Casimir energy & Casimir entropy

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The ideal Casimir force

A universal effect from confinement of vacuum energy, which depends only on $\hbar$, $c$, and geometry

$$F_{\text{Cas}} = -\frac{dE_{\text{Cas}}}{dL}, \quad E_{\text{Cas}} = -\frac{\hbar c \pi^2 A}{720 L^3}$$

- Written here in an idealized case
  - Perfectly parallel plane mirrors
  - Perfectly reflecting mirrors
  - Zero temperature

- Attractive force (negative pressure)
  $$F_{\text{Cas}} = P_{\text{Cas}} A, \quad P_{\text{Cas}} = -\frac{\hbar c \pi^2}{240 L^4}$$
  $$|P_{\text{Cas}}| \sim 1\,\text{mPa}$$
  at $L = 1\,\mu\text{m}$

The real Casimir force

- Experiments performed with Gold-covered plates
  - Force depends on non universal reflection properties of the metallic plates used in the experiments
- Experiments performed at room temperature
  - Effect of thermal and vacuum field fluctuations have to be taken into account
- Effect of geometry
  - Most precise experiments performed in the plane-sphere geometry
- Non ideality of surfaces
  - Roughness, electrostatic patches, contamination …

“Casimir Physics”, Lecture Notes in Physics 834 (Springer-Verlag, 2011)
Radiation pressure of quantum fluctuations

- Many ways to calculate the Casimir effect
- « Quantum Optics » approach
  - Quantum field fluctuations (vacuum and thermal fluctuations) pervade empty space → radiation pressure on mirrors
  - Force = pressure balance between inner and outer sides of the mirrors
- « Scattering theory »
  - Mirrors = scattering amplitudes depending on frequency, incidence, polarization
  - Solves the high-frequency problem
  - Gives results for real mirrors
  - Can be extended to other geometries
The Casimir force as a radiation pressure

- The Casimir force is the difference between inner and outer radiation pressures summed over all field modes

\[ F = \int_0^\infty \frac{d\omega}{2\pi c} 2\hbar \omega N(\omega) \left( g(\omega) - 1 \right) \]

Field fluctuation energy in the counter-propagating modes at frequency \( \omega \)

\[ 2\hbar \omega N = 2\hbar \omega \left( \frac{1}{2} + \bar{n}_\omega \right) = \frac{\hbar \omega}{\tanh \frac{\hbar \omega}{2k_B T}} \]

- The Casimir force can also be written in terms of causal amplitudes

\[ F = \mathcal{I}_r + (\mathcal{I}_r)^* \, , \, \mathcal{I}_r = \int_0^\infty \frac{d\omega}{2\pi c} 2\hbar \omega N(\omega) f(\omega) \, , \, f = \frac{re^{2ikL}}{1 - re^{2ikL}} \]
Casimir free energy and phase-shifts

- Casimir force obtained from the free energy through a differentiation wrt $L$

$$F = -\frac{\partial \mathcal{F}(L,T)}{\partial L}$$

$$\mathcal{F} = i\hbar \int_0^\infty \frac{d\omega}{2\pi} \left( \frac{1}{2} + \bar{n}_\omega \right) \left( \ln \left( 1 - re^{2ikL} \right) - \ln \left( 1 - r^*e^{-2ikL} \right) \right)$$

- with

$$\ln \left( \frac{1 - re^{2ikL}}{1 - re^{2ikL}} \right)^* = \ln \left( \det S_{12} \right) - \ln \left( \det S_1 \right) - \ln \left( \det S_2 \right)$$

- Casimir free energy can be written as a difference between changes of free energies calculated for different configurations

$$\mathcal{F} = \Delta \mathcal{F}_{12} - \Delta \mathcal{F}_1 - \Delta \mathcal{F}_2$$

The phase-shift interpretation

- Each of these free energies is given by the phase-shifts for the S-matrix associated with the scattering configuration

\[ \Delta \mathcal{F}_{12} = \frac{\hbar}{i} \int_0^\infty \frac{d\omega}{2\pi} \left( \frac{1}{2} + \tilde{n}_\omega \right) \ln \left( \det S_{12} \right) \]

Similar expression for configurations with mirrors 1 and 2 alone

- In fact, each such quantity is itself a difference of free energies calculated in the presence and in the absence of the scatterer

\[ \Delta \mathcal{F}_{12} \equiv \mathcal{F}_{12} - \mathcal{F}_{\text{vac}} \]

- In the end, the Casimir free energy is a “double difference” involving four different configurations

\[ \mathcal{F} = (\mathcal{F}_{12} - \mathcal{F}_{\text{vac}}) - (\mathcal{F}_1 - \mathcal{F}_{\text{vac}}) - (\mathcal{F}_2 - \mathcal{F}_{\text{vac}}) \]

\[ \mathcal{F} = \mathcal{F}_{12} - \mathcal{F}_1 - \mathcal{F}_2 + \mathcal{F}_{\text{vac}} \]
Casimir effect between two planes

- Derivation similar to that in the 1-d case

\[ \mathcal{F} = \left\{ i\hbar \int_0^\infty \frac{d\omega}{2\pi} \left( \frac{1}{2} + \bar{n}_\omega \right) \text{Tr} \ln d \right\} + \{ \}^* \]

\[ \text{Tr} \ln d \equiv \sum_p \left( \int \frac{A \, d^2k}{(2\pi)^2} \ln \, d_k^p \right) \]

\[ r \equiv r_1 r_2 \]

\[ d_k^p[\omega] = 1 - r_k^p[\omega] e^{2ik_zL} \]

- Casimir pressure obtained as

\[ P = -\frac{\partial \mathcal{F}(L, T)}{A \partial L} \]
Casimir effect and thermodynamics

- From the free energy, one derives the force
  \[ F = -\frac{\partial F(L, T)}{\partial L} \]

... as well as an entropy
  \[ S = -\frac{\partial F(L, T)}{\partial T} \]

... and an “internal energy”
  \[ \mathcal{E} = F + TS = F - T\frac{\partial F(L, T)}{\partial T} \]

- Usual thermodynamical relations are valid
  \[ dF = -PdV - SdT \]

\[ d\mathcal{E} = -PdV + TdS \]

\[ dV \equiv AdL \]

\[ F \equiv AP \]
Model for reflection amplitudes

- Lifshitz model (1956)
  - bulk mirror (very thick slab)
  - local dielectric response function $\varepsilon[\omega]$

- Reflection amplitudes on each mirror given by Fresnel laws

$$r_{1}^{TE}[\omega] = \frac{k_{z} - K_{z}}{k_{z} + K_{z}}, \quad r_{1}^{TM}[\omega] = \frac{K_{z} - \varepsilon k_{z}}{K_{z} + \varepsilon k_{z}}$$

$$K_{z} = \sqrt{\varepsilon \frac{\omega^2}{c^2} - k_{z}^2}, \quad k_{z} = \sqrt{\frac{\omega^2}{c^2} - k_{z}^2}$$


Models for metallic bulk plates

- Simple models for the (reduced) dielectric function for metals
  - bound electrons
    (inter-band transitions, tables of optical data)
  - conduction electrons
    - determined by (reduced) conductivity $\sigma$
  - Drude model for conductivity
    - plasma frequency $\omega_P$
    - relaxation parameter $\gamma$

- Drude parameters related to the density of conduction electrons and to the static conductivity
  - finite conductivity $\sigma_0 \Leftrightarrow$ non null $\gamma$

$$\varepsilon[\omega] = \bar{\varepsilon}[\omega] + \frac{\sigma[\omega]}{-i\omega}$$

$$\sigma[\omega] = \frac{\omega_P^2}{\gamma - i\omega}$$

$$\omega_P^2 = \frac{nq^2}{\varepsilon_0 m^*}$$

$$\sigma_0 = \frac{\omega_P^2}{\gamma}$$

Pressure between metallic mirrors (room T)

- Pressure different from the ideal Casimir formula
- Imperfect reflection
- Non zero temperature

Small losses lead to a large factor 2 at large distances

Negative contribution of thermal photons at intermediate distances for the Drude model

\[ \eta_P = \frac{P}{P_{\text{Cas}}} \]

\( \bar{\varepsilon} = 1 \) for this plot

Drude parameters of Gold

\[ \gamma = 0 \text{, } T = 300\text{K} \]

\[ \gamma = 0.004 \times \omega_P \text{, } T = 300\text{K} \]

\( L [\mu\text{m}] \)

\( \lambda_P = 136\text{nm} \)


Interaction entropy for metallic mirrors

- Interaction entropy found to be negative for intermediate products temperature * length

\[ \frac{(L^2/A)}{S/k_B} \]

\[ \gamma L/c = 0.01, 0.1, 1, 10, 100, 1000 \]

from bottom to top

Drude model
\[ \varepsilon = 1 \] for this plot
\[ \lambda_P = 136 \text{nm} \]
\[ L = 1 \mu\text{m} \]

Purdue measurements agree with predictions from the plasma model but deviate from predictions with dissipation accounted for!
\[ \delta P = P_{\text{exp}} - P_{\text{th}} \]

Difference of experimental and theoretical pressures

Theory: optical data for Gold, extrapolated to the Drude model when \( \omega \to 0 \)

\[ \left| \frac{\delta P}{P} \right| \sim 4\% \text{ at } L \sim 200\text{nm} \]

Experimental data kindly provided by R. Decca (IUPUI)
Theoretical pressure calculated by R. Behunin et al PRA 85 (2012) 012504
General scattering formula with big matrices mixing wavevectors and polarizations

\[ D = 1 - R_P e^{-iK_L} R_S e^{-iK_L} \]

- Reflection matrices on the plane written as Fresnel amplitudes in the plane waves basis \( \rightarrow R_P \)
- Reflection matrices on the sphere written as Mie amplitudes in the spherical waves basis \( \rightarrow R_S \)
- Transformation from plane to spherical waves (for electromagnetic fields)

We obtain an “exact” multipolar expansion of the energy

- Spherical waves labeling: \((\ell, m)\), \(|m| \leq \ell\)
- Sums truncated for the numerics \(\ell \leq \ell_{\text{max}}\)
- Results accurate for \(x \equiv \frac{L}{R} > x_{\text{min}}, \quad x_{\text{min}} \propto \frac{1}{\ell_{\text{max}}}\)

Correlation geometry - temperature

- Force between plane and spherical perfect reflectors at room or zero temperature
- Drawn as the ratio of force at $T \neq 0$ to force at $T = 0$
- Contribution of thermal photons repulsive at intermediate distances!

$$F(T) < F(0)$$
Casimir entropy in the plane-sphere case

- Casimir entropy at room temperature computed between perfectly reflecting sphere and plane, as a function of separation distance
- Drawn after division by the volume of the sphere
- Casimir entropy negative at some distances, for perfect mirrors here
- Features not seen for perfect plane mirrors
- Analytical expressions available for small spheres (dipolar approximation)

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Negative Casimir-Polder entropies in nanoparticle interactions

- Systematic study for the interaction with planes or between them of atoms or nanoparticles, in the limit $R \to 0$ (dipolar approximation)

- Negative interaction entropies obtained in many different configurations
  - Example of the interaction between two identical nanoparticles

- No contradiction with the principles of thermodynamics
- Phenomenon not exceptional
- In fact it is nearly ubiquitous