity

Crooks considered the forward (F) and reverse processes (R). During the F processes λ goes from A to B. During the R the inverse path is done.

Crooks derived the following identity:

$$\frac{P_F(W^J)}{P_R(-W^J)} = \exp(\frac{W^J - \Delta F}{K_B T}) = \exp(\frac{W_{dis}}{K_B T})$$

simple manipulation of this ratio and integration gives:

$$\int_{-\infty}^{\infty} P_F(W^J) \; \exp(-\frac{W^J}{K_B T}) \; dW^j = \exp(-\frac{\Delta F}{K_B T})$$

which is the Jarzynski equality:

$$\boxed{< \; \exp(-\beta \; W^J) \; > = \exp(-\beta \Delta F)}$$

The derivation has been argued by Cohen and Mauzerall cond-mat/0406128

The Jarzynski work

$$W^J = \int_0^\tau dt \, \frac{d\lambda}{dt} \, \frac{\partial H}{\partial \lambda}$$

- What is the meaning of λ in a real experiment ?

Connections between the macroscopic variables and the microscopic ones

- What is the quantity wich is controlled in an experiment ?

If λ is a displacement x then:

$$W^J = -\int_0^\tau dt \, \frac{dx}{dt} \, F = -W^{cl}$$

If λ is a

$$W^J = -\int_0^\tau dt \, \frac{dF}{dt} \, x = -\Big[F \; x\Big]_0^{t_s} + W^{cl}$$

The classical work

$$W^J = -\int_0^{t_s} \dot{M}\theta \; dt = -\Big[M\theta\Big]_0^{t_s} + W^{\mathsf{cl}},$$

where

$$W^{\mathsf{cl}} = \int_0^{t_s} M\dot{\theta} \; dt$$

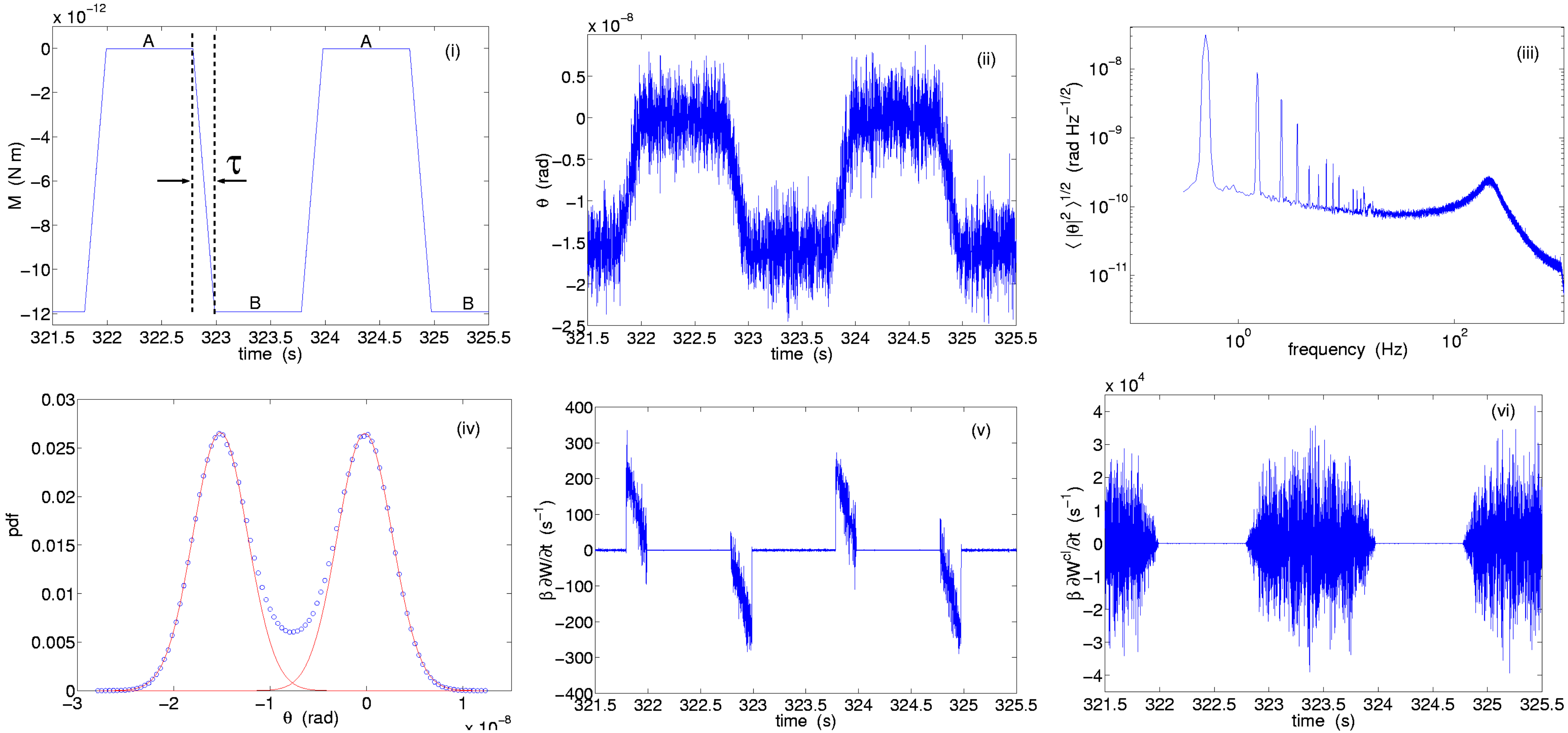
is the classical work

The ΔF computed by the JE in the case of a driven system, composed by the system Ξ plus the external driving, is the total free energy difference

$$\Delta F = \Delta F_0 - \Big[M\theta\Big]_A^B = \Delta F_0 - \Phi,$$

where ΔF_0 is the free energy of Ξ and $\Phi = \Big[M\theta\Big]_A^B$

Typical driving



Oscillator immersed in oil [case (a)]: (i) Applied external torque, (ii) Induced angular displacement, (iii) its psd, (iv) its pdf, (v) Injected power computed from the Jarzynski definition $\dot{W} = -\dot{M}\theta$, (vi) Injected power computed from the standard definition $\dot{W}^{\mathrm{cl}} = M\dot{\theta}$

The Free Energy for the torsion pendulum

The free energy difference of the oscillator alone is

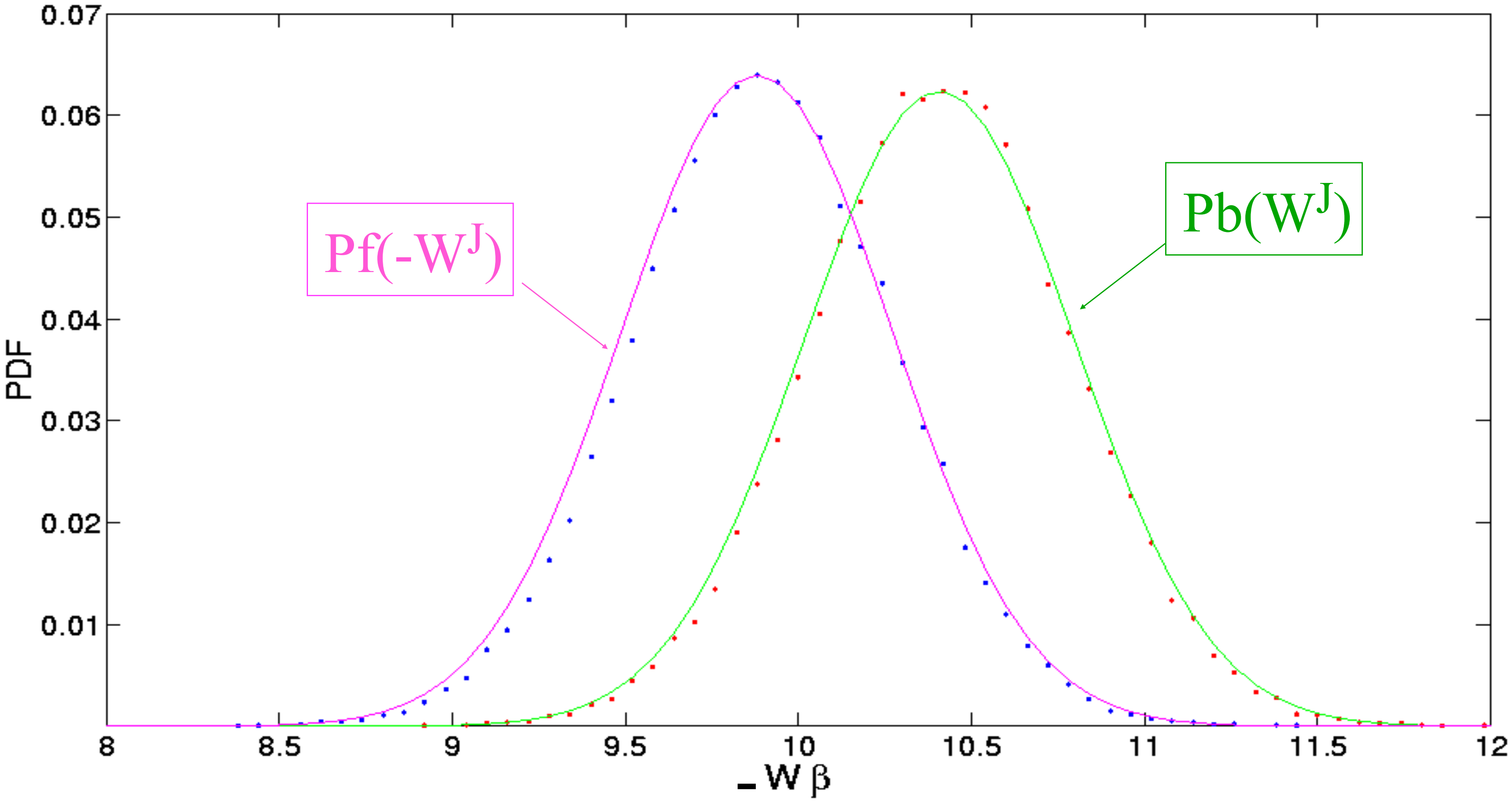
$$\Delta F_0 = \Delta U = \left[\frac{1}{2} C\theta^2\right]_A^B = \left[\frac{M^2}{2C}\right]_A^B,$$

whereas

$$\Delta F = \Delta F_0 - \left[\frac{M^2}{C}\right]_A^B,$$

i.e. for an harmonic potential $\Delta F = -\Delta F_0$.

Jarzynski Work PDF

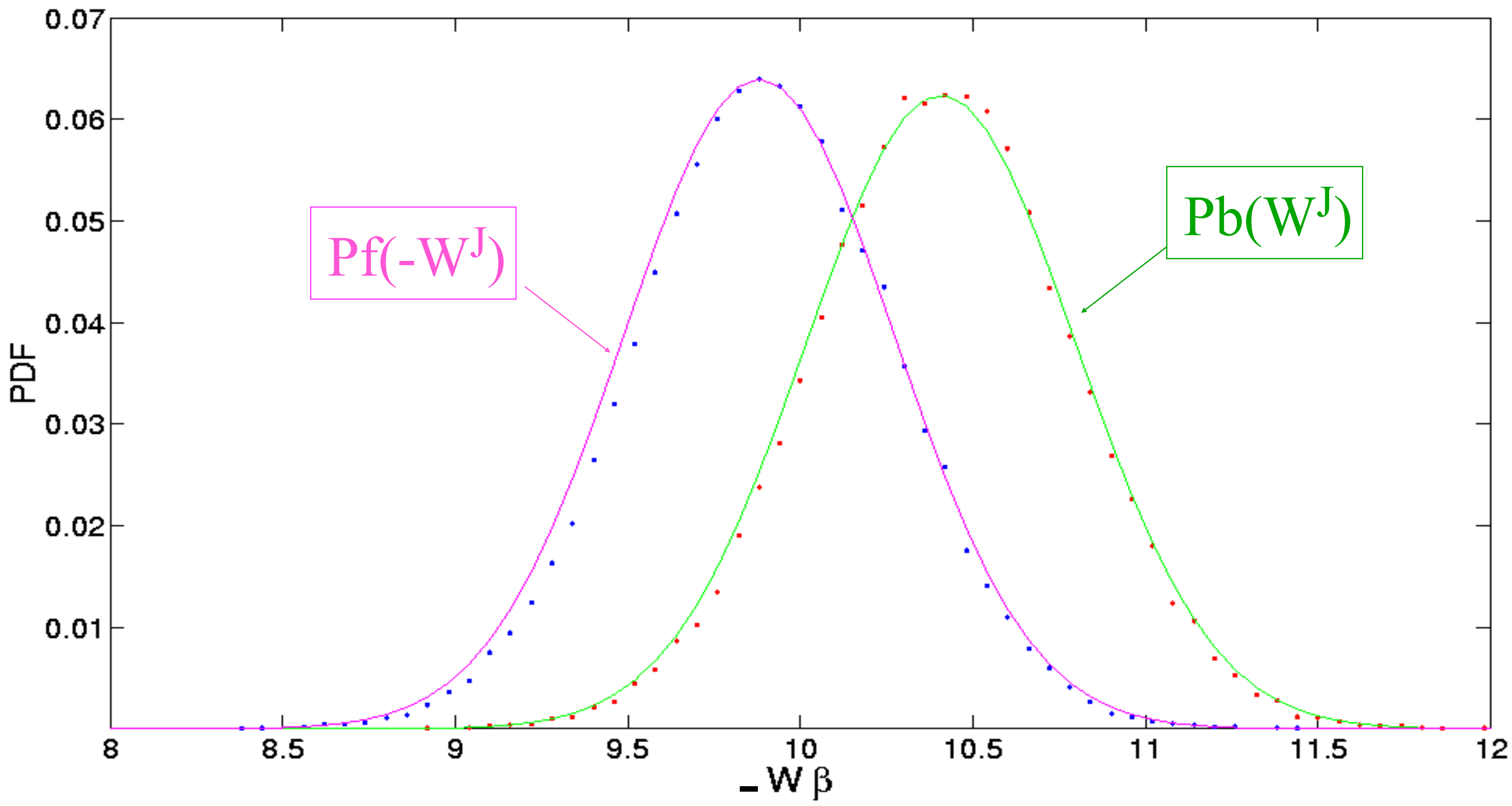


$\overline{W_f^J} = -9.88\ K_BT$, $\overline{W_b^J} = 10.40\ K_BT$, $\sigma_{W^J} = 0.45\ K_BT$

From the crossing point is $\Delta F = -10.15\ K_BT$

From JF $\Delta F \simeq -10.3\ K_BT$ and $\Delta F \simeq -10\ K_BT$

Jarzynski Work PDF

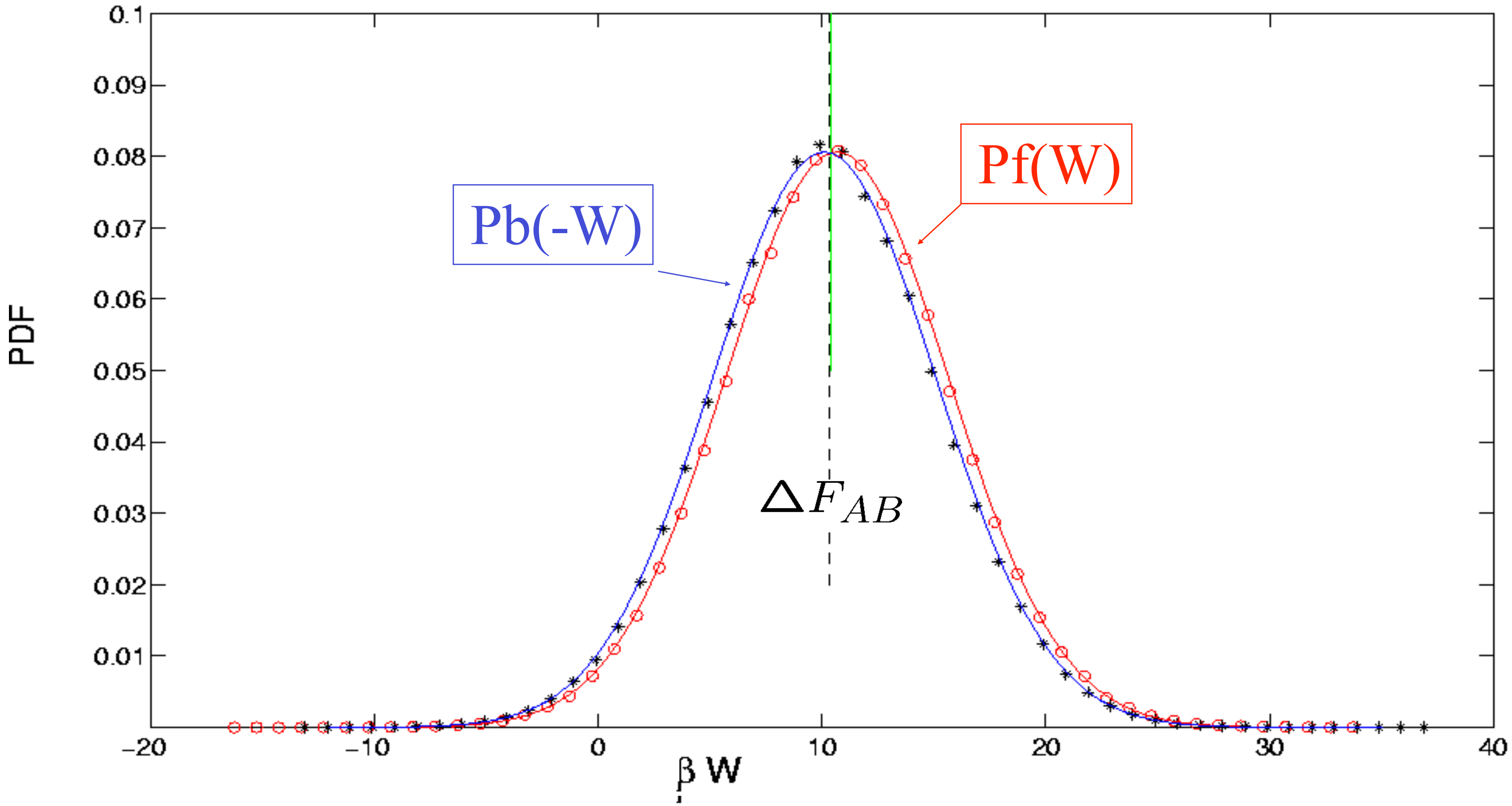


$\overline{W_f^J} = -9.88\ K_BT$, $\overline{W_b^J} = 10.40\ K_BT$, $\sigma_{W^J} = 0.45\ K_BT$

From the crossing point is $\Delta F = -10.15\ K_BT$

From JF $\Delta F \simeq -10.3\ K_BT$ and $\Delta F \simeq -10\ K_BT$

Classical Work PDF



$$\overline{W_f} = 10.67\ K_B T\ ,\ \overline{W_b} = -10.1\ K_B T,\ \sigma_W = 5\ K_B T$$

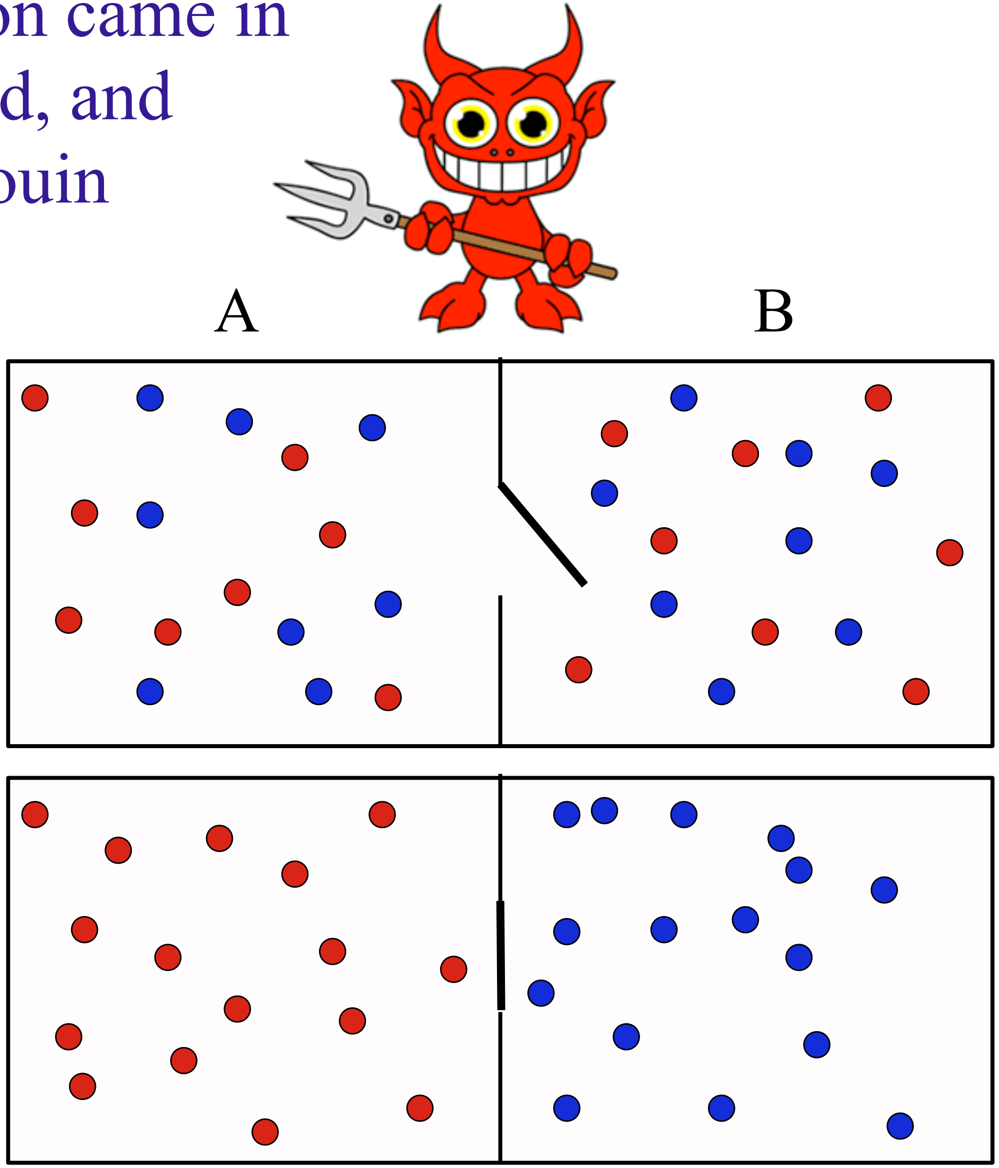
The crossing point of the PDF is the reversible work ΔF_{AB}

INFORMATION AND THERMODYNAMICS

- Landauer's principle
- How to realise it ?
- Experimental set-up
- Data analysis
- Comparison with numerical results
- Landauer's limit and the Jarzynski equality
- Conclusions

Landauer's Principle and The Maxwell's Demon

The first explanation came in 1929 by Leó Szilárd, and later by Léon Brillouin



- **slow molecules**
- **fast molecules**

ENS DE LYON

The Landauer's principle (I)

Any logically irreversible transformation of classical information is necessarily accompanied by the dissipation of at least $k_B T \cdot \ln 2$ of heat per lost bit (about $3 \cdot 10^{-21}$ Joules at room temperature)

Typical examples of logically irreversible transformations are Boolean functions such as AND, NAND, OR and NOR

They map several input states onto the same output state

The erasure of information, the RESET TO ONE operation, is logically irreversible and leads to an entropy production of $k_B \cdot \ln 2$ per erased bit

Landauer's principle II

Landauer's principle is a central result which exorcises the Maxwell's demon

It has been criticised and never tested in a real experiment

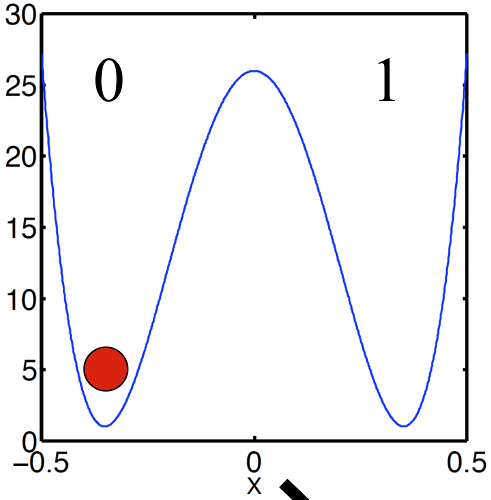
Questions

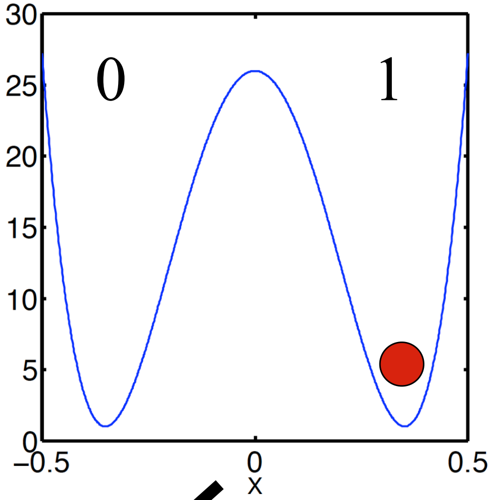
- Can the Landauer's limit be reached in any experiment ?
- Does any experimentally feasable procedure allow us to reach the limit ?

Following Bennett we use in our experiment the RESET to ONE operation

Bennett, C. H. The thermodynamics of computation, a review. Int. J. Theor. Phys. 21, 905-940 (1982).

The Bennett's erasure procedure

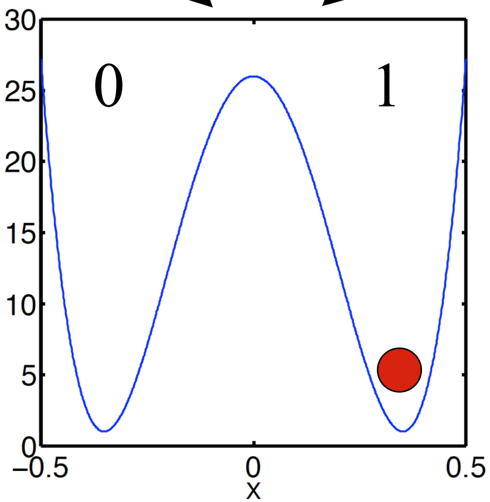




Initial state is 0 or 1 with equal probability 1/2

$S_i = k_B \cdot \ln 2$

Procedure



Final state is 1 with probability 1

$S_f = 0$

Thus $\Delta S_{min} = -k_B \cdot \ln 2$

Procedure

Quasi Static : $-T\Delta S=Q$

Energy variation : $\Delta U=0$

First principle : $\Delta U=-Q+W$

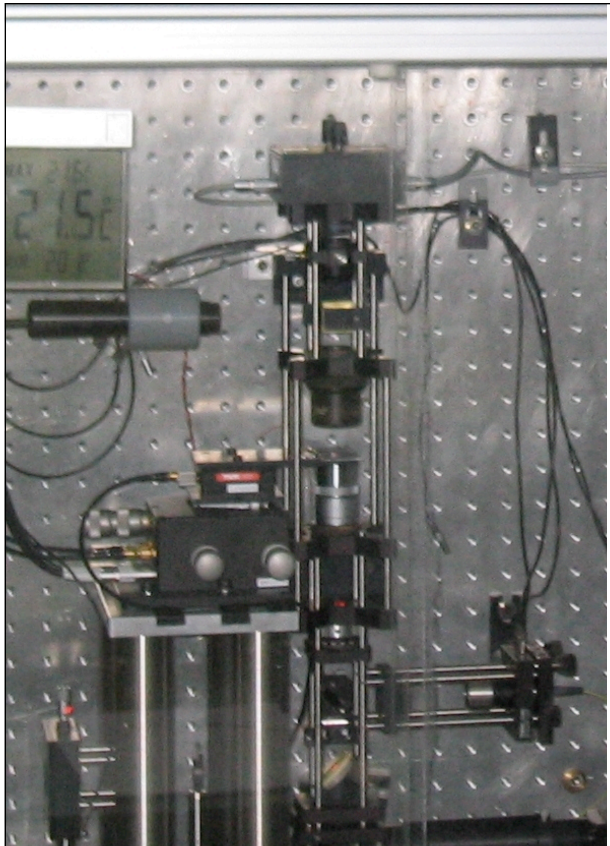
$$\Downarrow$$

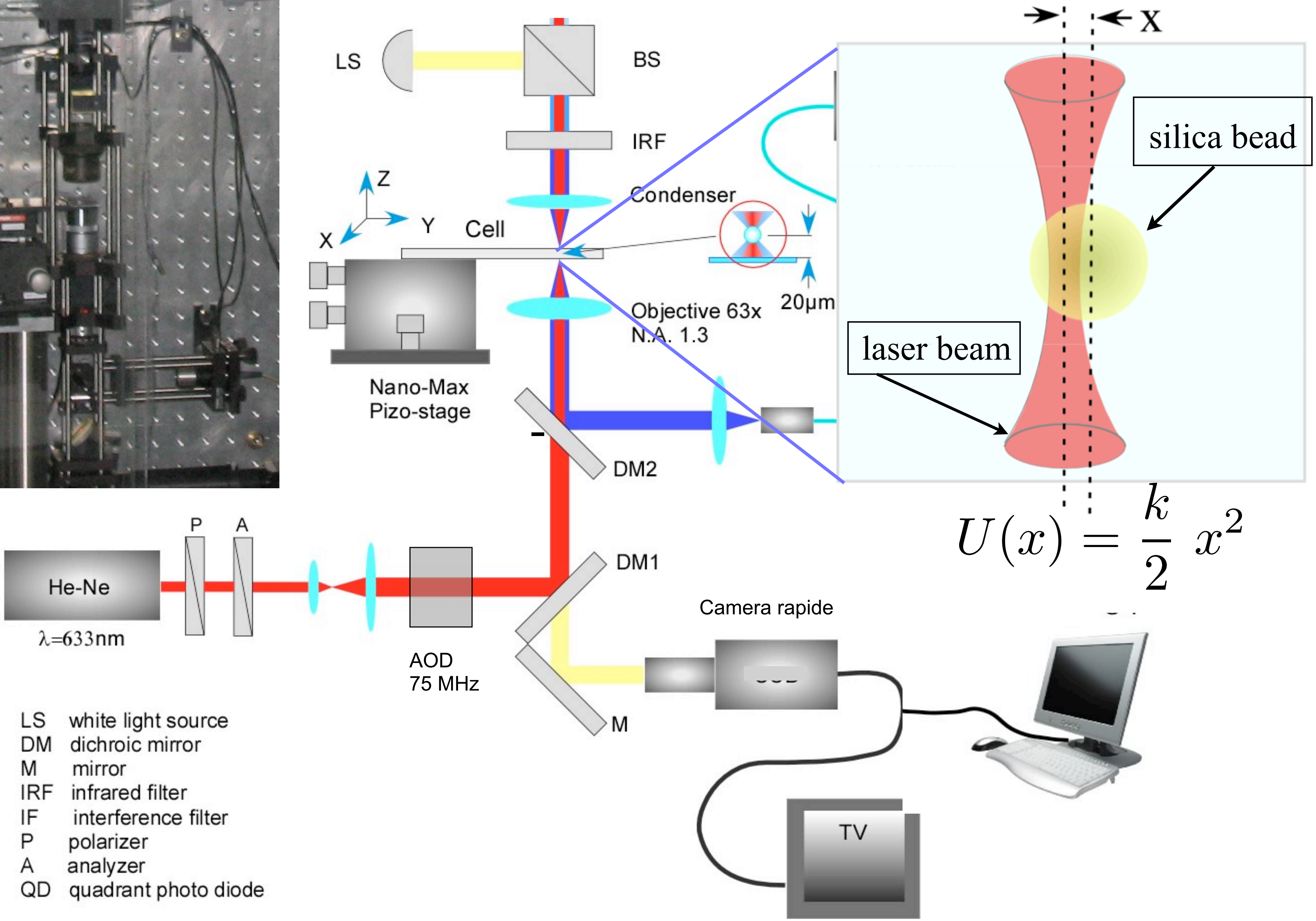
In average : $\langle W\rangle = \langle Q\rangle = -T\,\Delta S \geq k_B T \ln(2)$

Numerical result :

Memory Erasure in Small Systems,
R. Dillenschneiderand E. Lutz, Phys. Rev. Lett. 102, 210601 (2009)

Experimental set-up Optical trap

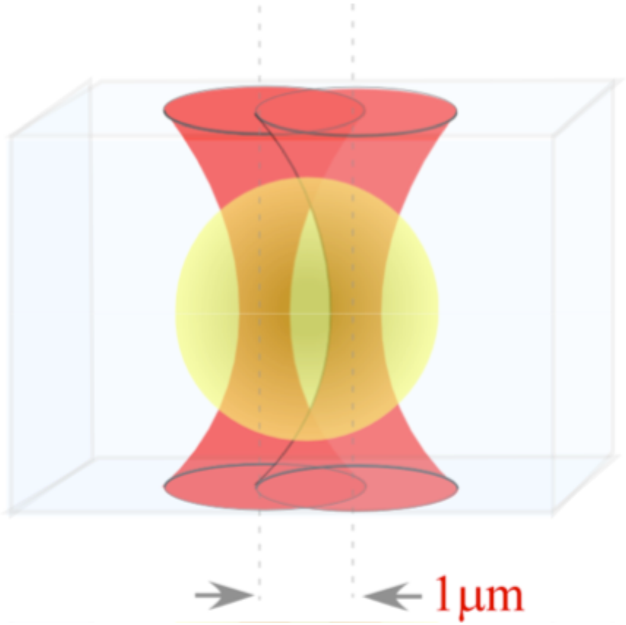


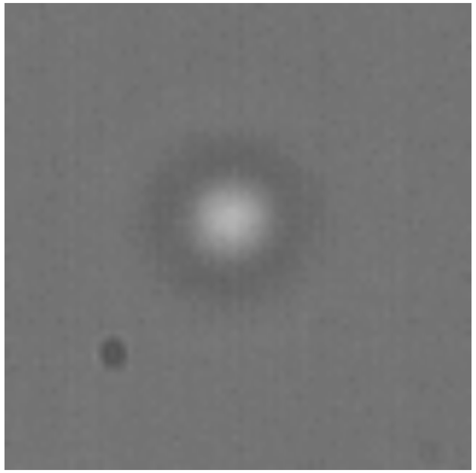


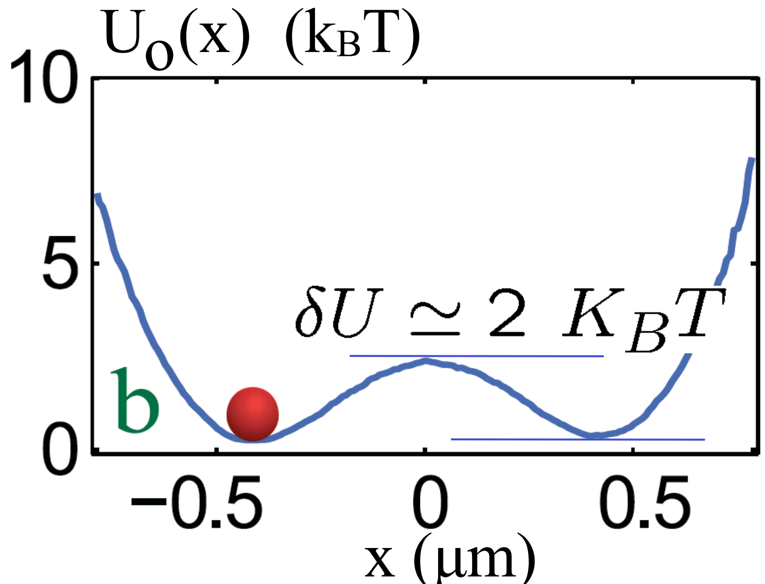
$$U(x) = \frac{k}{2}\, x^2$$

LS white light source
DM dichroic mirror
M mirror
IRF infrared filter
IF interference filter
P polarizer
A analyzer
QD quadrant photo diode

Brownian particle trapped by two laser beams







The Kramers time

$$\tau_K = \tau_o \ \exp[\frac{\delta U}{k_B T}]$$

with $\tau_o = 1\ s$

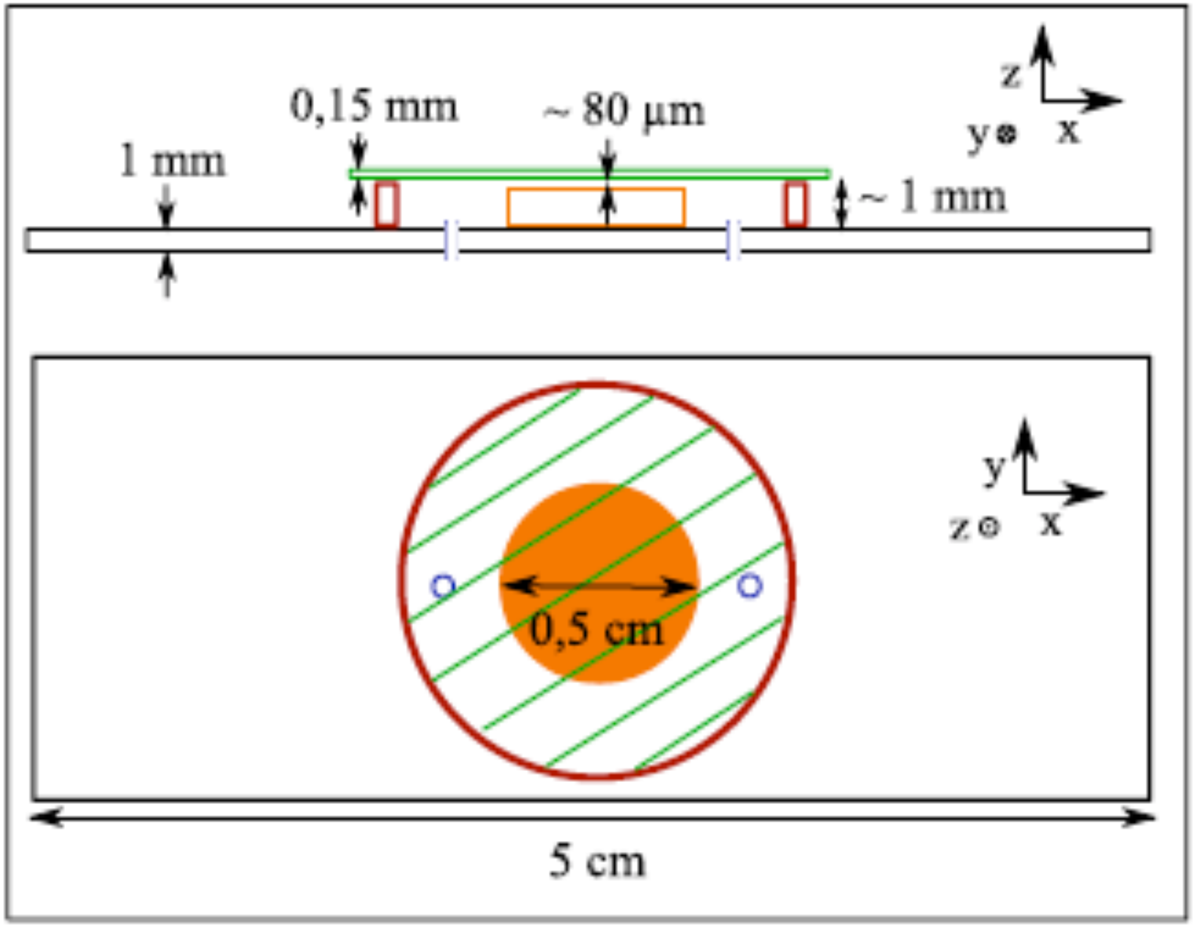
$$U_o(x) = a\ x^4 - b\ x^2 \ + \ d\ x$$

Potential measured using detailed balance

$$\frac{\omega_{i\to j}}{\omega_{j\to i}} = \ e^{-\frac{\Delta U_{ij}}{k_B T}}$$

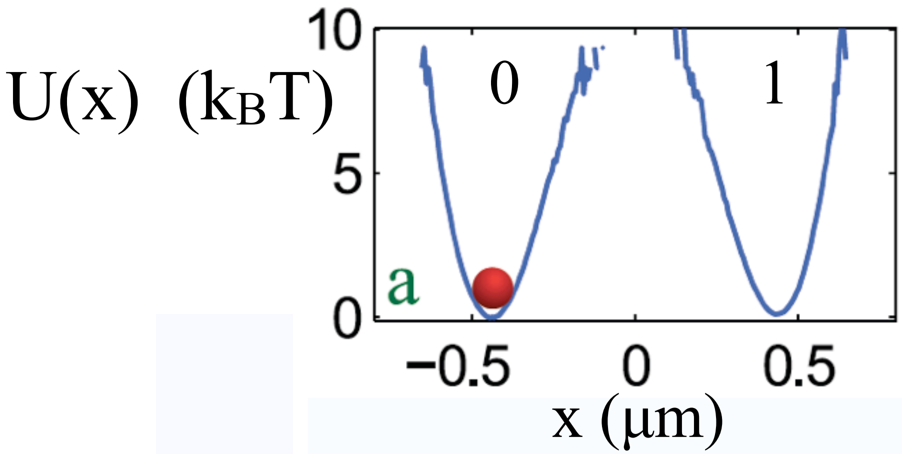
with $\Delta U_{i,j} = U(x_i) - U(x_j)$

The cell for the bead



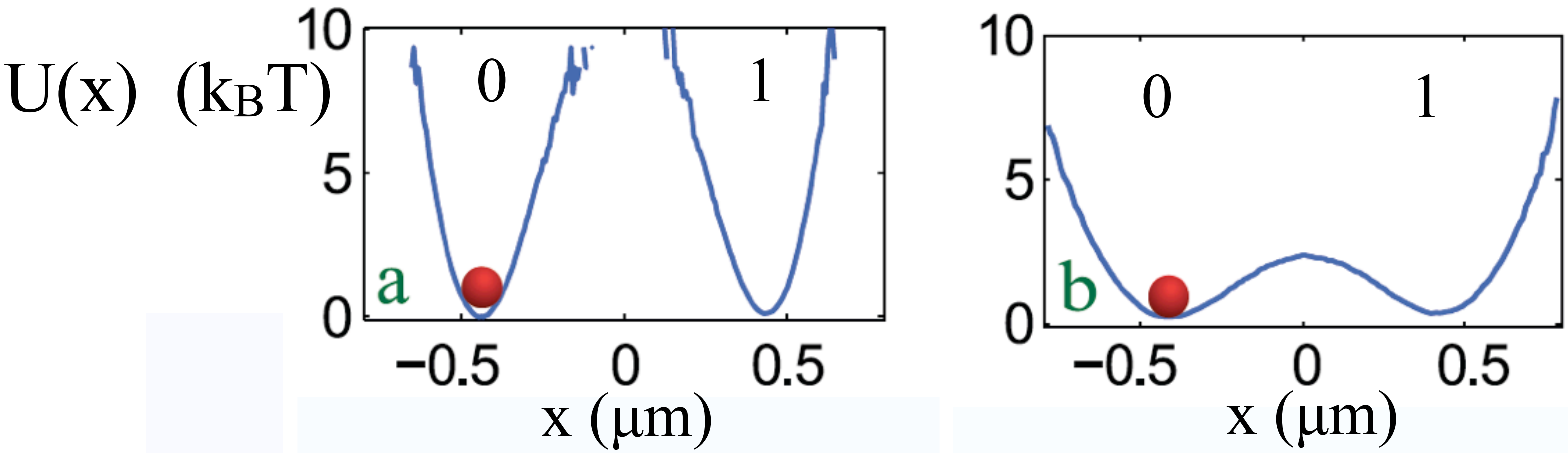
The Erasure Procedure

Initial state



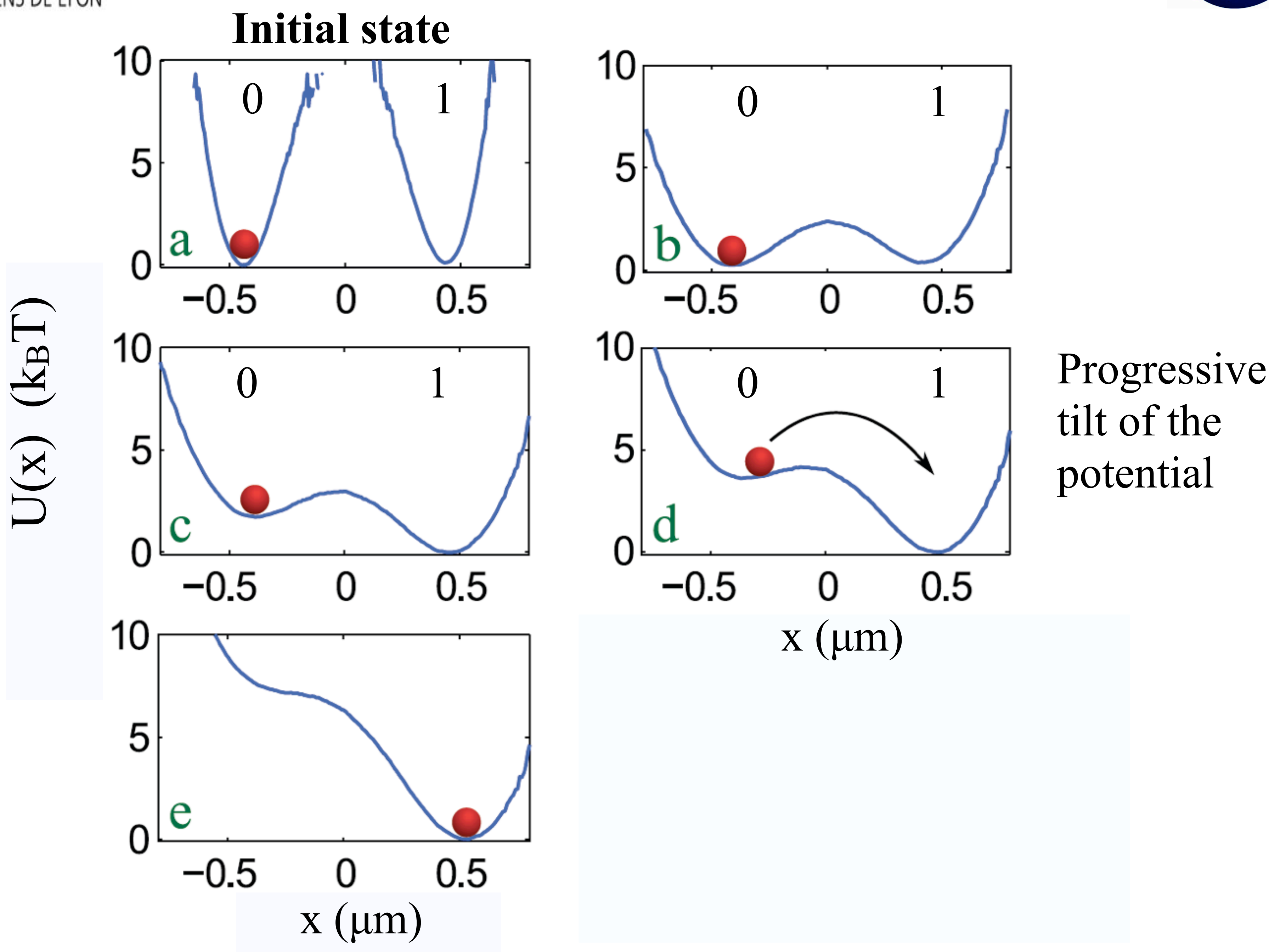
The Erasure Procedure

Initial state



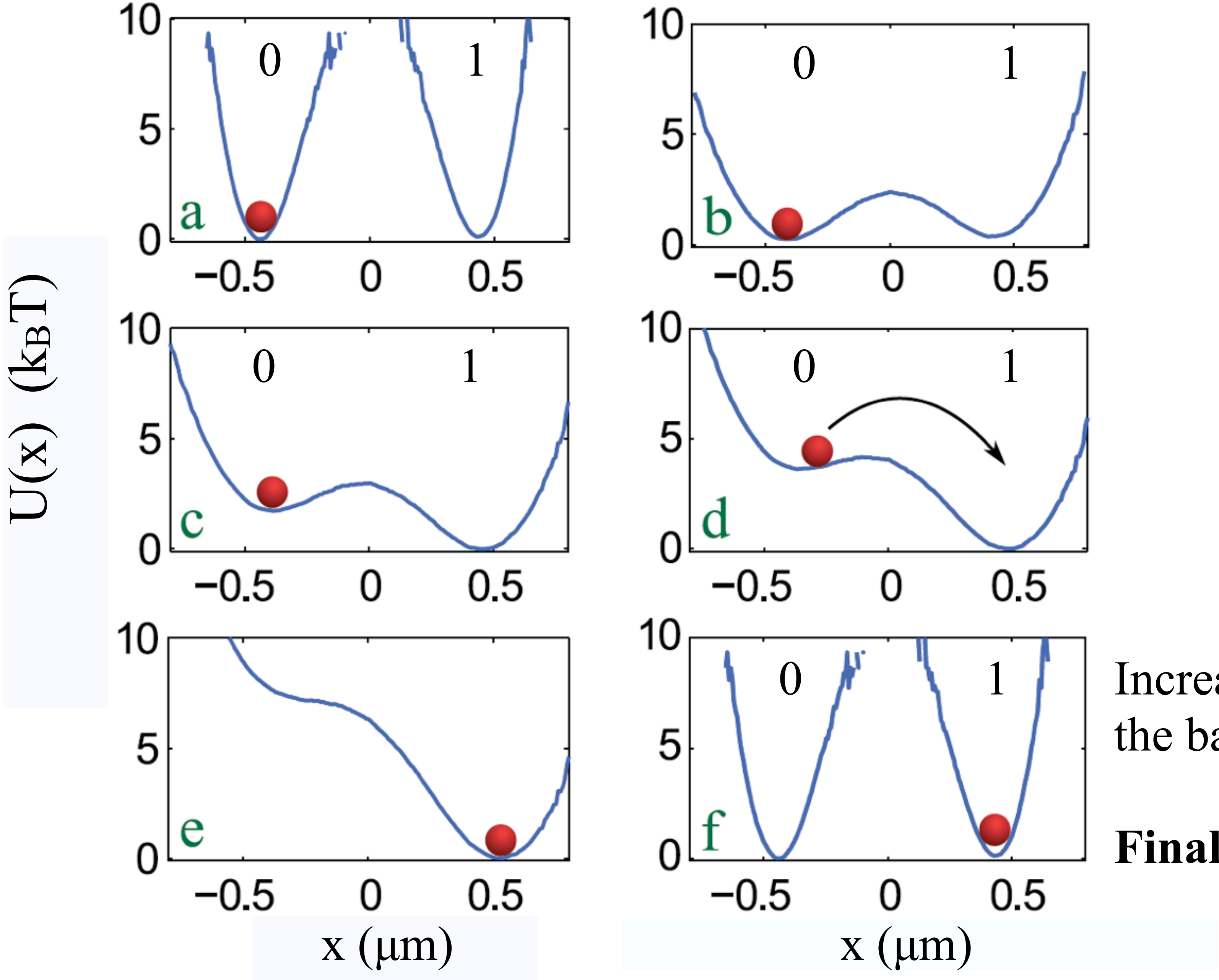
reduction of the barrier

The Erasure Procedure



The Erasure Procedure

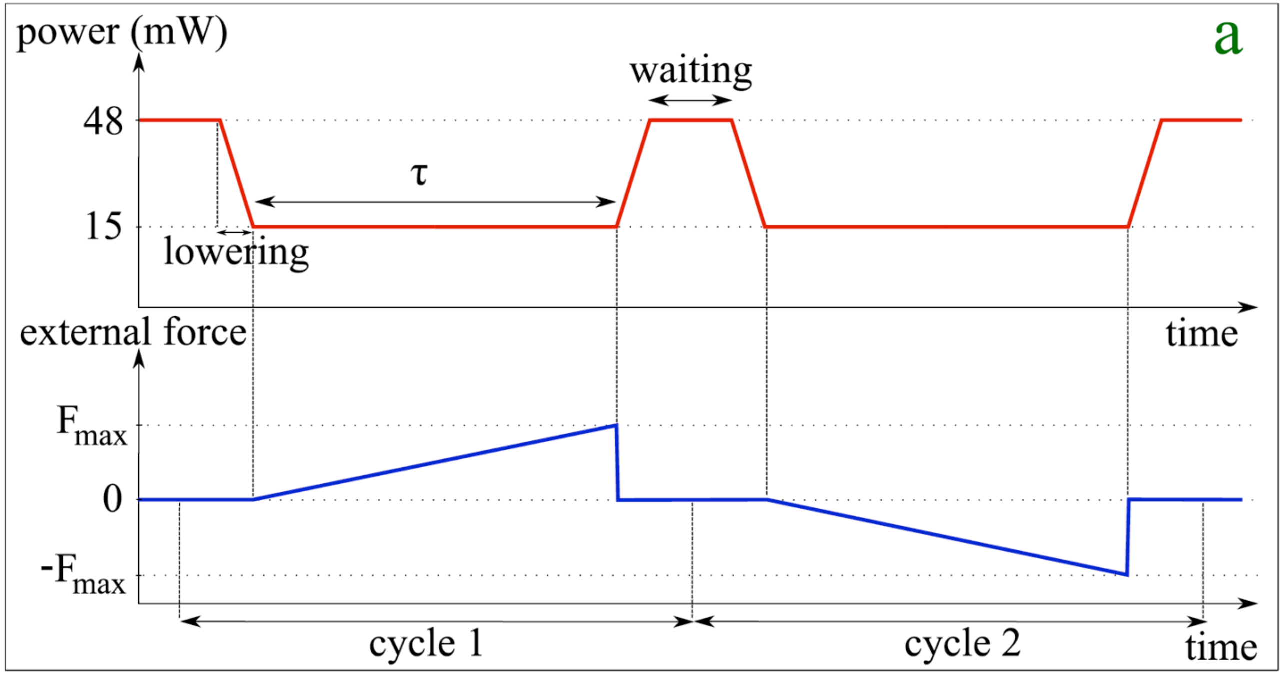
Initial state



Increasing of the barrier

Final state

Potential external control as a function of time



The laser intensity controls the barrier height

The potential tilt is produced by a linearly increasing external force f, applied on the time τ.

$\tau_{cycle} = \tau + 2$ s

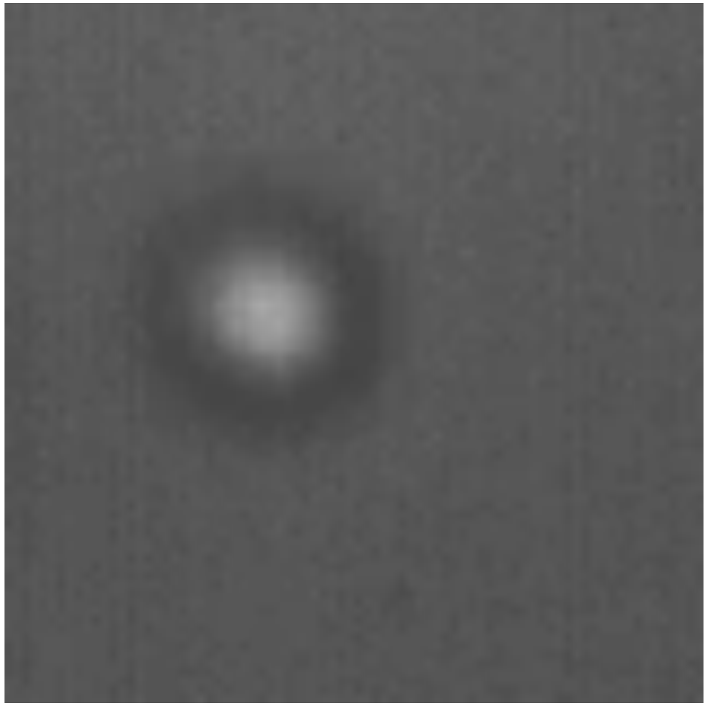
The force f is created by displacing the cell with respect to the laser, thus

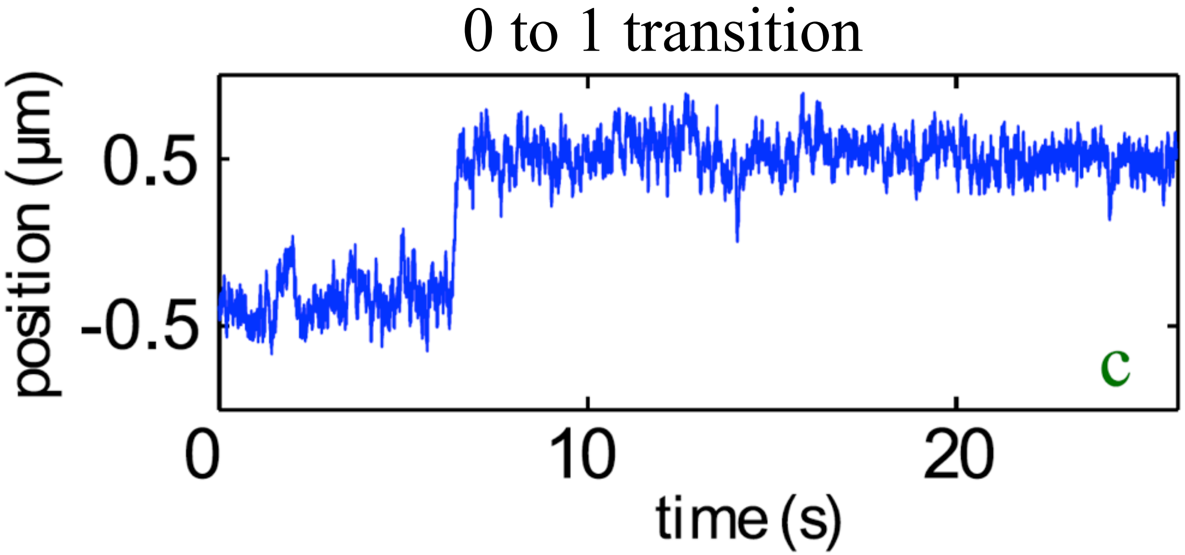
$$f = -\nu\ v \qquad \text{with} \qquad \nu = 6\pi R\mu$$

Two control parameters: τ the time of application of f

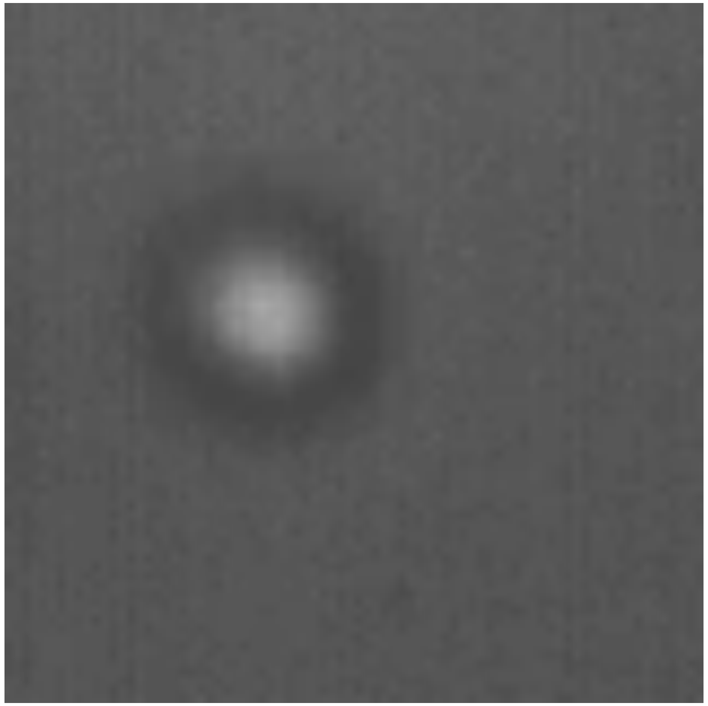
F_{max} the maximum applied force

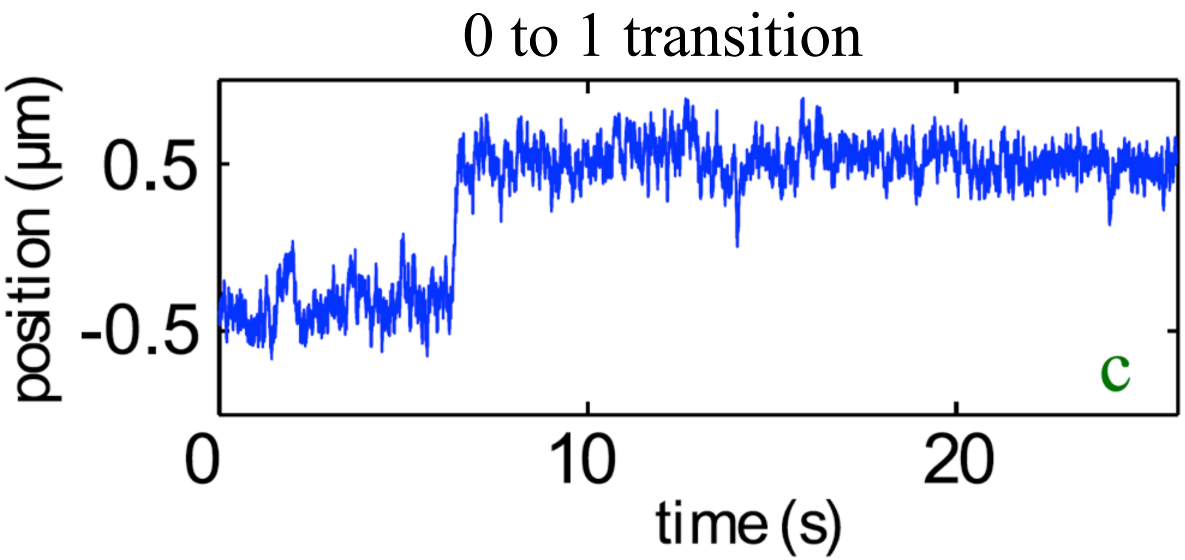
Bead trajectories

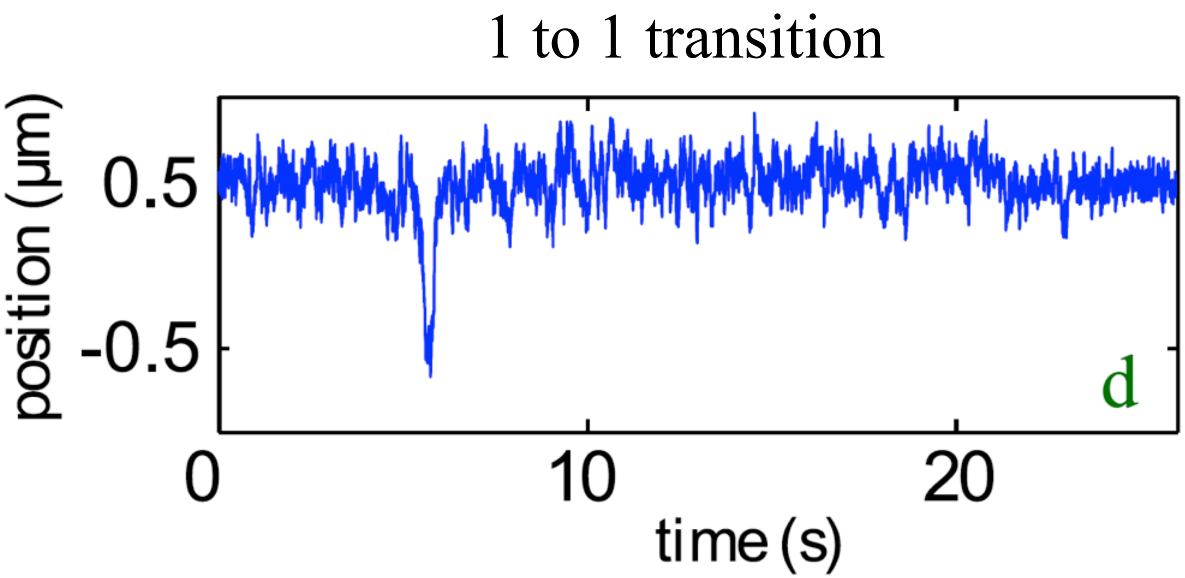




Bead trajectories







The work on the erasure cycle

$$\nu \dot{x} = -\frac{\partial U_o(x,t)}{\partial x} + f(t) + \eta$$

multiplying by ẋ and integrating for a time τ we get :

$$\Delta U_\tau = W_\tau - Q_\tau$$ Stochastic thermodynamics

$$\Delta U_\tau = -\int_0^\tau \frac{\partial U_o}{\partial x} \ \dot{x} \ dt \qquad W_\tau = \int_0^\tau f \ \dot{x} \ dt$$

$$Q_\tau = \int_0^\tau \nu \dot{x}^2 \ dt - \int_0^\tau \eta \dot{x} \ dt$$

Sekimoto K, Progress of Theoretical Phys. supplement (130), 17 (1998).

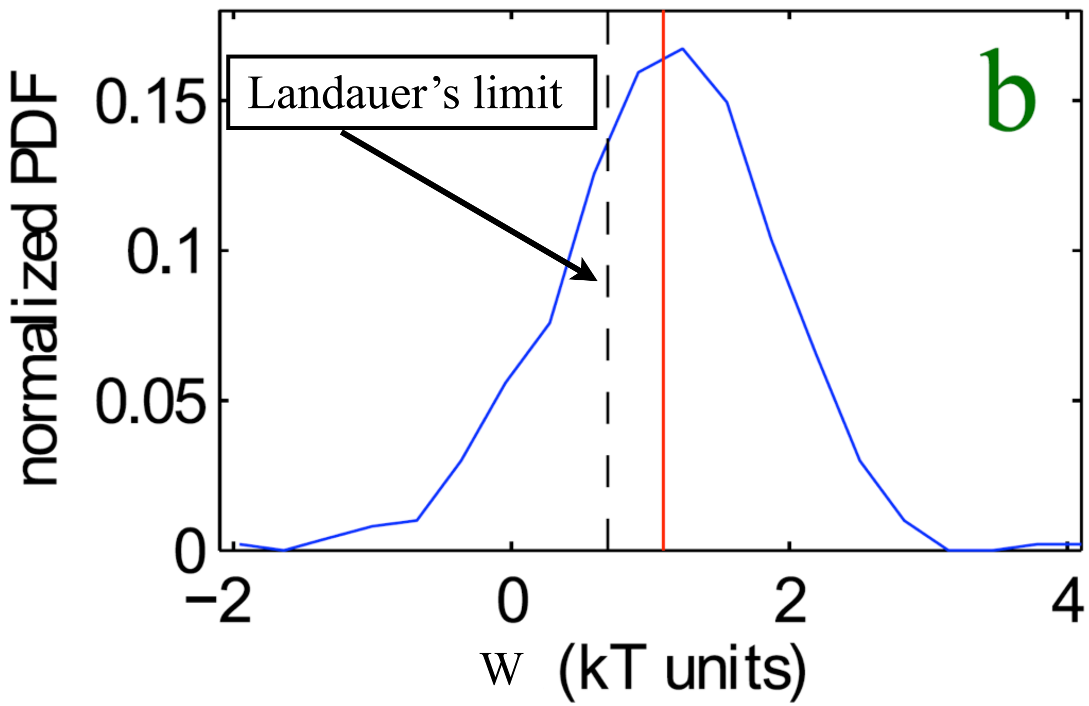
The work on the erasure cycle

The two erasure cycles have been considered

$$\begin{array}{l} 0 \searrow \\ \quad\ \ 1 \\ 1 \nearrow \end{array} \qquad \text{and} \qquad \begin{array}{l} 0 \searrow \\ \quad\ \ 0 \\ 1 \nearrow \end{array}$$

$$W_F = -\int_0^{\tau_{cycle}} \nu\ v(t)\dot{x}\ dt = \int_0^{\tau_{cycle}} F_{max}\frac{t}{\tau}\dot{x}\ dt$$

$$\cancel{\Delta U_\tau = -\int_0^{\tau_{cycle}} \frac{\partial U_o}{\partial x}\ \dot{x}\ dt}$$

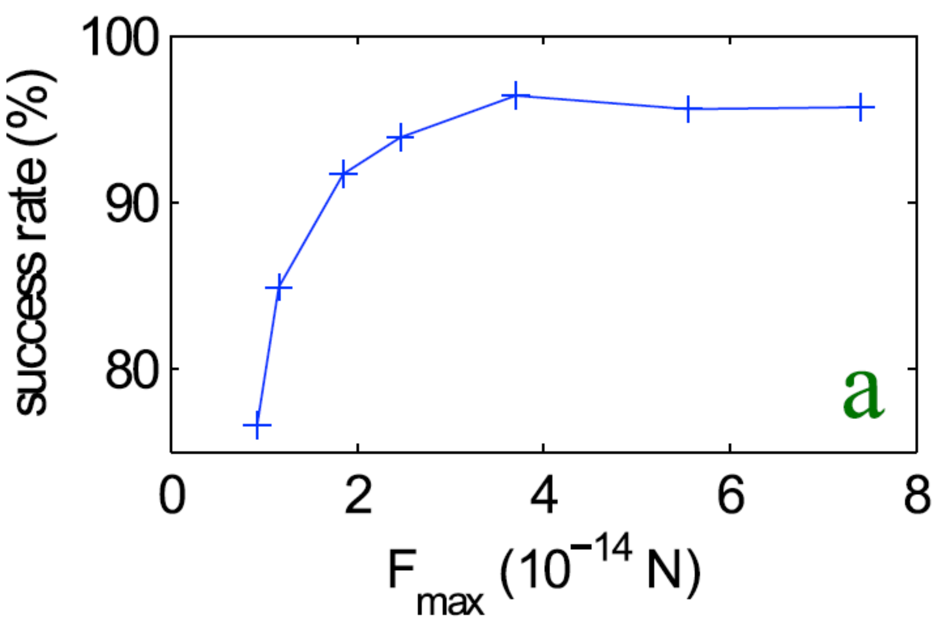


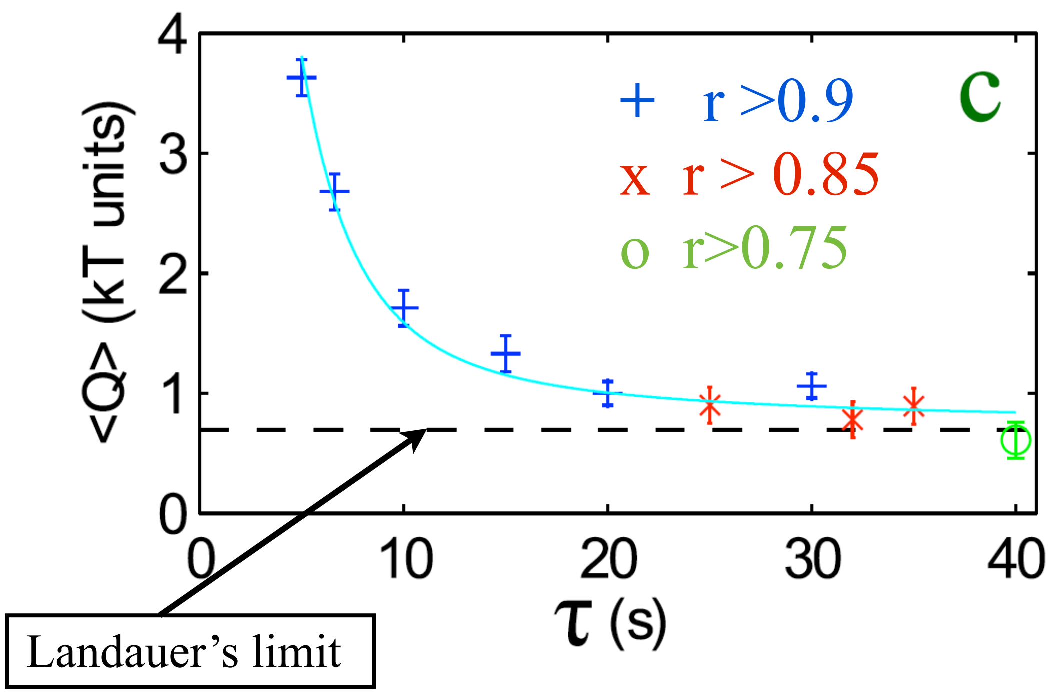
Results of the erasure procedure

$$\text{Success rate } r = \frac{\text{number of successful cycles}}{\text{Total number of cycle}}$$

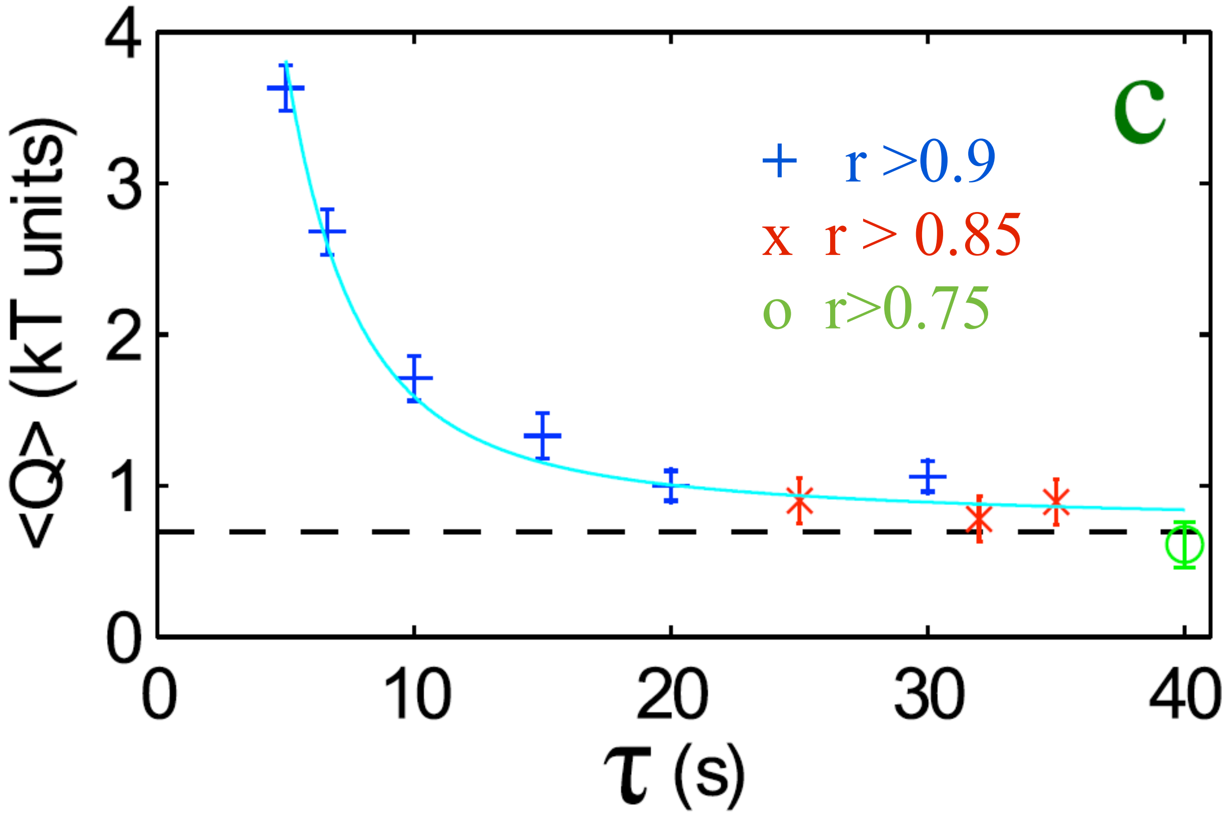
Qualitative observations :

- At constant τ : W and r increase with F_{max}
- At constant F_{max} : W decreases and r increases for increasing τ





Landauer's limit



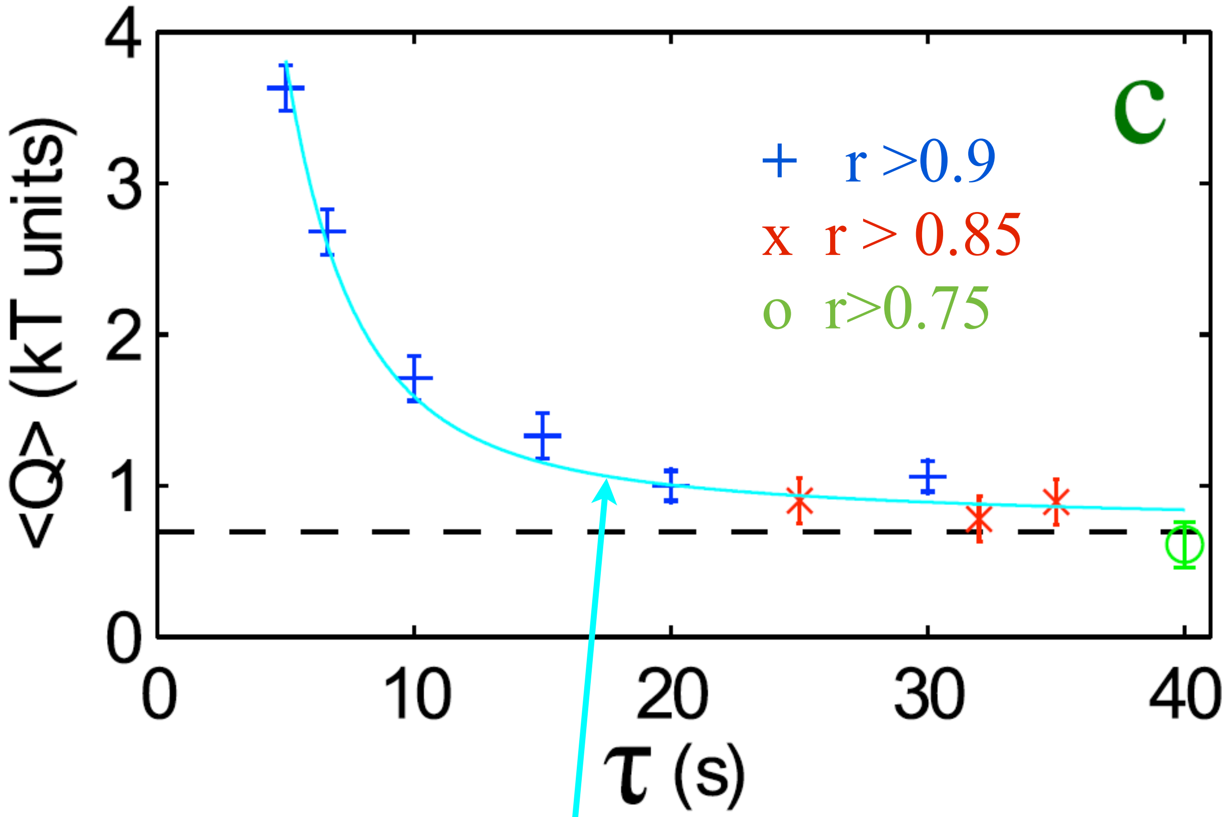
Landauer's limit as a function of r

$$< Q >^r_{\text{Landauer}} = kT[\ln 2 + r \ln r + (1-r)\ln(1-r)]$$

At r=0.5 $< Q >^r_{\text{Landauer}} = 0$

Indeed the Erasure Procedure left the initial state unchanged

Landauer's limit



Asymptotic behaviour

$$\tau \to \infty$$

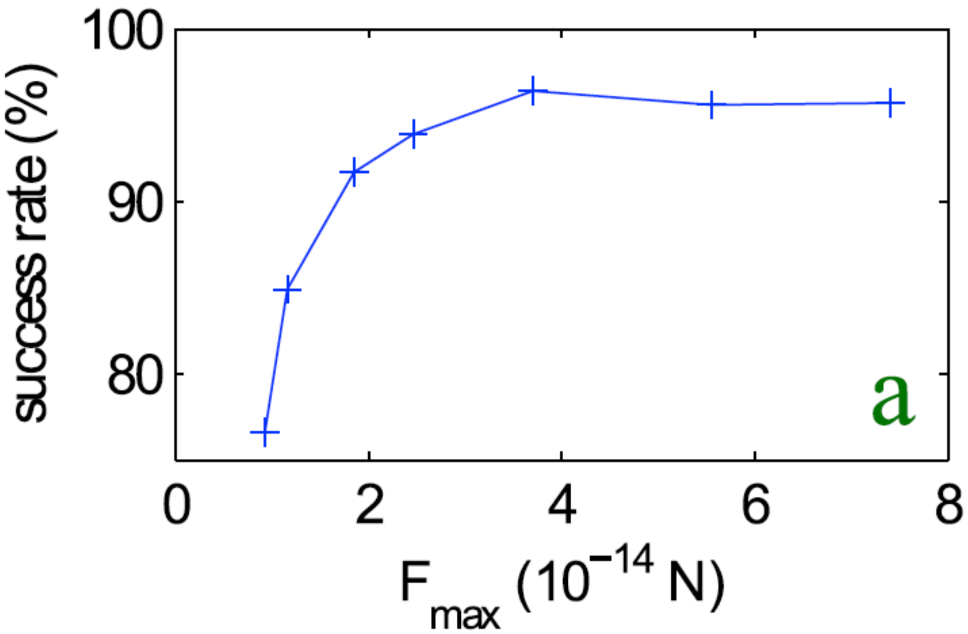
Sekimoto -Sasa J. Phys. Soc. Jpn. 66, 3326 (1997).

$$< W > \simeq \Delta F + B/\tau$$

As $< \Delta U >= 0$ then $\Delta F = -T\Delta S$ and $< Q >=< W > \simeq kT \ln 2 + B/\tau$

$$\boxed{< Q >=< Q >_{\text{Landauer}} + B/\tau}$$

The success rate r

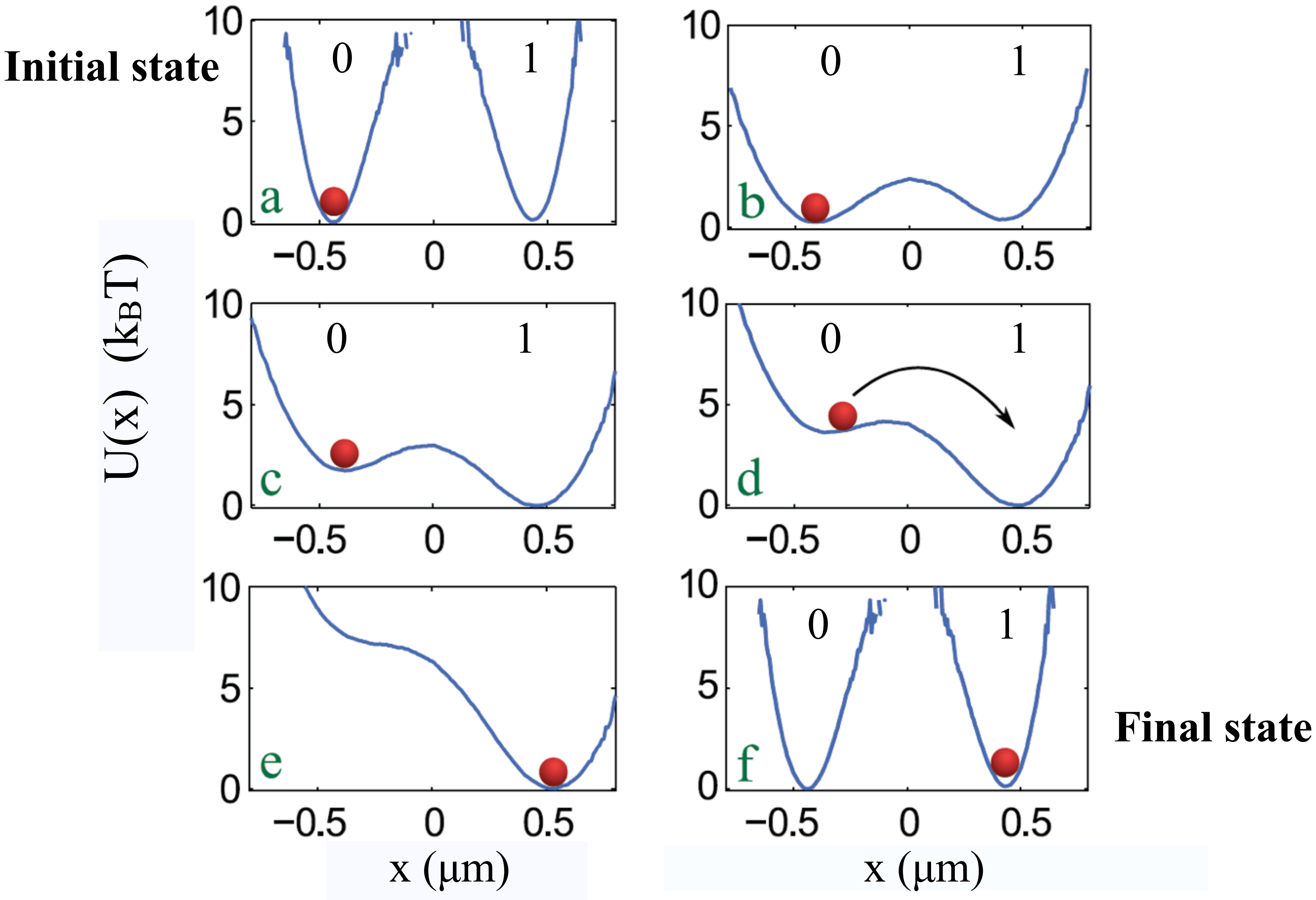


Why in the experiment r< 1 ?

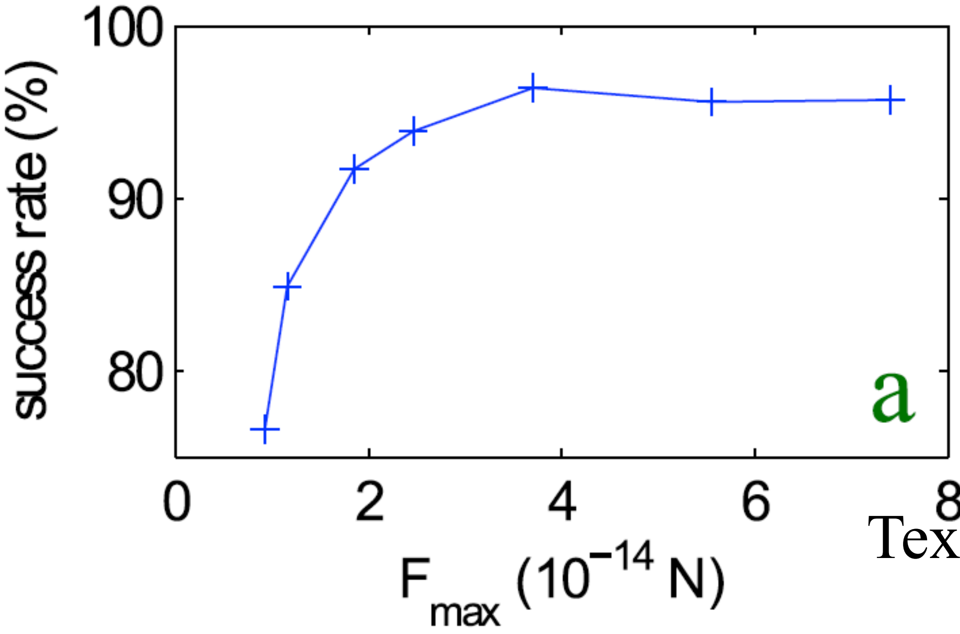
Is this result produced by 3D effects of the trap ?

Is the finite height of the initial barrier responsible of r<1 ?

The Erasure Procedure



The success rate r



Texte

Why in the experiment r< 1 ?

Is this result produced by 3D effects of the trap ?

Is the finite height of the initial barrier responsible of r<1 ?

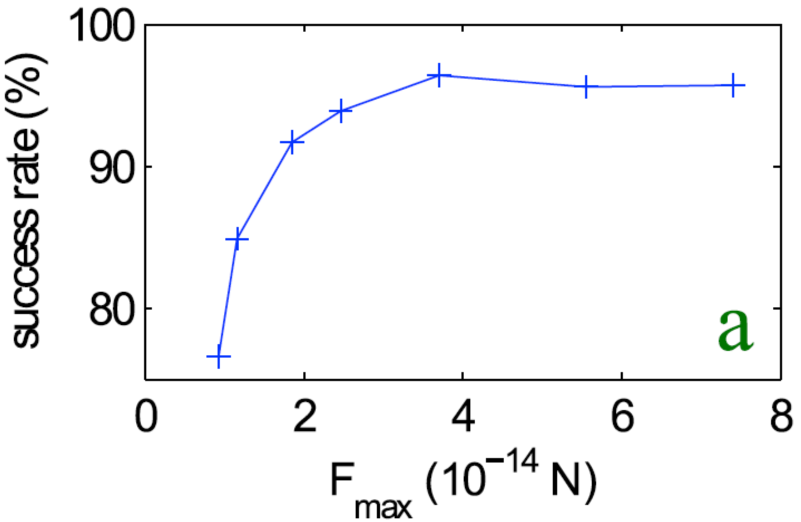
Numerical test

$$\nu \dot{x} = -\frac{\partial U_o(x,t)}{\partial x} + \eta$$

We use all the experimental parameters and procedure

with two different initial barriers $8k_BT$ and $15k_BT$

The success rate r



Why in the experiment r< 1 ?

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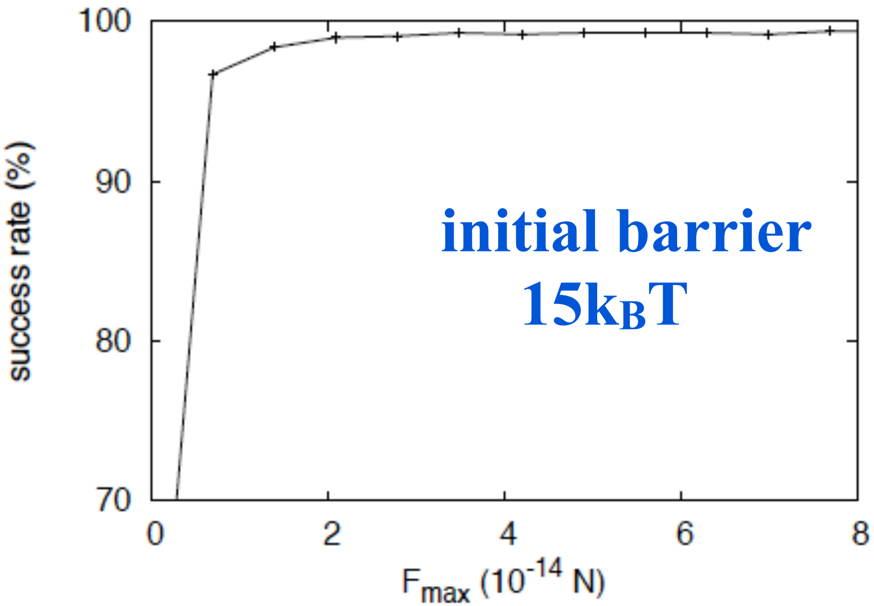
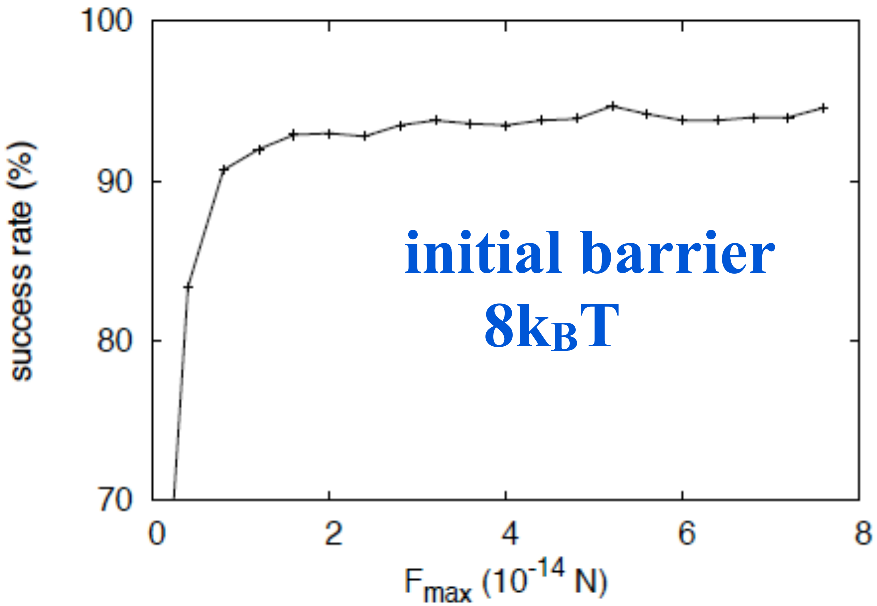
$$\nu \dot{x} = -\frac{\partial U_o(x,t)}{\partial x} + \eta$$

Numerical test





Conclusions (partials)

- Our experimental results indicate that the thermodynamic limit to information erasure, the Landauer bound, can be approached in the quasistatic regime, but not exceeded.

- The asymptotic limit is reached in $1/\tau$ for $\tau > 3\,\tau_k$

- The fact that $r<1$ is due to the finite height of the initial barrier

- Thermal fluctuations play an important role to reach the limit

Question: Does Jarzynski equality compute the right ΔF?

Landauer's limit and the Jarzynski equality

$$< \exp(-W_s) >= \exp(-\Delta F)$$

with $$W_s = -\int_o^{\tau_{cycle}} \dot{\lambda} \frac{\partial H(x,\lambda)}{\partial \lambda} dt$$

In our case this equality transforms

$$W_s = \int_o^{\tau} \dot{f}\ x\ dt = [f\ x]_o^{\tau} - \int_o^{\tau} f\ \dot{x}\ dt = -W_f$$

Since the height of the barrier is always finite there is no change in the ***equilibrium* F** of the system between **the beginning and the end of the procedure**.

$$< \exp(-W_s) >= \frac{\rho_{eq}(\tau)}{\rho(\tau)} \exp(-\Delta F)$$

S. Vaikuntanathan and C. Jarzynski, Euro. Phys. Lett. 87, 60005 (2009).

Generalized Jarzynski

Landauer's limit and the Jarzynski equality

We consider the erasure procedure

$$\begin{matrix} 0 \searrow & \\ & 0 \\ 1 \nearrow & \end{matrix}$$

If the final state is 0 then $\rho = r \simeq 1, \quad \rho_{eq} = 1/2, \quad \Delta F = 0$

and the Generalized Jarzynski is : $$< \exp(-W_s) >_{\to 0} = \frac{1/2}{r}$$

from Jensen inequality $$< W_s >_{\to 0} \geq (\ln 2 \ + \ \ln r)$$

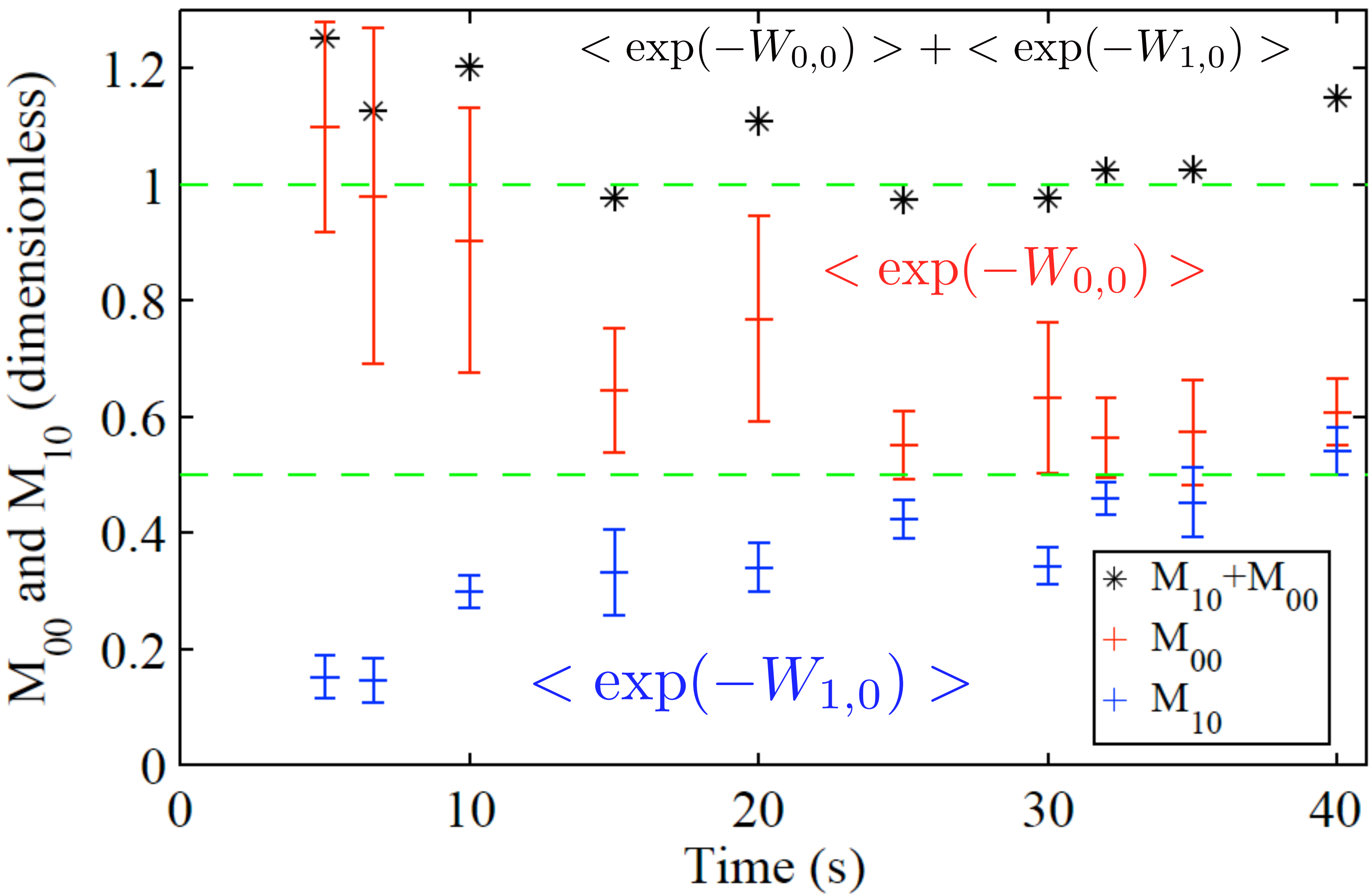
$$\frac{1}{2} < \exp(-W_{1,0}) > + \frac{1}{2} < \exp(-W_{0,0}) > = \frac{1}{2}$$

Work done if the particle makes the jump from 1 to 0

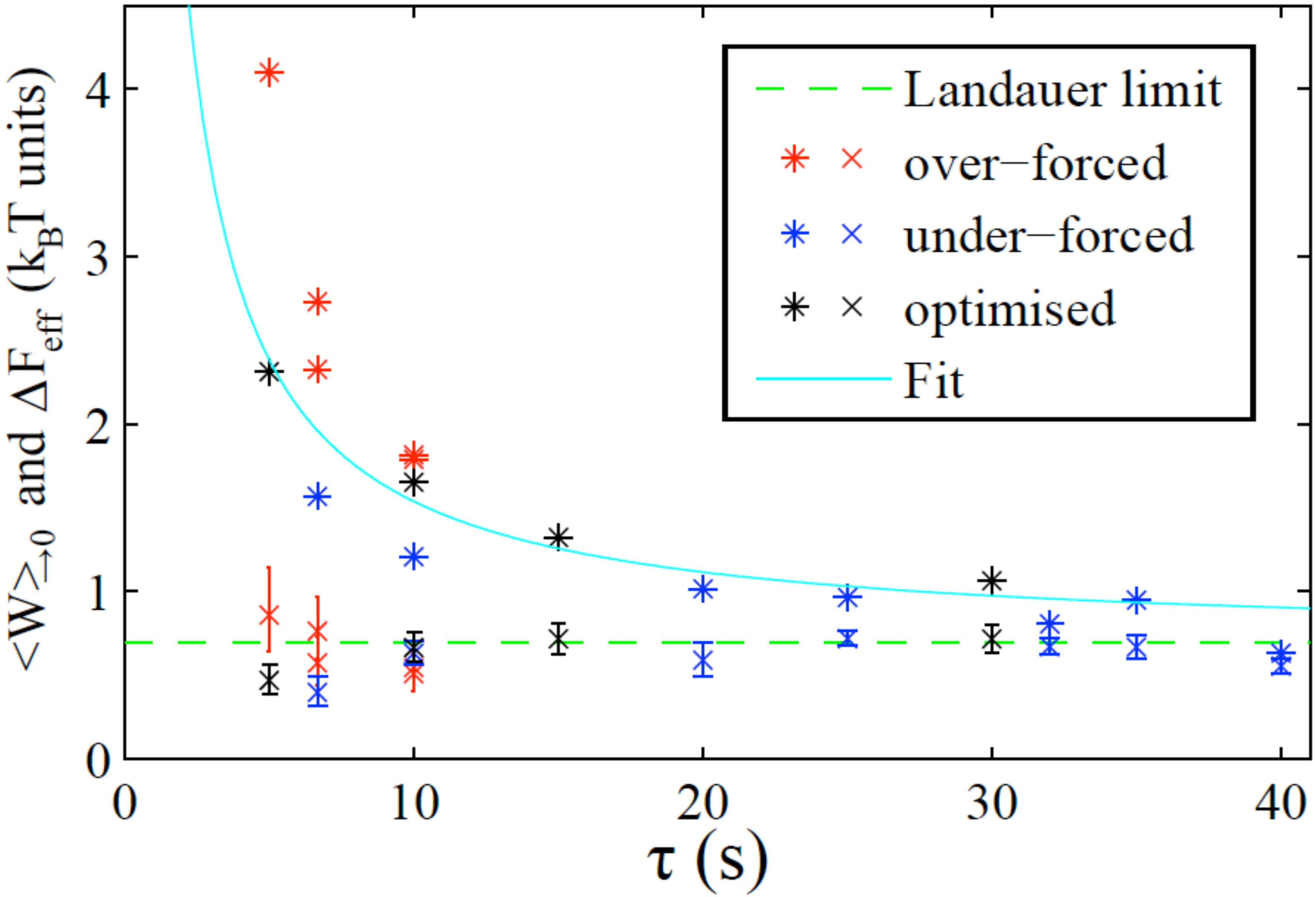
Work done when the particle starts in the final state

Landauer's limit and the Jarzynski equality

$$-\ln\left(\frac{<\exp(-W_{0,0})> + <\exp(-W_{1,0})>}{2}\right) = \Delta F_{eff}$$



Landauer's limit and the Jarzynski equality



Landauer's limit and the Jarzynski equality

We consider the erasure procedure $\begin{matrix} 0 \searrow \\ \quad 0 \\ 1 \nearrow \end{matrix}$

If the final state is 0 **then** $\rho = r \simeq 1, \quad \rho_{eq} = 1/2, \quad \Delta F = 0$

$$< \exp(-W_s) >_{\to 0} = \frac{1/2}{r} \qquad \text{and} \qquad < W_s >_{\to 0} \geq (\ln 2 \ + \ \ln r)$$

If the final state is 1 **then** $\rho = (1-r) \simeq 0, \quad \rho_{eq} = 1/2, \quad \Delta F = 0$

$$< \exp(-W_s) >_{\to 1} = \frac{1/2}{1-r} \qquad \text{and} \quad < W_s >_{\to 1} \geq \ln 2 \ + \ \ln(1-r)$$

Total work $\quad < W_s > = r \ < W_s >_{\to 0} + (1-r) \ < W_s >_{\to 1}$

using the inequalities $\quad \boxed{< W_s > \geq \ln 2 \ + r \ \ln r + (1-r) \ \ln(1-r)}$

The generalized Landauer's bound

Conclusions

- Our experimental results indicate that the thermodynamic limit to information erasure, the Landauer bound, can be approached in the quasistatic regime, but not exceeded.

- The asymptotic limit is reached in $1/\tau$

- The fact that $r<1$ is due to the finite height of the initial barrier

- Thermal fluctuations play an important role to reach the limit

- Jarzinsky equality computes the Landauer limit independently of the rapidity of the procedure

Question : Does any procedure allows us to reach the Landauer's limit ?

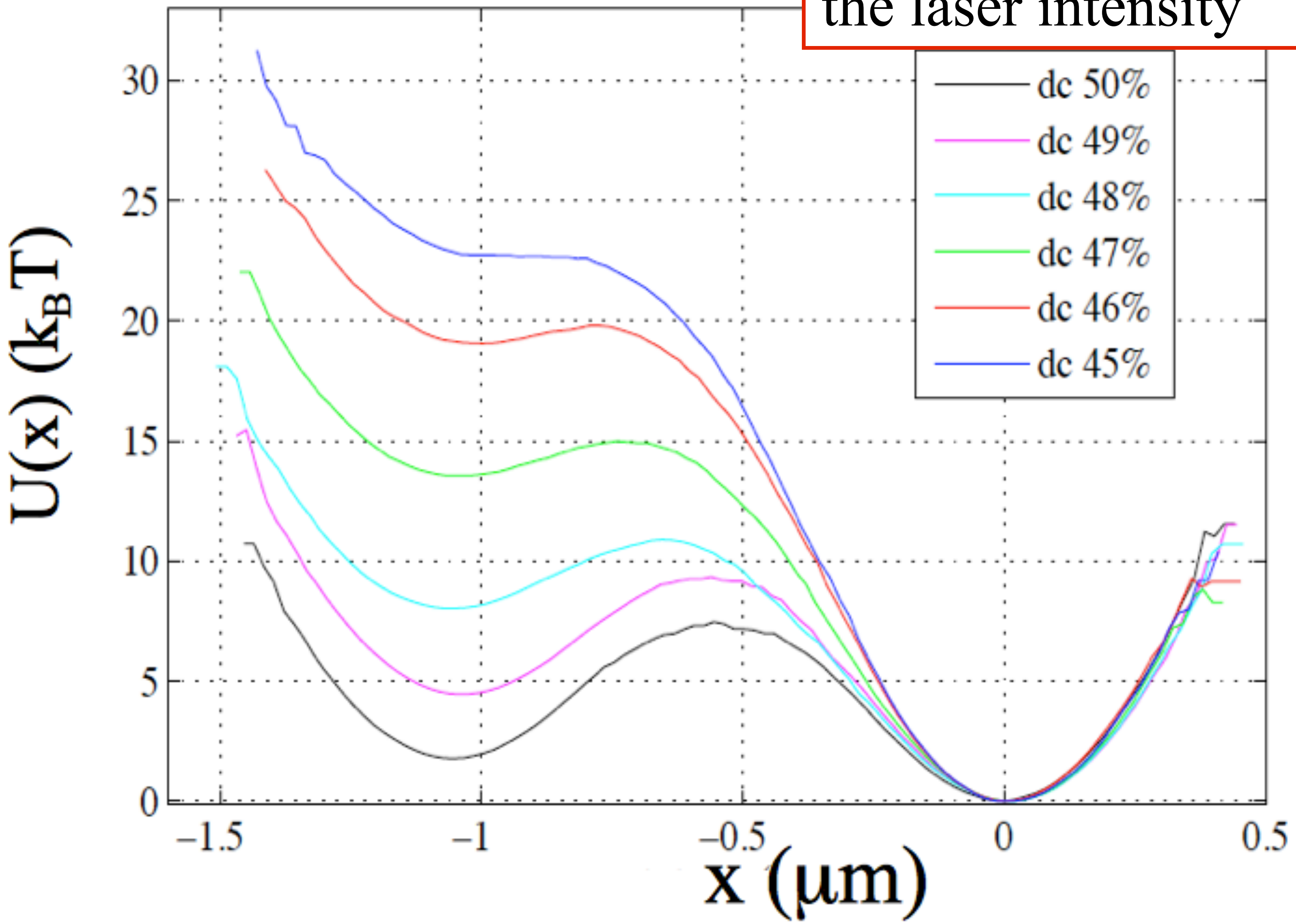
Answer : NO. The barrier reduction and tilt must be two separate process

See recent paper on optimisation :

E. Aurell, et al. J. Stat. Phys.147, 487-505 (2012).

Other procedure (I)

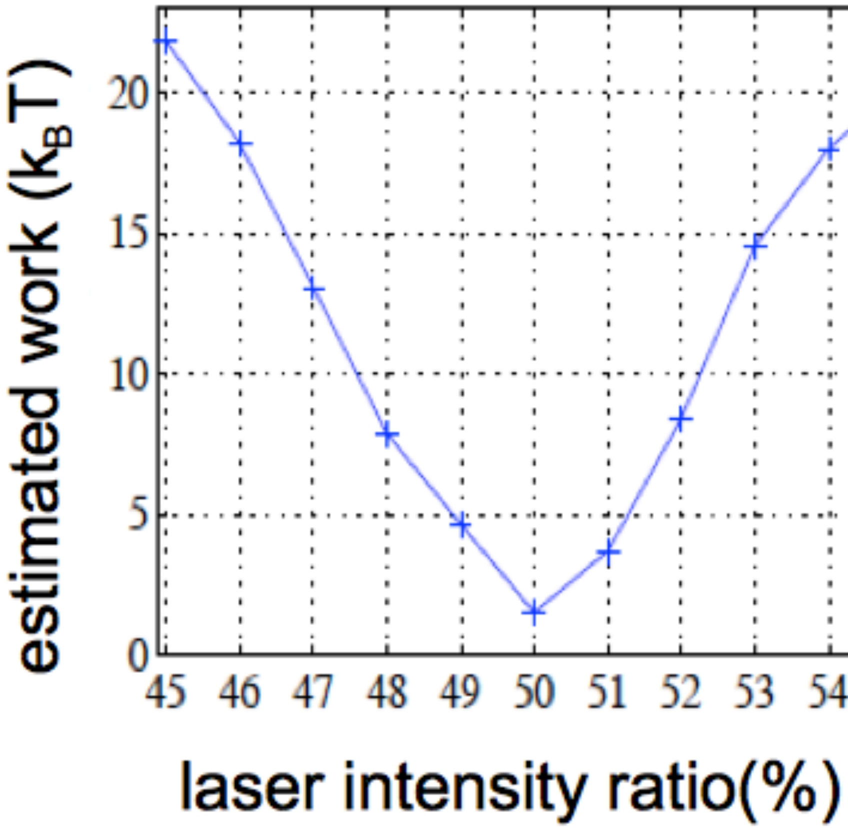
Relative change of the laser intensity



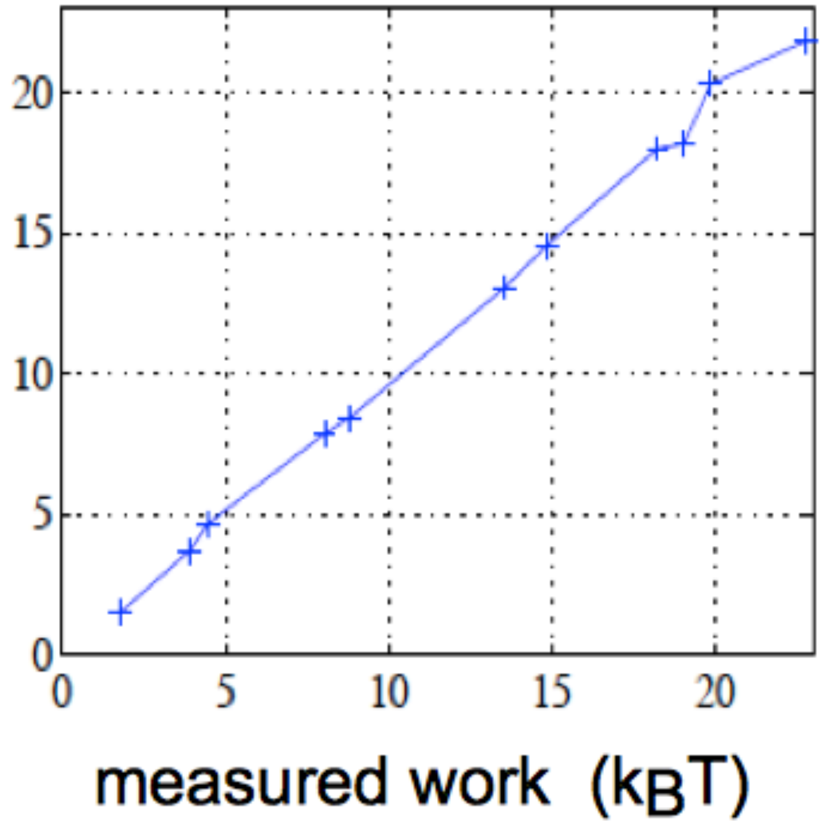
The ramping time of the laser intensity has been changed from 1s to 50s

Other procedure (II)

Fixed intensity ratio



Ramp of intensity ratio



The work is mainly due to the jump of the particle

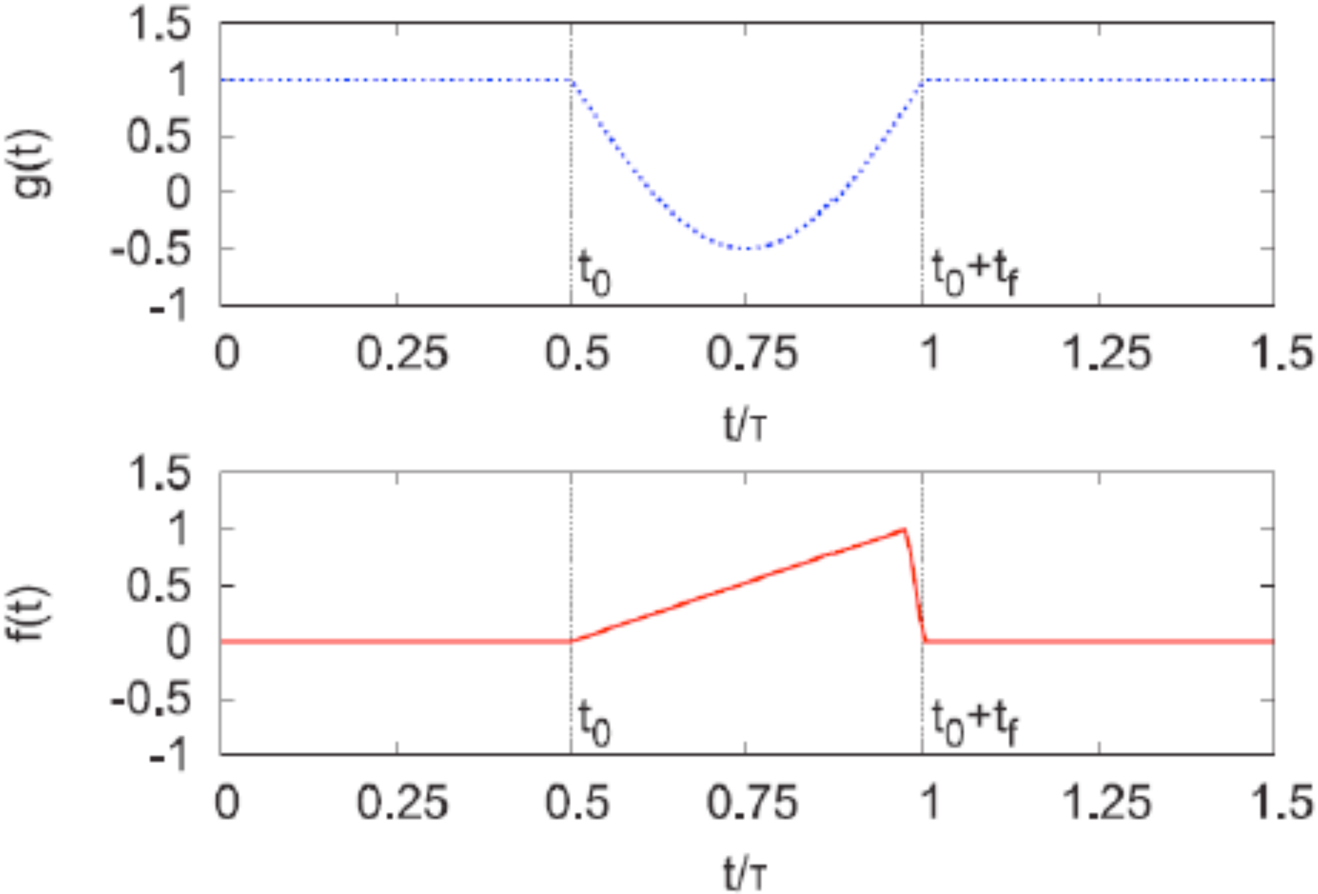
The Landauer limit can never be reached

Numerical results

Memory Erasure in Small Systems,
R. Dillenschneiderand E. Lutz, Phys. Rev. Lett. 102, 210601 (2009)

Potential : $V(x,t) = -\frac{1}{2}g(t)x^2 + \frac{1}{4}x^4$

External force : $Af(t)$



Non-dimensional numbers and the success rate

$$\bar{\tau} = \frac{\tau}{\tau_k}$$

Possibility of jumping the barrier without force

$$\bar{F} = \frac{\delta x \ F_{max}}{\Delta U}$$

The maximum external work overcomes the barrier

$\tau_K = \tau_o \exp[\frac{\Delta U}{k_B T}]$ is the Kramers time with $\tau_o \simeq 1s$

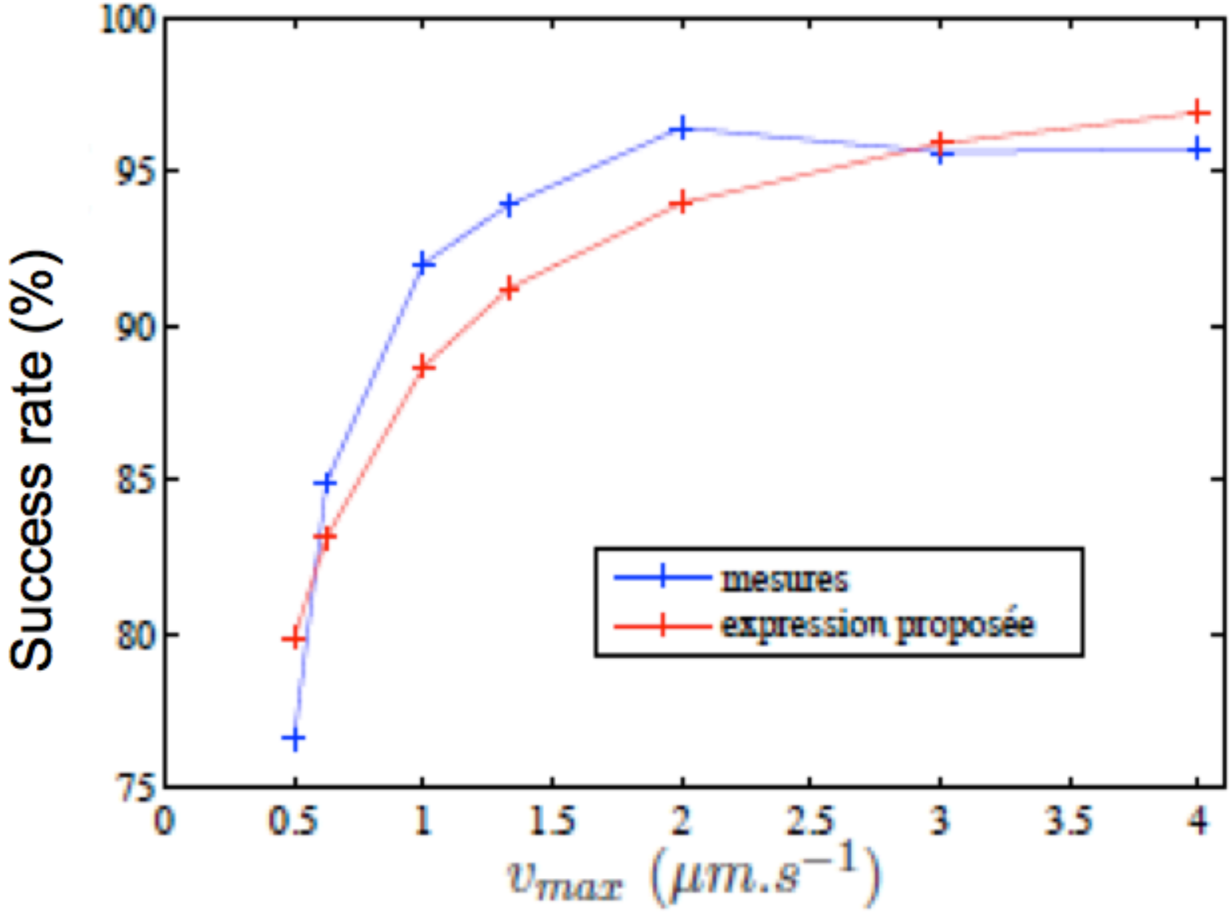
δx is the distance of the potential minima

One can think that the success rate is :

$$r = \frac{1}{2}[1 + \exp(-\frac{1}{\bar{\tau}\bar{F}})]$$

Non-dimensional numbers and the success rate

Experimentally $$r = \frac{1}{2}[1 + \exp(-\frac{a}{\bar{\tau}\bar{F}^2})]$$



Conclusions

- Our experimental results indicate that the thermodynamic limit to information erasure, the Landauer bound, can be approached in the quasistatic regime, but not exceeded.

- The asymptotic limit is reached in $1/\tau$ for $\tau > 3\,\tau_k$

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Answer : NO. The barrier reduction and tilt must be two separate process